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ABSTRACT. Let R be a discrete valuation ring with maximal ideal generated by a prime element π and let R_π denote the completion of R with respect to the π-adic topology on R. Assume that the residue field $R/(\pi)$ is a finite field. Let $f \in R_\pi[x_1, \ldots, x_n]$. For each $m \geq 1$, let c_m denote the number of solutions to the congruence $f(x_1, \ldots, x_n) \equiv 0 \bmod \pi^m$. The Poincaré series of f is the formal power series

$$P_f(y) = 1 + \sum_{m=1}^{\infty} c_m y^m.$$

In this paper we compute $P_f(y)$ for an arbitrary diagonal polynomial f given by

$$f(x_1, \ldots, x_n) = \epsilon_1 x_1^{t_1} + \cdots + \epsilon_n x_n^{t_n} + b$$

where $\epsilon_1, \cdots, \epsilon_n \in R_\pi$, $t_1, \ldots, t_n$ are positive integers and $b \in R_\pi$. We thus extend results of Goldman, Wang and Han and also give a rather explicit description of the rational function $P_f(y)$ that specializes to the results of Wang and Han.

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