

1. (1 point) Library/FortLewis/Calc3/16-2-Iterated-integrals/H  
 GM4-16-2-06-Iterated-integrals.pg

Evaluate the following integral.

$$\int_1^3 \int_0^4 (3x^2 + y^2) dx dy = \underline{\hspace{2cm}}$$

Correct Answers:

- 162.667

2. (1 point) Library/272/setStewart15\_2/problem\_12.pg

Evaluate the integral  $\int_0^{\pi/4} \int_5^6 (y \cos x + 1) dy dx$ .

Correct Answers:

- 4.67448545992346

3. (1 point) Library/UMN/calculusStewartET/s\_15\_2\_35.pg

Find the average value of  $f(x, y) = x^2 y$  over the rectangle  $R$  with vertices  $(-4, 0), (-4, 5), (2, 5), (2, 0)$ .

Answer:  $\underline{\hspace{2cm}}$

Correct Answers:

- $5/[6*(2--4)]*[2^3-(-4)^3]$

4. (1 point) Library/WHFreeman/Rogawski\_Calculus\_Early\_Transce  
 ndentals\_Second\_Edition/15\_Multiple\_Integration/15.1\_Integrati  
 on\_in\_Two\_Variables/15.1.29.pg

Evaluate the iterated integral:

$$\int_0^6 \int_3^5 \sqrt{x+4y} dx dy$$

Answer :  $\underline{\hspace{2cm}}$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

**Solution:** We compute the inner integral, treating  $y$  as a constant. Then we evaluate the resulting integral with respect to  $y$ :

$$\begin{aligned} & \int_0^6 \int_3^5 \sqrt{x+4y} dx dy \\ &= \int_0^6 \left. \frac{2}{3}(x+4y)^{3/2} \right|_{x=3}^5 dy \\ &= \int_0^6 \frac{2}{3} \left( (5+4y)^{3/2} - (3+4y)^{3/2} \right) dy \\ &= \frac{2}{3} \left( \frac{2}{5.4} (5+4y)^{5/2} - \frac{2}{5.4} (3+4y)^{5/2} \right) \Big|_0^6 \\ &= \frac{1}{15} \left( (29^{5/2} - 27^{5/2}) - (5^{5/2} - 3^{5/2}) \right) \\ &= \frac{1}{15} \left( 29^{5/2} - 27^{5/2} - 5^{5/2} + 3^{5/2} \right) \approx 46.7077 \end{aligned}$$

Correct Answers:

- 46.7077

5. (1 point) Library/272/setStewart15\_2/problem\_5.pg

Calculate the double integral  $\iint_{\mathbf{R}} x \cos(x+y) dA$  where  $\mathbf{R}$  is the region:  $0 \leq x \leq \frac{\pi}{6}, 0 \leq y \leq \frac{\pi}{4}$

Correct Answers:

- 0.0767515510438744

6. (1 point) Library/Michigan/Chap16Sec2/Q35.pg

Evaluate the integral

$$\int_0^1 \int_y^1 \sin(x^2) dx dy$$

by reversing the order of integration.

With order reversed,

$$\int_a^b \int_c^d \sin(x^2) dy dx,$$

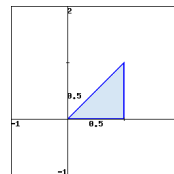
where  $a = \underline{\hspace{1cm}}, b = \underline{\hspace{1cm}}, c = \underline{\hspace{1cm}},$  and  $d = \underline{\hspace{1cm}}.$

Evaluating the integral,  $\int_0^1 \int_y^1 \sin(x^2) dx dy =$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

**SOLUTION**

The function  $\sin(x^2)$  has no elementary antiderivative, so we try integrating with respect to  $y$  first. The region of integration is shown in the figure below.



Changing the order of integration, we get

$$\int_0^1 \int_y^1 \sin(x^2) dx dy = \int_0^1 \int_0^x \sin(x^2) dy dx,$$

which we can easily evaluate:

$$\begin{aligned} \int_0^1 \int_0^x \sin(x^2) dy dx &= \int_0^1 \sin(x^2) \cdot y \Big|_0^x dx = \int_0^1 \sin(x^2) \cdot x dx \\ &= \frac{1}{2} (-\cos(x^2)) \Big|_0^1 = \frac{1}{2} (1 - \cos 1). \end{aligned}$$

Correct Answers:

- 0
- 1
- 0
- x
- $1/2*[1-\cos(1*1)]$

7. (1 point) Library/UMN/calculusStewartET/s\_15\_3\_4.pg

Evaluate the iterated integral  $\int_0^2 \int_y^{2y} xy \, dx \, dy$ .

Answer: \_\_\_\_\_

Correct Answers:

- $2^4 * (2^2 - 1) / 8$

8. (1 point) Library/ASU-topics/setCalculus/stef/stef15\_3p2.pg

Consider the integral  $\int_0^1 \int_{3x}^3 f(x,y) \, dy \, dx$ . Sketch the region of integration and change the order of integration.

$$\int_a^b \int_{g_1(y)}^{g_2(y)} f(x,y) \, dx \, dy$$

$a =$  \_\_\_\_\_  $b =$  \_\_\_\_\_

$g_1(y) =$  \_\_\_\_\_  $g_2(y) =$  \_\_\_\_\_

Correct Answers:

- 0
- 3
- 0
- $y/3$

9. (1 point) Library/FortLewis/Calc3/16-2-Iterated-integrals/H

GM4-16-2-28-Iterated-integrals.pg

Suppose  $R$  is the triangle with vertices  $(-1, 0)$ ,  $(0, 1)$ , and  $(1, 0)$ .

(a) As an iterated integral,  $\iint_R (3x + 3y)^2 \, dA = \int_A^B \int_C^D (3x + 3y)^2 \, dx \, dy$  with limits of integration

$A =$  \_\_\_\_\_

$B =$  \_\_\_\_\_

$C =$  \_\_\_\_\_

$D =$  \_\_\_\_\_

(b) Evaluate the integral in part (a). Hint: substitution may make the integral easier.

Integral = \_\_\_\_\_

Correct Answers:

- 0
- 1
- $y-1$
- $1-y$
- $(3^2 + 3^2) / 6$

10. (1 point) Library/Hope/Multi2/11-07-Scalar-surface-integr

als/Surface-integral-05.pg

Evaluate the integral with respect to surface area  $\iint_T 12x \, dA$ , where  $T$  is the part of the plane  $x + y + 4z = 9$  in the first octant.

$$\iint_T 12x \, dA =$$

Correct Answers:

- $2 * \text{sqrt}(18) * 9^{3/4}$

11. (1 point) Library/Michigan/Chap16Sec2/Q23.pg

For the integral

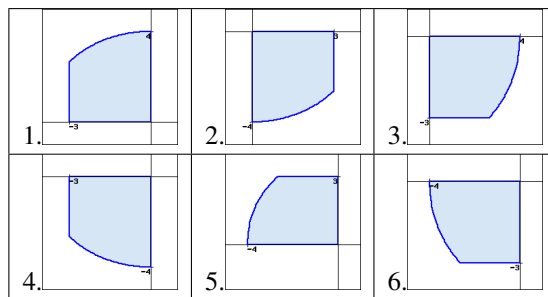
$$\int_{-3}^0 \int_{-\sqrt{16-x^2}}^0 3xy \, dy \, dx,$$

sketch the region of integration and evaluate the integral.

Your sketch should be approximately the same as one of the graphs shown below; which is the correct region? Graph [?/1/2/3/4/5/6]

Then  $\int_{-3}^0 \int_{-\sqrt{16-x^2}}^0 3xy \, dy \, dx =$  \_\_\_\_\_

Graphs:



**Solution:** ( Instructor solution preview: show the student solution after due date. )

**SOLUTION**

The region of integration is bounded by  $x = -3$  and  $x = 0$ , below by  $y = -\sqrt{16-x^2}$  and above by  $y = 0$ . The correct region is therefore 4. Evaluating the integral, we have

$$\int_{-3}^0 \int_{-\sqrt{16-x^2}}^0 3xy \, dy \, dx = \int_{-3}^0 \frac{3}{2} xy^2 \Big|_{-\sqrt{16-x^2}}^0 \, dx = \int_{-3}^0 -\frac{3}{2} x(16-x^2) \, dx$$

$$= -\frac{3}{2} \int_{-3}^0 16x - x^3 \, dx = -\frac{3}{2} \left( 16 \left( \frac{1}{2} \right) (x^2) - \left( \frac{1}{4} \right) (x^4) \right) \Big|_{-3}^0$$

$$= \frac{27}{4} 16 - \frac{243}{8} = \frac{621}{8}.$$

Correct Answers:

- 4
- $1/4 * 3^3 * 16 - 1/8 * 3^5$

12. (1 point) Library/UMN/calculusStewartET/s\_15\_3\_17.pg

Evaluate the double integral  $\iint_D x \cos y \, dA$ , where  $D$  is bounded by  $y = 0$ ,  $y = x^2$ , and  $x = 2$ .

