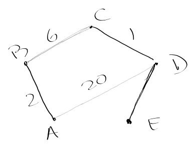
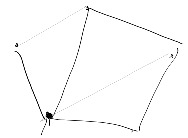




Traveling Salesman Problem

Dijkstra's Algorithm

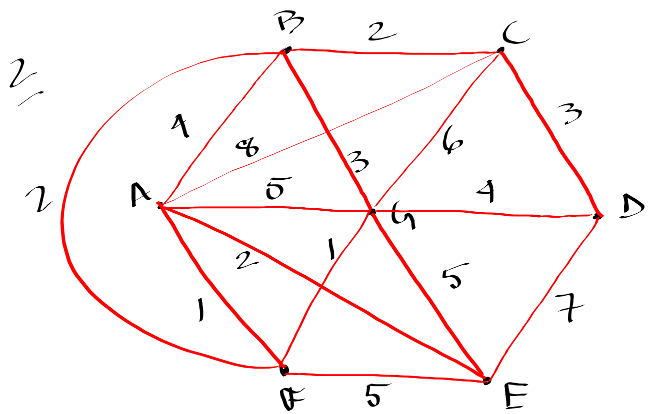
The Dijkstra's Algorithm is used to find the shortest path between two vertices in a weighted graph (Edges have weights).

Step 1 : Set $v_i = A$ and assign to this vertex the permanent label 0. Assign every other vertex a temporary label of ∞.

Step 2 : Until E has been assigned a permanent label or no temporary labels are changed in (a) or (b) below, do the following.

(a) Take the vertex v_i that most recently acquired a permanent label say d. For each vertex v that is adjacent to v_i and has not received a permanent label, if $d + w(v_i, v) < t$ (temporary label of v) change the temporary label of v to $d + w(v_i, v)$

(b) Take a vertex v that has a temporary label smallest among all temporary labels in the graph. Set $v_{i+1} = v$ and make its temporary label permanent. If there are several vertices v that tie for the smallest temporary label, make any choice.



V	A	B	C	D	E	F	G
A	$\boxed{0_A}$	4_A	8_A	∞	2_A	1_A	5_A
F	X	3_F	8_A	∞	2_A	$\boxed{1_A}$	2_F
E	X	3_F	8_A	9_E	$\boxed{2_A}$	X	2_F
G	X	3_F	8_A	6_G	X	X	$\boxed{2_F}$
B	X	$\boxed{3_F}$	5_B	6_G	X	X	X
C	X	X	$\boxed{5_B}$	6_G	X	X	X
D	X	X	X	$\boxed{6_G}$	X	X	X
	X	X	X	X	X	X	X

$A \to C$

AFBC

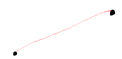
$A \to D$

AFGD

23(b) G connected graph
$n > 1$ vertices
Prove that G has at least $n-1$ edges.

We will use induction.

Base Case: $n = 2$ vertices

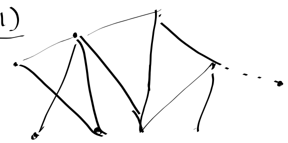


# of edges $= 1 = 2-1$

$\therefore P(2)$ is true.

Inductive Step: Suppose $P(k)$ is true for some integer $k > 1$, i.e any connected graph of k vertices has at least $k-1$ edges.

$P(k+1)$



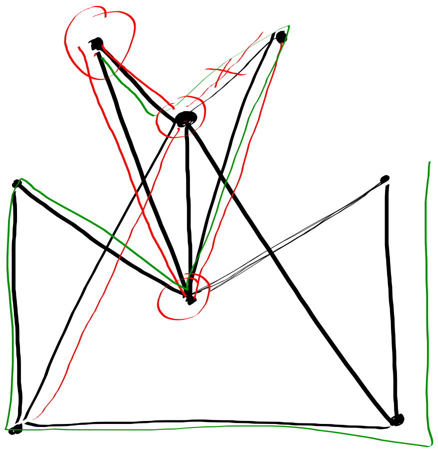
$k+1$ vertices.

