

# POWERS IN COMPLETE DISCRETE VALUATION RINGS

DIBYAJYOTI DEB AND DAVID B. LEEP

ABSTRACT. Let  $R$  be a discrete valuation ring with maximal ideal generated by a prime element  $\pi$  and let  $R_\pi$  denote the completion of  $R$  with respect to the  $\pi$ -adic topology on  $R$ . Let  $U$  denote the group of units of  $R_\pi$ . Let  $m$  be a positive integer. For each positive integer  $i$ , there is a surjective group homomorphism  $f_i : U \rightarrow R/(\pi^i)^\times / (R/(\pi^i)^\times)^m$ . It is clear that  $U^m \in \ker(f_i)$  for all  $i \geq 1$ . In this paper we determine the least value of  $i$ , if one exists, such that  $\ker(f_j) = U^m$  for all  $j \geq i$ . In other words, when can one say that if  $\alpha \in R_\pi$  is an  $m^{\text{th}}$  power modulo  $\pi^i$ , then  $\alpha$  is an  $m^{\text{th}}$  power in  $R_\pi$ ?

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF KENTUCKY, LEXINGTON KY 40506-0027

*E-mail address:* `ddeb@ms.uky.edu`

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF KENTUCKY, LEXINGTON KY 40506-0027

*E-mail address:* `leep@ms.uky.edu`