Answer: \_\_\_\_\_

Correct Answers:

- $1/2 * [1 - \cos(4)]$

13. (1 point) Library/UMN/calculusStewartET/s\_15\_3\_18.pg

Evaluate the double integral  $\iint_D (x^2 + 2y) dA$ , where  $D$  is bounded by  $y = x$ ,  $y = x^3$ , and  $x \geq 0$ .

Answer: \_\_\_\_\_

Correct Answers:

- $4/21 * 1 + 1/12$

14. (1 point) Library/UMN/calculusStewartET/s\_15\_3\_22.pg

Evaluate the double integral  $\iint_D 8xy dA$ , where  $D$  is the triangular region with vertices  $(0,0)$ ,  $(1,2)$ , and  $(0,3)$ .

Answer: \_\_\_\_\_

Correct Answers:

- $7/8 * 8$

15. (1 point) Library/CSUN/Calculus/Cartesian\_to\_polar\_integra  
l\_1.pg

Convert the integral

$$I = \int_0^{2/\sqrt{2}} \int_y^{\sqrt{4-y^2}} e^{2x^2+2y^2} dx dy$$

to polar coordinates, getting

$$\int_C^D \int_A^B h(r, \theta) dr d\theta,$$

where

$h(r, \theta) =$  \_\_\_\_\_

$A =$  \_\_\_\_\_

$B =$  \_\_\_\_\_

$C =$  \_\_\_\_\_

$D =$  \_\_\_\_\_

and then evaluate the resulting integral to get

$I =$  \_\_\_\_\_.

Correct Answers:

- $r * e^{(2 * r^2)}$
- 0
- 2
- 0
- $\pi/4$
- $(\pi/16) * (e^{(8)} - 1)$

16. (1 point) Library/FortLewis/Calc3/16-4-Polar-integrals/HG  
M5-16-4-14-Double-integrals-polar.pg

Sketch the region of integration for the following integral.

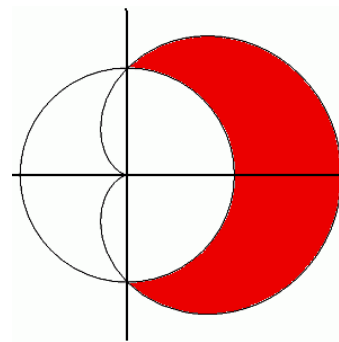
$$\int_0^{\pi/4} \int_0^{4/\cos(\theta)} f(r, \theta) r dr d\theta$$

The region of integration is bounded by

- A.  $y = 0, x = \sqrt{16 - y^2}$ , and  $y = 4$
- B.  $y = 0, y = x$ , and  $x = 4$
- C.  $y = 0, y = x$ , and  $y = 4$
- D.  $y = 0, y = \sqrt{16 - x^2}$ , and  $x = 4$
- E. None of the above

Correct Answers:

• B



Use a double integral in polar coordinates to find the area of the region bounded on the inside by the circle of radius 2 and on the outside by the cardioid  $r = 2(1 + \cos(\theta))$

Correct Answers:

- $2^2 * (2 + \pi/4)$

18. (1 point) Library/Michigan/Chap16Sec4/Q21.pg

Convert the integral

$$\int_0^{\sqrt{4}} \int_{-x}^x dy dx$$

to polar coordinates and evaluate it (use  $t$  for  $\theta$ ):

With  $a =$  \_\_\_\_\_,  $b =$  \_\_\_\_\_,  $c =$  \_\_\_\_\_ and  $d =$  \_\_\_\_\_,

$$\int_0^{\sqrt{4}} \int_{-x}^x dy dx = \int_a^b \int_c^d \text{_____} dr dt$$

$$= \int_a^b \text{_____} dt$$

$$= \text{_____} \Big|_a^b$$

$$= \text{_____}$$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

SOLUTION

In polar coordinates,  $-\pi/4 \leq \theta \leq \pi/4$ . Also, the outer (right) boundary of the region is  $\sqrt{4} = x = r \cos \theta$ . Hence,  $0 \leq r \leq \sqrt{4}/\cos \theta$ . The integral becomes

$$\int_0^{\sqrt{4}} \int_{-x}^x dy dx = \int_{-\pi/4}^{\pi/4} \int_0^{\sqrt{4}/\cos \theta} r dr d\theta.$$

Then

$$\int_{-\pi/4}^{\pi/4} \int_0^{\sqrt{4}/\cos \theta} r dr d\theta = \int_{-\pi/4}^{\pi/4} \left( \frac{r^2}{2} \Big|_0^{\sqrt{4}/\cos \theta} \right) d\theta = \int_{-\pi/4}^{\pi/4} \frac{2}{\cos^2 \theta} d\theta.$$

And

$$\int_{-\pi/4}^{\pi/4} \frac{2}{\cos^2 \theta} d\theta = 2 \tan \theta \Big|_{-\pi/4}^{\pi/4} = 2 \cdot (1 - (-1)) = 4.$$

Notice that we can check this answer because the integral gives the area of the shaded triangular region which is  $\frac{1}{2} \cdot \sqrt{4} \cdot (2\sqrt{4}) = 4$ .

Correct Answers:

- -pi/4
- pi/4
- 0
- [sqrt(4)]/[cos(t)]
- r
- 2/([cos(t)]^2)
- 2\*tan(t)
- 4

**19. (1 point)** Library/Rochester/setVMultIntegrals2Polar/UR\_VC\_9\_1.pg

Using polar coordinates, evaluate the integral  $\iint_R \sin(x^2 + y^2) dA$  where  $R$  is the region  $9 \leq x^2 + y^2 \leq 81$ .

Correct Answers:

- -5.30243111246558

**20. (1 point)** Library/UMN/calculusStewartET/s\_15\_4\_6.pg  
Sketch the region whose area is given by the integral and evaluate it.

$$\int_{\pi/2}^{\pi} \int_0^{2\sin\theta} r dr d\theta$$

Answer: \_\_\_\_\_

Correct Answers:

- pi/2\*(2/2)^2

**21. (1 point)** Library/CSUN/Calculus/Triple\_iterated\_integral\_2.pg

Find the limits of integration  $l_y, u_y, l_x, u_x, l_z, u_z$  (some of which will involve variables  $x, y, z$ ) so that

$$\int_{l_y}^{u_y} \int_{l_x}^{u_x} \int_{l_z}^{u_z} dx dz dy$$

represents the volume of the region in the first octant that is inside the paraboloid  $y = x^2 + 6z^2$  and between the planes  $y = 9$  and  $y = 10$ .