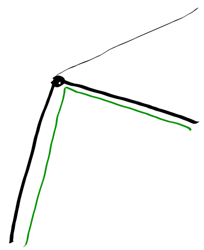
Any connected graph of $k+1$ vertices has at least k edges.

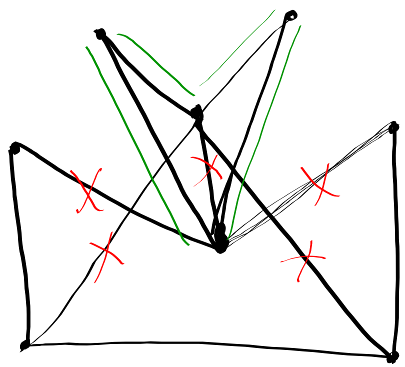
Remove a vertex v from this graph, and the resulting subgraph G_1 has k vertices. This means G_1 has at least $k-1$ edges by the inductive hypothesis. Since G is connected hence, there is a walk between v and any other vertex. So, there is an edge between v and an adjacent vertex.

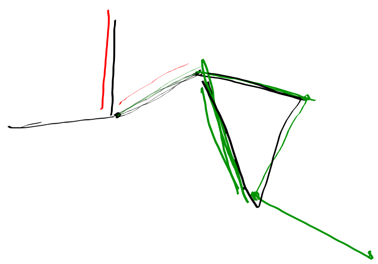
$\therefore$ G has atleast $(k-1)+1$ edges $= k$ edges.

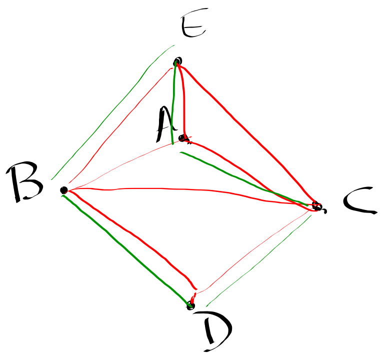
Hence, by induction, the result is true for all $n > 1$.











Is it Hamiltonian?

ACDBEA

$a_n = 5a_{n-1}$ $\qquad a_1 = 2$

$a_2 = 5a_1 = 5\cdot 2 \qquad = 5\cdot 2$

$a_3 = 5a_2 = 5(5\cdot 2) = 5^2\cdot 2$

$a_4 = 5a_3 = 5(5\cdot 5\cdot 2) = 5^3\cdot 2$

$\vdots$

$a_n = 5^{n-1}\cdot 2$

P(1): LHS: $a_1 = 2$. ✓

RHS: $5^0\cdot 2 = 2$ ✓

P(k) is true: i.e $a_k = 5^{k-1}\cdot 2$

$$\begin{aligned} \underline{a_{k+1}} &= 5\cdot a_k. \\ &= 5(5^{k-1})\cdot 2 \\ &= 5^k\cdot 2 \\ &= 5^{\underbrace{(k+1)}-1}\cdot 2 \end{aligned}$$

$a_n = r a_{n-1} + s a_{n-2} + f(n).$

14(b) $a_n = -6a_{n-1} - 9a_{n-2} + n^2 + 3n$

$a_0 = \frac{179}{128}$, $a_1 = \frac{-21}{128}$, $n \geq 2$

$a_n = -6a_{n-1} - 9a_{n-2}$

Characteristic Equation.

$r^2 + 6r + 9 = 0$

$(r+3)(r+3) = 0$

$r = -3, -3$

$a_n = c_1(-3)^n + c_2 n(-3)^n$

Suppose $p_n = an^2 + bn + c$ is a particular solution of the given relation.