$l_x =$  \_\_\_\_\_

$u_x =$  \_\_\_\_\_

$l_z =$  \_\_\_\_\_

$u_z =$  \_\_\_\_\_

$l_y =$  \_\_\_\_\_

$u_y =$  \_\_\_\_\_

Correct Answers:

- 0
- sqrt(y-6\*z^2)
- 0
- sqrt(y/6)
- 9
- 10

**22. (1 point)** Library/FortLewis/Calc3/16-3-Triple-integrals/HGM4-16-3-21-Triple-integrals.pg

Suppose  $R$  is the solid bounded by the plane  $z = 2x$ , the surface  $z = x^2$ , and the planes  $y = 0$  and  $y = 2$ . Write an iterated integral in the form below to find the volume of the solid  $R$ .

$$\iiint_R f(x,y,z) dV = \int_A^B \int_C^D \int_E^F \text{_____} dz dy dx$$

with limits of integration

A = \_\_\_\_\_

B = \_\_\_\_\_

C = \_\_\_\_\_

D = \_\_\_\_\_

E = \_\_\_\_\_

F = \_\_\_\_\_

Correct Answers:

- 1
- 0
- 2
- 0
- 2
- x^2
- 2\*x

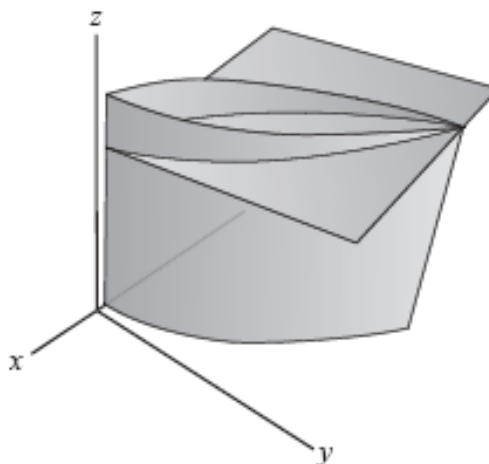
**23. (1 point)** Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/15\_Multiple\_Integration/15.3\_Triple\_Integrals/15.3.20.pg

Find the volume of the solid in  $\mathbf{R}^3$  bounded by  $y = x^2$ ,  $x = y^2$ ,  $z = x + y + 21$ , and  $z = 0$ .

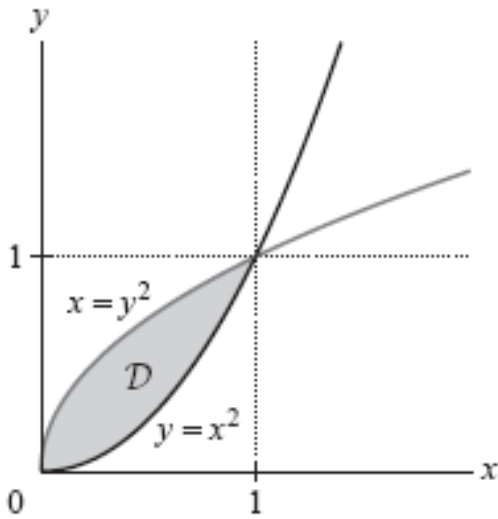
$V =$  \_\_\_\_\_

**Solution:** ( Instructor solution preview: show the student solution after due date. )

**Solution:** The solid  $\mathcal{W}$  is shown in the following figure:



The upper surface is the plane  $z = x + y + 21$  and the lower surface is the plane  $z = 0$ . The projection of  $\mathcal{W}$  onto the  $xy$ -plane is the region in the first quadrant enclosed by the curves  $y = x^2$  and  $x = y^2$ .



We use the formula for the volume as a triple integral to write

$$\text{Volume}(\mathcal{W}) = \iiint_{\mathcal{W}} 1 \, dV$$

The triple integral is equal to the following iterated integral:

$$\begin{aligned} \text{Volume}(\mathcal{W}) = \iiint_{\mathcal{W}} 1 \, dV &= \iint_{\mathcal{D}} \left( \int_0^{x+y+21} 1 \, dz \right) dA \\ &= \iint_{\mathcal{D}} z \Big|_{z=0}^{x+y+21} dA \\ &= \iint_{\mathcal{D}} (x+y+21) \, dA \\ &= \int_0^1 \left( \int_{x^2}^{\sqrt{x}} (x+y+21) \, dy \right) dx \\ &= \int_0^1 \left. xy + \frac{y^2}{2} + 21y \right|_{y=x^2}^{\sqrt{x}} dx \\ &= \int_0^1 \left( x\sqrt{x} + \frac{x}{2} + 21\sqrt{x} - \left( x^3 + \frac{x^4}{2} + 21x^2 \right) \right) dx \\ &= \int_0^1 \left( -\frac{x^4}{2} - x^3 - 21x^2 + x^{3/2} + \frac{x}{2} + 21x^{1/2} \right) dx \\ &= \left. -\frac{x^5}{10} - \frac{x^4}{4} - 7x^3 + \frac{2}{5}x^{5/2} + \frac{x^2}{4} + 14x^{3/2} \right|_0^1 \\ &= -\frac{1}{10} - \frac{1}{4} - 7 + \frac{2}{5} + \frac{1}{4} + 14 = \frac{146}{20} = 7.3 \end{aligned}$$

Correct Answers:

- 7.3

**24. (1 point)** Library/UMN/calculusStewartET/s\_15\_7\_7.pg

Evaluate the iterated integral  $\int_0^{\pi/3} \int_0^y \int_0^x \cos(x+y+z) \, dz \, dx \, dy$ .

Answer: \_\_\_\_\_

Correct Answers:

- $-(1/2) \cdot \sin(\pi/3) + 1/2 \cdot \sin(2\pi/3) - 1/6 \cdot \sin(3\pi/3)$

**25. (1 point)** Library/UMN/calculusStewartET/s\_15\_7\_18.pg

Evaluate the triple integral  $\iiint_E z \, dV$ , where  $E$  is bounded by the cylinder  $y^2 + z^2 = 16$  and the planes  $x = 0$ ,  $y = 4x$ , and  $z = 0$  in the first octant.

Answer: \_\_\_\_\_

Correct Answers:

- $4^{3/8}$

**26. (1 point)** Library/FortLewis/Calc3/16-5-Cylindrical-integrals/HGM4-16-5-26-Cylindrical-integrals.pg

Let  $W$  be the top half of the unit sphere centered at the origin. Without calculating, determine whether each integral below is positive, negative, or zero.

1.  $\iiint_W (z^2 - z) \, dV$

2.  $\iiint_W -xz \, dV$

Correct Answers:

- Negative
- Zero

**27. (1 point)** Library/FortLewis/Calc3/16-5-Cylindrical-integrals/HGM5-16-5-33-Cylindrical-integrals.pg

Evaluate the integral.

$$\int_0^5 \int_{-5}^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \frac{1}{(x^2+y^2)^{1/2}} \, dy \, dx \, dz = \underline{\hspace{2cm}}$$

Correct Answers:

- $50\pi$

Integrate the function  $(x^2 + y^2)^{1/3}$  over the region  $E$  that is bounded by the  $xy$  plane below and above by the paraboloid  $z = 8 - 4x^2 - 4y^2$  using cylindrical coordinates.

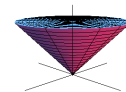
$$\iiint_E (x^2 + y^2)^{1/3} \, dV = \int \int \int \dots$$

Correct Answers:

- 0
- $2\pi$
- 0
- 1.41421
- 0
- $8-4r^2$
- $r^{1.66667}$
- 20.3562

**29. (1 point)** Library/Michigan/Chap16Sec5/Q32.pg

The region  $W$  is the cone shown below.



The angle at the vertex is  $\pi/2$ , and the top is flat and at a height of 3.

Write the limits of integration for  $\int_W dV$  in the following coordinates (do **not** reduce the domain of integration by taking advantage of symmetry):

(a) Cartesian:

With  $a = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$ ,

$c = \underline{\hspace{2cm}}$ ,  $d = \underline{\hspace{2cm}}$ ,

$e = \underline{\hspace{2cm}}$ , and  $f = \underline{\hspace{2cm}}$ ,

Volume =  $\int_a^b \int_c^d \int_e^f \underline{\hspace{2cm}} d\underline{\hspace{2cm}} d\underline{\hspace{2cm}} d\underline{\hspace{2cm}}$

(b) Cylindrical:

With  $a = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$ ,

$c = \underline{\hspace{2cm}}$ ,  $d = \underline{\hspace{2cm}}$ ,

$e = \underline{\hspace{2cm}}$ , and  $f = \underline{\hspace{2cm}}$ ,

Volume =  $\int_a^b \int_c^d \int_e^f \underline{\hspace{2cm}} d\underline{\hspace{2cm}} d\underline{\hspace{2cm}} d\underline{\hspace{2cm}}$

(c) Spherical:

With  $a = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$ ,

$c = \underline{\hspace{2cm}}$ ,  $d = \underline{\hspace{2cm}}$ ,

$e = \underline{\hspace{2cm}}$ , and  $f = \underline{\hspace{2cm}}$ ,

Volume =  $\int_a^b \int_c^d \int_e^f \underline{\hspace{2cm}} d\underline{\hspace{2cm}} d\underline{\hspace{2cm}} d\underline{\hspace{2cm}}$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

SOLUTION

(a) Since the cone has an angle of  $\pi/2$  at its vertex, it has equation

$$z = \sqrt{x^2 + y^2}.$$

The top of the cone, the plane with equation  $z = 3$ , intersects the cone in the circle  $x^2 + y^2 = 9$ . Thus, in cartesian coordinates we have

$$\int_W dV = \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^3 1 dz dy dx.$$

(b) In cylindrical coordinates the cone has equation  $z = r$ , so the integral becomes

$$\int_W dV = \int_0^{2\pi} \int_0^3 \int_r^3 r dz dr d\theta.$$

(c) In spherical coordinates, the cone has equation  $\phi = \pi/4$  and the plane  $z = 3$  is  $\rho \cos \phi = 3$ . Thus we have

$$\int_W dV = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{3/\cos \phi} \rho^2 \sin \phi d\rho d\phi d\theta.$$

Correct Answers:

- -3
- 3

- $-\sqrt{3^2-x^2}$
- $\sqrt{3^2-x^2}$
- $1*\sqrt{x^2+y^2}$
- 3
- 1
- z
- y
- x
- 0
- $2*\pi$
- 0
- 3
- $1*r$
- 3
- r
- z
- r
- theta
- 0
- $2*\pi$
- 0
- $\pi/4$
- 0
- $3/[\cos(\phi)]$
- $\rho^2*\sin(\phi)$
- rho
- phi
- theta

30. (1 point) Library/Michigan/Chap16Sec5/Q09.pg

Evaluate the triple integral of  $f(x,y,z) = \sin(x^2 + y^2)$  over the solid cylinder with height 4 and with base of radius 1 centered on the  $z$  axis at  $z = -1$ .

Integral =  $\underline{\hspace{4cm}}$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

SOLUTION

We have

$$\int_W f dV = \int_{-1}^3 \int_0^{2\pi} \int_0^1 (\sin(r^2)) r dr d\theta dz.$$

Integrating,

$$\int_{-1}^3 \int_0^{2\pi} \int_0^1 (\sin(r^2)) r dr d\theta dz = \int_{-1}^3 \int_0^{2\pi} \left. \frac{1}{2} \cdot -\cos(r^2) \right|_{r=0}^{r=1} d\theta dz = \int_{-1}^3 \int_0^{2\pi} \frac{1}{2} (1 - \cos(1)) d\theta dz = 4\pi(1 - \cos(1)).$$

Correct Answers:

- $4*\pi*[1-\cos(1*1)]$

1. (1 point) Library/Dartmouth/setMTWCh5S6/problem\_5.pg

A lamina occupies the part of the disk  $x^2 + y^2 \leq 25$  in the first quadrant and the density at each point is given by the function  $\rho(x, y) = 3(x^2 + y^2)$ .

- What is the total mass? \_\_\_\_\_
- What is the moment about the x-axis? \_\_\_\_\_
- What is the moment about the y-axis? \_\_\_\_\_
- Where is the center of mass? (\_\_\_\_\_, \_\_\_\_\_)
- What is the moment of inertia about the origin?

Correct Answers:

- 736.310778185108
- 1875
- 1875
- 2.54647908947033
- 2.54647908947033
- 12271.8463030851

2. (1 point) Library/Rochester/setIntegrals24Centroid/centroid\_6\_5.1.pg

Find the centroid  $(\bar{x}, \bar{y})$  of the region bounded by:

$$y = 5x^2 + 4x, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 7.$$

$\bar{x} =$  \_\_\_\_\_

$\bar{y} =$  \_\_\_\_\_

Correct Answers:

- 5.16463414634146
- 82.0365853658536

3. (1 point) Library/272/setStewart15\_5/problem\_5a.pg

A lamina occupies the part of the disk  $x^2 + y^2 \leq 1$  in the first quadrant and the density at each point is given by the function  $\rho(x, y) = 4(x^2 + y^2)$ .

- What is the total mass? \_\_\_\_\_
- Where is the center of mass? (\_\_\_\_\_, \_\_\_\_\_)

Correct Answers:

- 1.5707963267949
- 0.509295817894065
- 0.509295817894065

4. (1 point) Library/272/setStewart15\_5/problem\_4a.pg

A lamina occupies the part of the rectangle  $0 \leq x \leq 8$ ,  $0 \leq y \leq 6$  and the density at each point is given by the function  $\rho(x, y) = 4x + 1y + 7$ .

- What is the total mass? \_\_\_\_\_
- Where is the center of mass? (\_\_\_\_\_, \_\_\_\_\_)

Correct Answers:

- 1248
- 4.82051282051282
- 3.11538461538462

5. (1 point) Library/272/setStewart15\_5/problem\_3.pg

Find the mass of the rectangular region  $0 \leq x \leq 1$ ,  $0 \leq y \leq 3$  with density function  $\rho(x, y) = 3 - y$ .

Correct Answers:

- 4.5

6. (1 point) Library/272/setStewart15\_5/problem\_3a.pg

Find the mass of the triangular region with vertices  $(0, 0)$ ,  $(2, 0)$ , and  $(0, 5)$ , with density function  $\rho(x, y) = x^2 + y^2$ .

Correct Answers:

- 24.1666666666667

7. (1 point) Library/272/setStewart15\_5/problem\_5.pg

A lamina occupies the region inside the circle  $x^2 + y^2 = 8y$  but outside the circle  $x^2 + y^2 = 16$ . The density at each point is inversely proportional to its distance from the origin.

Where is the center of mass?  
 (\_\_\_\_\_, \_\_\_\_\_)

Correct Answers:

- 0
- 5.05816623182515

8. (1 point) Library/FortLewis/Calc3/16-3-Triple-integrals/HGM5-16-3-35-Triple-integrals.pg

Set up a triple integral to find the mass of the solid tetrahedron bounded by the xy-plane, the yz-plane, the xz-plane, and the plane  $x/4 + y/3 + z/12 = 1$ , if the density function is given by  $\delta(x, y, z) = x + y$ . Write an iterated integral in the form below to find the mass of the solid.

$$\iiint_R f(x, y, z) dV = \int_A^B \int_C^D \int_E^F \text{_____} dz dy dx$$

with limits of integration

A = \_\_\_\_\_

B = \_\_\_\_\_

C = \_\_\_\_\_

D = \_\_\_\_\_

E = \_\_\_\_\_

F = \_\_\_\_\_

Correct Answers:

- x+y
- 0

- 4
- 0
- $3-0.75*x$
- 0
- $12-3*x-4*y$

9. (1 point) Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/15\_Multiple\_Integration/15.3\_Triple\_Integrals/15.3.37.pg

Find the center mass of the solid bounded by planes  $x + y + z = 1$ ,  $x = 0$ ,  $y = 0$ , and  $z = 0$ , assuming a mass density of  $\rho(x, y, z) = 3\sqrt{z}$ .

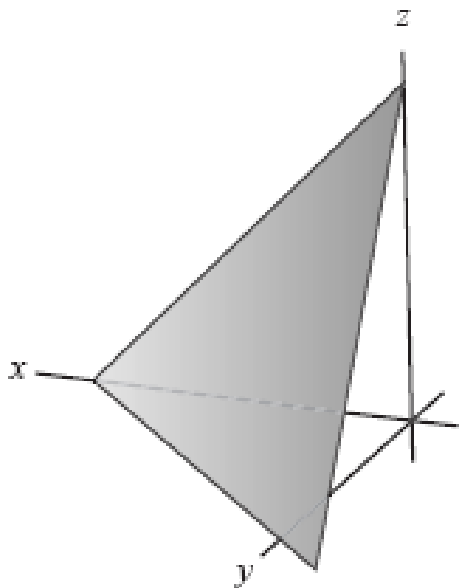
$$(x_{CM}, y_{CM}, z_{CM}) = \underline{\hspace{2cm}}$$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

**Solution:** The coordinates of the center of mass are defined by

$$\begin{aligned} x_{CM} &= \frac{\iiint_{\mathcal{W}} 3x\sqrt{z}dV}{\iiint_{\mathcal{W}} 3\sqrt{z}dV} \\ y_{CM} &= \frac{\iiint_{\mathcal{W}} 3y\sqrt{z}dV}{\iiint_{\mathcal{W}} 3\sqrt{z}dV} \quad (1) \\ z_{CM} &= \frac{\iiint_{\mathcal{W}} 3z\sqrt{z}dV}{\iiint_{\mathcal{W}} 3\sqrt{z}dV} \end{aligned}$$