$p_{n-1} = a(n-1)^2 + b(n-1) + c$

$p_{n-2} = a(n-2)^2 + b(n-2) + c$

$\therefore p_n = -6p_{n-1} - 9p_{n-2} + n^2 + 3n$

$\therefore an^2 + bn + c = -6\left(a(n-1)^2 + b(n-1) + c\right) - 9\left(a(n-2)^2 + b(n-2) + c\right) + n^2 + 3n$

$= -6\left(an^2 - 2an + a + bn - b + c\right) - 9\left(an^2 - 4an + 4a + bn - 2b + c\right) + n^2 + 3n$

$an^2 + bn + c = -6an^2 + 12an - 6a - 6bn + 6b - 6c - 9an^2 + 36an - 36a - 9bn + 18b - 9c + n^2 + 3n$

$= n^2(-15a+1) + n(48a - 15b + 3) + (-42a + 24b - 15c)$

Equating coefficients,

$a = -15a + 1$

$16a = 1 \Rightarrow a = 1/16$

$b = 48a - 15b + 3$

$b = 3 - 15b + 3$

$16b = 6$

$b = \frac{6}{16}$

$c = -42a + 24b - 15c$

$= -42\left(\frac{1}{16}\right) + 24\left(\frac{3}{8}\right) - 15c$

$16c = -\frac{21}{8} + 9$

$16c = \frac{51}{8}$

$c = \frac{51}{128}$

$$\therefore\ p_n = \frac{1}{16}n^2 + \frac{3}{8}n + \frac{51}{128}$$

$\therefore$ General solution of the recurrence relation

$$a_n = c_1(-3)^n + c_2 n(-3)^n + \frac{1}{16}n^2 + \frac{3}{8}n + \frac{51}{128}$$

$$a_0 = c_1 + \frac{51}{128} = \frac{179}{128}$$

$$\therefore\ c_1 = \frac{128}{128} = 1$$

$$a_1 = -3c_1 - 3c_2 + \frac{1}{16} + \frac{3}{8} + \frac{51}{128} = -\frac{21}{128}$$

$$\Rightarrow -3 - 3c_2 + \frac{8+48+51}{128} = -\frac{21}{128}$$

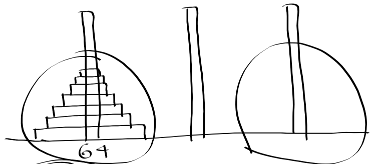
$$\Rightarrow -3 + \frac{107}{128} - 3c_2 = -\frac{21}{128}$$

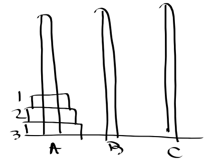
$$\Rightarrow -3 - 3c_2 = \frac{-21}{128} - \frac{107}{128} = \frac{-128}{128} = -1$$

$$\Rightarrow -3c_2 = 2$$

$$c_2 = -2/3$$

$$\therefore\ \boxed{a_n = (-3)^n - \frac{2}{3}n(-3)^n + \frac{1}{16}n^2 + \frac{3}{8}n + \frac{51}{128}}$$

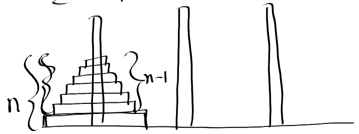




$2^n - 1$

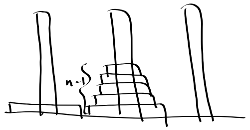
$2^{64} - 1$

1 → C
2 → B
1 → B

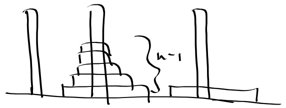


a_n = # of moves needed to move n disks from one peg to another.

First move the top $n-1$ disks to peg 2.
This takes a_{n-1} steps



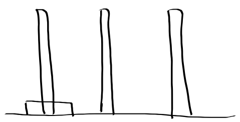
Move the largest disk from Peg 1 to Peg 3.
This takes 1 move



Now we move the $n-1$ disks from Peg 2 to Peg 3. This takes a_{n-1} moves.

∴ Total no. of moves $= a_n = a_{n-1} + 1 + a_{n-1}$

$\Rightarrow a_n = 2a_{n-1} + 1, \quad n \geq 1$



$a_2 = 2\cdot 1 + 1 = 3 = 2^2 - 1$, $\boxed{a_1 = 1} = 2^1 - 1$

$a_3 = 2\cdot 3 + 1 = 7 = 2^3 - 1$

$a_4 = 2\cdot 7 + 1 = 15 = 2^4 - 1$

⋮

$a_n = 2^n - 1$

$\underline{5}, 4, 3, 2, 1, 1$