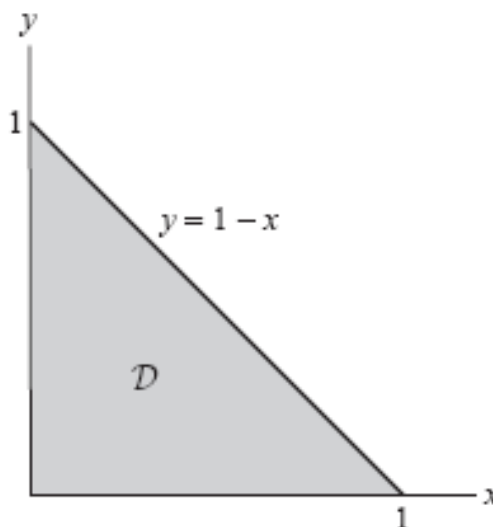
$\mathcal{W}$  is the tetrahedron shown in the figure:



The projection  $\mathcal{D}$  of  $\mathcal{W}$  onto the  $xy$ -plane is the triangle determined by the line  $AB: y = 1 - x$  and the positive  $x$  and  $y$  axes. The upper surface is the plane  $z = 1 - x - y$  and the lower surface is  $z = 0$ .

Therefore, we describe  $\mathcal{W}$  by the following inequalities:

$$0 \leq x \leq 1, \quad 0 \leq y \leq 1 - x, \quad 0 \leq z \leq 1 - x - y \quad (2)$$



We compute the mass of the solid  $\mathcal{W}$ :

$$\begin{aligned} \iiint_{\mathcal{W}} 3\sqrt{z}dV &= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 3\sqrt{z}dzdydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2}{3} z^{3/2} \Big|_{z=0}^{1-x-y} dydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2}{3} (1-x-y)^{3/2} dydx \\ &= 3 \int_0^1 -\frac{2}{3} \cdot \frac{2}{5} (1-x-y)^{5/2} \Big|_{y=0}^{1-x} dx \\ &= 3 \int_0^1 -\frac{4}{15} \left( (1-x-(1-x))^{5/2} - (1-x)^{5/2} \right) dx \\ &= 3 \int_0^1 \frac{4}{15} (1-x)^{5/2} dx \\ &= 3 \left( -\frac{4}{15} \cdot \frac{2}{7} (1-x)^{7/2} \Big|_0^1 \right) \\ &= 3 \cdot \frac{4}{15} \cdot \frac{2}{7} = \frac{24}{105} \end{aligned}$$

That is,

$$\iiint_{\mathcal{W}} 3\sqrt{z}dV = \frac{24}{105} \quad (3)$$

We now compute the triple integrals in the numerators of (1). Using (2) we have

$$\begin{aligned} \iiint_{\mathcal{W}} 3x\sqrt{z}dV &= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 3x\sqrt{z}dzdydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2x}{3} z^{3/2} \Big|_{z=0}^{1-x-y} dydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2x}{3} (1-x-y)^{3/2} dydx \\ &= 3 \int_0^1 \frac{2x}{3} \cdot \left(-\frac{2}{5}\right) (1-x-y)^{5/2} \Big|_{y=0}^{1-x} dx \\ &= 3 \int_0^1 -\frac{4x}{15} \left( (1-x-(1-x))^{5/2} - (1-x)^{5/2} \right) dx \\ &= 3 \int_0^1 \frac{4x}{15} (1-x)^{5/2} dx \end{aligned}$$

We compute the integral using the substitution  $u = 1 - x$ ,  $du = -dx$ . We get

$$\begin{aligned} \iiint_{\mathcal{W}} 3x\sqrt{z}dV &= 3 \int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{4}{15} (1-u)u^{5/2}(-du) \\ &= 3 \int_0^1 \int_0^{1-x} \frac{4}{15} (u^{5/2} - u^{7/2}) du \\ &= 3 \left( \frac{4}{15} \left( \frac{2}{7}u^{7/2} - \frac{2}{9}u^{9/2} \right) \Big|_{u=0}^1 \right) \\ &= 3 \left( \frac{4}{15} \left( \frac{2}{7} - \frac{2}{9} \right) \right) = \frac{48}{945} \quad (4) \end{aligned}$$

We compute the numerator of  $y_{CM}$  in (1), using (2). This gives

$$\begin{aligned} \iiint_{\mathcal{W}} 3y\sqrt{z}dV &= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 3y\sqrt{z}dzdydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2y}{3} z^{3/2} \Big|_{z=0}^{1-x-y} dydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2y}{3} (1-x-y)^{3/2} dydx \quad (5) \end{aligned}$$

We compute the inner integral using the substitution  $u = 1 - x - y$ ,  $du = -dy$ . We get

$$\begin{aligned} \int_0^{1-x} 3 \cdot \frac{2y}{3} (1-x-y)^{3/2} dy &= \int_{1-x}^0 -3 \cdot \frac{2}{3} (1-x-u)u^{3/2} du \\ &= \int_0^{1-x} 3 \cdot \frac{2}{3} ((1-x)u^{3/2} - u^{5/2}) du \\ &= 3 \left( \frac{2}{3} \left( \frac{2(1-x)}{5} u^{5/2} - \frac{2}{7} u^{7/2} \right) \Big|_{u=0}^{1-x} \right) \\ &= 3 \left( \frac{4}{15} (1-x)(1-x)^{5/2} - \frac{4}{21} (1-x)^{7/2} \right) \\ &= \frac{24}{105} (1-x)^{7/2} \end{aligned}$$

Combining with (5) we get

$$\begin{aligned} \iint_{\mathcal{W}} 3y\sqrt{z}dV &= \int_0^1 \frac{24}{105} (1-x)^{7/2} dx = \\ &= -\frac{24}{105} \cdot \frac{2}{9} (1-x)^{9/2} \Big|_0^1 = \frac{48}{945} \quad (6) \end{aligned}$$

Finally, we compute the denominator of  $z_{CM}$  in (1), using (2):

$$\begin{aligned} \iiint_{\mathcal{W}} 3z\sqrt{z}dV &= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 3z^{3/2} dzdydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2}{5} z^{5/2} \Big|_{z=0}^{1-x-y} dydx \\ &= \int_0^1 \int_0^{1-x} 3 \cdot \frac{2}{5} (1-x-y)^{5/2} dydx \\ &= 3 \int_0^1 -\frac{2}{5} \cdot \frac{2}{7} (1-x-y)^{7/2} \Big|_{y=0}^{1-x} dx \\ &= 3 \int_0^1 -\frac{4}{35} \left( (1-x-(1-x))^{7/2} - (1-x)^{7/2} \right) dx \\ &= 3 \int_0^1 \frac{4}{35} (1-x)^{7/2} dx \\ &= 3 \left( -\frac{4}{35} \cdot \frac{2}{9} (1-x)^{9/2} \Big|_0^1 \right) \\ &= \frac{24}{315} \quad (7) \end{aligned}$$

Substituting (3), (4), (6) and (7) in (1) we obtain the following center of mass:

$$(x_{CM}, y_{CM}, z_{CM}) = \left( \frac{48}{945}, \frac{48}{945}, \frac{24}{315} \right) = \left( \frac{2}{9}, \frac{2}{9}, \frac{1}{3} \right)$$

Correct Answers:

- (0.222222, 0.222222, 0.333333)

**10. (1 point)** Library/Hope/Multi2/11-04-MV-Int-change-var/Chan ge-variables-02.pg

Suppose a change of coordinates  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  from the  $uv$ -plane to the  $xy$ -plane is given by

$$x = 3v - 3u - 2, \quad y = -2 - 2u - 3v.$$

(a) Find the absolute value of the determinant of the Jacobian for this change of coordinates.

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \left| \det \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \right| = \underline{\hspace{2cm}}$$

(b) If a region  $D^*$  in the  $uv$ -plane has area 7.22, find the area of the region  $T(D^*)$  in the  $xy$ -plane.

Area =                     

Correct Answers:

- `<table border='0' cellspacing='10'>`
- `<tr><td> -3 </td><td> 3 </td></tr>`
- `<tr><td> -2 </td><td> -3 </td></tr>`
- `</table>`
- 15
- 7.22\*15

**11. (1 point)** Library/Hope/Multi2/11-04-MV-Int-change-var/Chan ge-variables-05.pg

Compute the area of the region  $D$  bounded by

$$xy = 1, \quad xy = 16, \quad x^2 = 1, \quad y^2 = 49$$

in the first quadrant of the  $xy$ -plane.

(a) Graph the region  $D$ .

(b) Using the non-linear change of variables  $u = xy$  and  $v = x^2$ , find  $x$  and  $y$  as functions of  $u$  and  $v$ .

$$\begin{aligned} x &= x(u,v) = \underline{\hspace{2cm}} \\ y &= y(u,v) = \underline{\hspace{2cm}} \end{aligned}$$

(c) Find the determinant of the Jacobian for this change of variables.

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \det \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} = \underline{\hspace{2cm}}$$

(d) Using the change of variables, set up a double integral for calculating the area of the region  $D$ .

$$\begin{aligned} \iint_D dx dy &= \int_a^b \int_c^d \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du dv = \\ \int_{\underline{\hspace{1cm}}}^{\underline{\hspace{1cm}}} \int_{\underline{\hspace{1cm}}}^{\underline{\hspace{1cm}}} \underline{\hspace{2cm}} du dv \end{aligned}$$

(e) Evaluate the double integral and compute the area of the

region  $D$ .

Area = \_\_\_\_\_

Correct Answers:

- |         |
|---------|
| $u^2/v$ |
| $v/u$   |
- |            |              |
|------------|--------------|
| $2u/v$     | $-u^2/(v^2)$ |
| $-v/(u^2)$ | $1/u$        |
- |      |
|------|
| $49$ |
| $1$  |

$16$
$1$

$1/v$
-------
- $(16-1) \cdot \ln(49)$

**12. (1 point)** Library/Hope/Multi2/11-04-MV-Int-change-var/Change-variables-06.pg

Compute the double integral

$$\int \int_D 2xy^2 \, dx \, dy$$

over the region  $D$  bounded by

$$xy = 1, \quad xy = 16, \quad x - y = 1, \quad x - y = 49$$

in the first quadrant of the  $xy$ -plane. Hint: make a change of variables  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  that converts a rectangular region  $D^*$  in the  $uv$ -plane into the region of integration  $D = T(D^*)$  in the  $xy$ -plane.

Double integral = \_\_\_\_\_

Correct Answers:

- $2 \cdot (16-1) \cdot (49-1)$

**13. (1 point)** Library/Michigan/Chap16Sec7/Q05.pg

Find the Jacobian.  $\frac{\partial(x,y,z)}{\partial(s,t,u)}$ , where  $x = 3t - 3s - 2u, y = -(3s + 2t + 2u), z = 3s - 5t - 4u$ .

$$\frac{\partial(x,y,z)}{\partial(s,t,u)} = \underline{\hspace{2cm}}$$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

**SOLUTION**

We have

$$\left| \frac{\partial(x,y,z)}{\partial(s,t,u)} \right| = \begin{vmatrix} x_s & x_t & x_u \\ y_s & y_t & y_u \\ z_s & z_t & z_u \end{vmatrix} = \begin{vmatrix} -3 & 3 & -2 \\ -3 & -2 & -2 \\ 3 & -5 & -4 \end{vmatrix}.$$

This  $3 \times 3$  determinant is computed the same way as for the cross product, with the entries  $-3, 3, -2$  in the first row playing the same role as  $\vec{i}, \vec{j}, \vec{k}$ . We get

$$\frac{\partial(x,y,z)}{\partial(s,t,u)} = ((-2)(-4) - (-2)(-5)) - 3 + ((-2)(3) - (-3)(-4))3 + ((-3)(-3) - (-3)(-4))3 + ((-3)(-3) - (-3)(-4))3 + ((-3)(-3) - (-3)(-4))3 + ((-3)(-3) - (-3)(-4))3$$

Correct Answers:

- $-3 \cdot (-2 \cdot -4 - -2 \cdot -5) - 3 \cdot (-3 \cdot -4 - -2 \cdot 3) + -2 \cdot (-3 \cdot -5 - -2 \cdot 3)$

**14. (1 point)** Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/15\_Multiple\_Integration/15.6\_Change\_of\_Variables/15.6.41.pg

Let  $I = \iint_D (x^2 - y^2) \, dx \, dy$ , where

$$D = \{(x,y) : 1 \leq xy \leq 5, 0 \leq x - y \leq 5, x \geq 0, y \geq 0\}$$

Show that the mapping  $u = xy, v = x - y$  maps  $D$  to the rectangle  $\mathcal{R} = [1, 5] \times [0, 5]$ .

(a) Compute  $\partial(x,y)/\partial(u,v)$  by first computing  $\partial(u,v)/\partial(x,y)$ .

(b) Use the Change of Variables Formula to show that  $I$  is equal to the integral of  $f(u,v) = v$  over  $\mathcal{R}$  and evaluate.

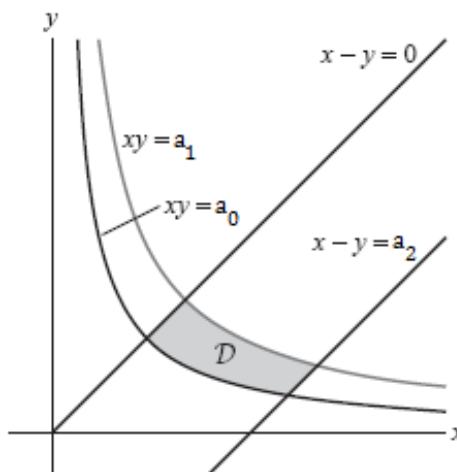
(a)  $\frac{\partial(x,y)}{\partial(u,v)} = \underline{\hspace{2cm}}$

(b)  $I = \underline{\hspace{2cm}}$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

**Solution:** The domain  $D$  is defined by the inequalities

$$D : 1 \leq xy \leq 5, 0 \leq x - y \leq 5, x \geq 0, y \geq 0$$



With  $a_0 = 1, a_1 = 5$  and  $a_2 = 5$

Since  $u = xy$  and  $v = x - y$ , the image of  $D$  under this mapping is the rectangle defined by

$$D_0 : 1 \leq u \leq 5, 0 \leq v \leq 5$$

That is,  $D_0 = [1, 5] \times [0, 5]$ .

(a) We compute the Jacobian  $\frac{\partial(u,v)}{\partial(x,y)}$  and then use the formula for the Jacobian of the inverse mapping

to compute  $\frac{\partial(x,y)}{\partial(u,v)}$ .

Since  $u = xy$  and  $v = x - y$ , we have

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} =$$

$$\begin{vmatrix} y & x \\ 1 & -1 \end{vmatrix} = -y - x = -(x+y)$$

Therefore,

$$\frac{\partial(x,y)}{\partial(u,v)} = \left( \frac{\partial(u,v)}{\partial(x,y)} \right)^{-1} = -\frac{1}{x+y}$$

(b) In  $\mathcal{D}$ ,  $x \geq 0$  and  $y \geq 0$ , hence  $\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \frac{1}{x+y}$ .

Using the change of variable formula gives:

$$\begin{aligned} I &= \iint_{\mathcal{D}_0} (x^2 - y^2) \cdot \frac{1}{x+y} du dv = \iint_{\mathcal{D}_0} (x-y) du dv \\ &= \int_0^5 \int_1^5 v du dv \\ &= \left( \int_0^5 v dv \right) \left( \int_1^5 du \right) \\ &= \left( \frac{v^2}{2} \Big|_0^5 \right) \left( u \Big|_1^5 \right) \\ &= \frac{25}{2} \cdot 4 = 50 \end{aligned}$$

Correct Answers:

- $-[1/(x+y)]$
- 50

15. (1 point) Library/Hope/Multi2/12-01-Vector-fields/Vector-fields-02b.pg

Suppose  $\vec{F} = \langle 5x, 5y \rangle$ . Complete the following table of values of  $\vec{F}$ .

	Values of $\vec{F}$				
$y = 2$	_____	_____	_____	_____	_____
$y = 1$	_____	_____	_____	_____	_____
$y = 0$	_____	_____	_____	_____	_____
$y = -1$	_____	_____	_____	_____	_____
$y = -2$	_____	_____	_____	_____	_____
	$x = -2$	$x = -1$	$x = 0$	$x = 1$	$x = 2$

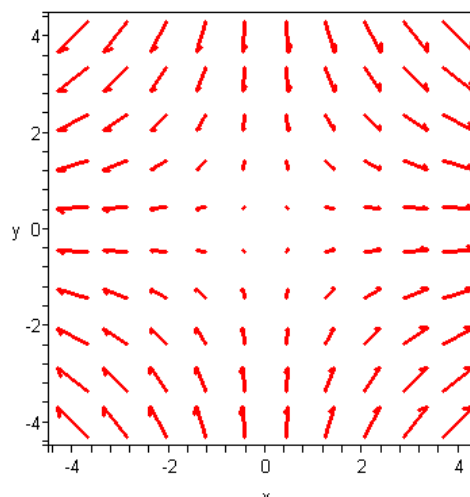
Using your table of values as a starting point, sketch this vector field on a piece of paper for  $-2 \leq x \leq 2$  and  $-2 \leq y \leq 2$ .

Correct Answers:

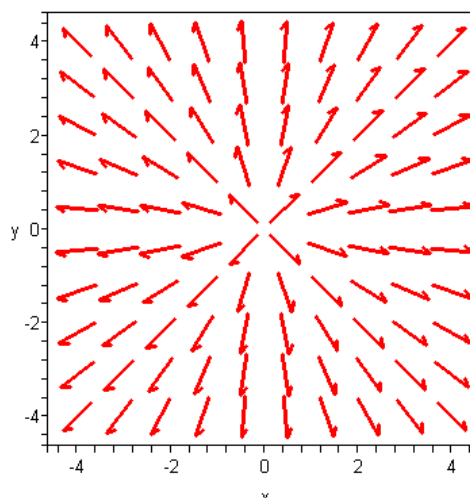
- |                                     |   |   |
|-------------------------------------|---|---|
| <table border='0' cellspacing='10'> | <tr><td> <-10,10> </td><td> <-5,10> </td><td> <0,10>  | > |
|                                     | <tr><td> <-10,5> </td><td> <-5,5> </td><td> <0,5> </t | > |
|                                     | <tr><td> <-10,0> </td><td> <-5,0> </td><td> <0,0> </t | > |
|                                     | <tr><td> <-10,-5> </td><td> <-5,-5> </td><td> <0,-5>  | > |
|                                     | <tr><td> <-10,-10> </td><td> <-5,-10> </td><td> <0,-1 | > |
|                                     | </table>  | > |

16. (1 point) Library/ASU-topics/setCalculus/stef/stef16\_1p2/stef16\_1p2.pg

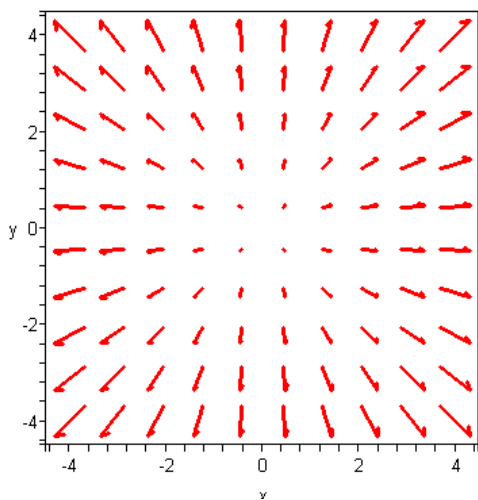
Match the functions  $f$  with the plots of their gradient vector fields labeled A-D.



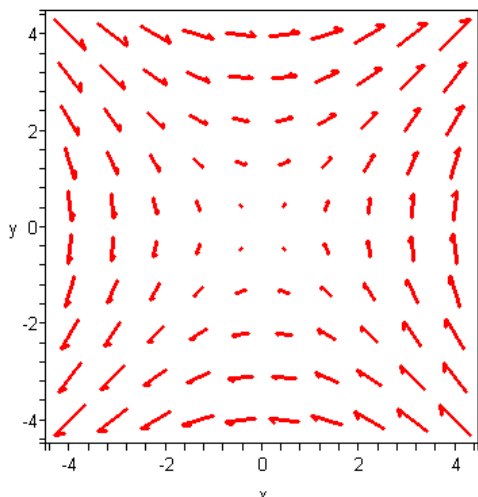
A.



B.



C.



D.

- \_\_\_ 1.  $f(x,y) = x^2 - y^2$
- \_\_\_ 2.  $f(x,y) = \sqrt{x^2 + y^2}$
- \_\_\_ 3.  $f(x,y) = x^2 + y^2$
- \_\_\_ 4.  $f(x,y) = xy$

Correct Answers:

- A
- B
- C
- D

17. (1 point) Library/FortLewis/Calc3/17-3-Vector-fields/HGM4-17-3-10-Vector-fields.pg

Sketch the vector field  $\vec{F}(\vec{r}) = 2\vec{r}$  in the plane, where  $\vec{r} = \langle x,y \rangle$ . Select all that apply.

- A. The vectors increase in length as you move away from the origin.

- B. All the vectors point away from the origin.
- C. The vectors decrease in length as you move away from the origin.
- D. All the vectors point toward the origin.
- E. The length of each vector is 2.
- F. All the vectors point in the same direction.

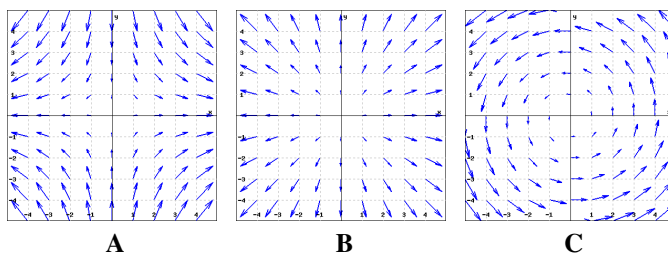
Correct Answers:

- AB

18. (1 point) Library/FortLewis/Calc3/17-3-Vector-fields/HGM5-17-3-27-Vector-fields.pg

Match each vector field with its graph.

- ? 1.  $\vec{F}(x,y) = x\vec{i} + y\vec{j}$
- ? 2.  $\vec{H}(x,y) = x\vec{i} - y\vec{j}$
- ? 3.  $\vec{G}(x,y) = -y\vec{i} + x\vec{j}$



(Click on a graph to enlarge it)

Correct Answers:

- B
- A
- C

19. (1 point) Library/Michigan/Chap17Sec3/Q21.pg  
Give an example of a vector field  $\vec{F}(x,y)$  in 2-space, where  $\vec{F}$  is constant.

$\vec{F}(x,y) =$  \_\_\_\_\_

**Solution:** ( Instructor solution preview: show the student solution after due date. )

SOLUTION

$\vec{F}(x,y) = a\vec{i} + b\vec{j}$  for any real numbers  $a$  and  $b$  is a constant vector field. For example,  $\vec{F}(x,y) = 3\vec{i} - 4\vec{j}$ .

Correct Answers:

- $i + j$

20. (1 point) Library/Hope/Multi2/11-06-Scalar-line-integrals/Scalar-line-integral-04.pg

Find the line integral with respect to arc length  $\int_C (6x + 4y) ds$ , where  $C$  is the line segment in the  $xy$ -plane with endpoints  $P = (3,0)$  and  $Q = (0,8)$ .

(a) Find a vector parametric equation  $\vec{r}(t)$  for the line segment  $C$  so that points  $P$  and  $Q$  correspond to  $t = 0$  and  $t = 1$ , respectively.

$\vec{r}(t) =$  \_\_\_\_\_

(b) Using the parametrization in part (a), the line integral with respect to arc length is

$$\int_C (6x + 4y) ds = \int_a^b \text{_____} dt$$

with limits of integration  $a = \text{_____}$  and  $b = \text{_____}$

(c) Evaluate the line integral with respect to arc length in part (b).

$$\int_C (6x + 4y) ds = \text{_____}$$

Correct Answers:

- $(3, 0) + t\langle -3, 8 \rangle$
- $[6*(3-3*t) + 4*8*t] * \text{sqrt}(3^2 + 8^2)$
- 0
- 1
- 213.6

21. (1 point) Library/ASU-topics/setCalculus/stef16\_2p1.pg

Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where  $\mathbf{F}(x, y, z) = 3x\mathbf{i} + 4y\mathbf{j} - 2z\mathbf{k}$  and  $C$  is given by the vector function  $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$ ,  $0 \leq t \leq 3\pi/2$ .

Correct Answers:

- -22.7066099024511

22. (1 point) Library/Hope/Multi2/11-06-Scalar-line-integrals/Scalar-line-integral-05.pg

If  $C$  is the part of the circle  $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$  in the first quadrant, find the following line integral with respect to arc length.

$$\int_C (5x - 6y) ds = \text{_____}$$

Correct Answers:

- $5*4^2 - 6*4^2$

23. (1 point) Library/AlfredUniv/anton8e/chapter13/13\_2/defineintegral.pg

Evaluate

$$\int_{-1}^3 (t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}) dt = \text{_____}$$

Correct Answers:

- $\langle 4, 9.33333, 20 \rangle$

24. (1 point) Library/UBC/calculusStewart/lineIntegrals21.pg  
Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F} = \langle -3\sin x, 2\cos y, xz \rangle$  and  $C$  is the path given by  $\mathbf{r}(t) = \langle -3t^3, 2t^2, -3t \rangle$  for  $0 \leq t \leq 1$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \text{_____}$$

**Solution:** ( Instructor solution preview: show the student solution after due date. )

The relevant vectors are:

$$\mathbf{F}(\mathbf{r}(t)) = \langle -3\sin(-3t^3), 2\cos(2t^2), 9t^4 \rangle$$

$$\mathbf{r}'(t) = \langle -9t^2, 4t, -3 \rangle$$

$$\begin{aligned} \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) &= \langle -3\sin(-3t^3), 2\cos(2t^2), 9t^4 \rangle \cdot \langle -9t^2, 4t, -3 \rangle \\ &= 27t^2 \sin(-3t^3) + 8t \cos(2t^2) - 27t^4 \end{aligned}$$

The line integral is then:

$$\begin{aligned} \int_C \mathbf{F} \cdot d\mathbf{r} &= \int_0^1 \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt = \int_0^1 27t^2 \sin(-3t^3) + 8t \cos(2t^2) - 27t^4 dt \\ &= \left[ 3\cos(-3t^3) + 2\sin(2t^2) + \frac{-27}{5}t^5 \right]_0^1 \\ &= 3\cos(-3) + 2\sin(2) + \frac{-42}{5} \end{aligned}$$

Correct Answers:

- -9.55138

25. (1 point) Library/ASU-topics/setCalculus/stef16\_3p1.pg

For each of the following vector fields  $\mathbf{F}$ , decide whether it is conservative or not by computing the appropriate first order partial derivatives. Type in a potential function  $f$  (that is,  $\nabla f = \mathbf{F}$ ) with  $f(0, 0) = 0$ . If it is not conservative, type N.

A.  $\mathbf{F}(x, y) = (-2x - 8y)\mathbf{i} + (-8x + 8y)\mathbf{j}$

$$f(x, y) = \text{_____}$$

B.  $\mathbf{F}(x, y) = -y\mathbf{i} + 0x\mathbf{j}$

$$f(x, y) = \text{_____}$$

C.  $\mathbf{F}(x, y) = (-\sin y)\mathbf{i} + (-16y - x\cos y)\mathbf{j}$

$$f(x, y) = \text{_____}$$

Note: Your answers should be either expressions of  $x$  and  $y$  (e.g. " $3xy + 2y$ "), or the letter "N"

Correct Answers:

- $4*y^2 - (x^2 + 8*x*y)$
- N
- $-[x*\sin(y) + 8*y^2]$

Show that  $\int_{(8,-7)}^{(9,-1)} f dx + g dy = \int_{(8,-7)}^{(9,-1)} (10xy^3 dx + 15y^2 x^2 dy)$  is independent of path:

$$\frac{\partial f}{\partial y} = \text{_____}$$

$$\frac{\partial g}{\partial x} = \text{_____}$$

$$\int_{(8,-7)}^{(9,-1)} (10xy^3 dx + 15y^2 x^2 dy) = \text{_____}$$

Correct Answers:

- $-(5*2*x*3*y^2)$
- $-(5*3*y^2*2*x)$
- 405-109760

27. (1 point) Library/272/setStewart16\_1/problem\_2.pg

Consider the vector field  $F(x, y, z) = (6y, 6x, -z)$ . Show that  $F$  is a gradient vector field  $F = \nabla V$  by determining the function  $V$  which satisfies  $V(0, 0, 0) = 0$ .

$V(x, y, z) =$  \_\_\_\_\_

Correct Answers:

- $6*x*y + -1*z*z/2$

Show that  $\int_{(-5,6,0)}^{(-2,9,3)} f dx + g dy + h dz = \int_{(-5,6,0)}^{(-2,9,3)} (3x^2y^3z^5 dx + 3y^2xz^3 dy + 5z^4y^3x^3 dz)$  is independent of path:

$\frac{\partial h}{\partial y} =$  \_\_\_\_\_

$\frac{\partial g}{\partial z} =$  \_\_\_\_\_

$\frac{\partial f}{\partial z} =$  \_\_\_\_\_

$\frac{\partial h}{\partial x} =$  \_\_\_\_\_

$\frac{\partial g}{\partial x} =$  \_\_\_\_\_

$\frac{\partial f}{\partial y} =$  \_\_\_\_\_

Therefore curl  $F = \int_{(-5,6,0)}^{(-2,9,3)} - (3x^2y^3z^5 dx + 3y^2xz^3 dy + 5z^4y^3x^3 dz) =$  \_\_\_\_\_

Correct Answers:

- $-(5*z^4*3*y^2*x^3)$
- $-(3*y^2*x^3*5*z^4)$
- $-(3*x^2*y^3*5*z^4)$
- $-(5*z^4*y^3*x^2)$
- $-(3*y^2*x^3*2*z^5)$
- $-(3*x^2*3*y^2*z^5)$
- $\langle 0, 0, 0 \rangle$
- $(1.41718E+06) - 0$

29. (1 point) Library/AlfredUniv/anton8e/chapter16/16\_3/prob8.pg

Let  $F = 10xyi + 3y^2j$  be a vector field in the plane, and  $C$  the path  $y = 3x^2$  joining  $(0,0)$  to  $(1,3)$  in the plane.

A. Evaluate  $\int_C F \cdot dr$  \_\_\_\_\_

B. Does the integral in part (A) depend on the path joining  $(0,0)$  to  $(1,3)$ ? \_\_\_\_\_ (y/n)

Correct Answers:

- 34.5
- y

30. (1 point) Library/Michigan/Chap18Sec3/Q23.pg

For each of the following decide whether the vector field could be a gradient vector field. Be sure that you can justify your answer.

(a)  $\vec{F}(x, y, z) = \frac{2z}{\sqrt{x^2+z^2}} \vec{i} + \frac{2y}{\sqrt{x^2+z^2}} \vec{j} + \frac{2x}{\sqrt{x^2+z^2}} \vec{k}$

$\vec{F}(x, y, z)$  is

- ?
- a gradient vector field
- not a gradient vector field

(b)  $\vec{F}(x, y, z) = 4x\vec{i} + 4y\vec{j} + 4z\vec{k}$

$\vec{F}(x, y, z)$  is

- ?
- a gradient vector field
- not a gradient vector field

(c)  $\vec{F}(x, y, z) = x\sqrt{3x^2+y^2+z^2}\vec{i} + y\sqrt{3x^2+y^2+z^2}\vec{j} + z\sqrt{3x^2+y^2+z^2}\vec{k}$

$\vec{F}(x, y, z)$  is

- ?
- a gradient vector field
- not a gradient vector field

(d)  $\vec{F}(x, y, z) = 4x\vec{i} + 4z\vec{j}$

$\vec{F}(x, y, z)$  is

- ?
- a gradient vector field
- not a gradient vector field

Solution: ( Instructor solution preview: show the student solution after due date. )

SOLUTION

(a)  $\vec{F}$  is not a gradient vector field. If  $\vec{F}$  were a gradient vector field, then we would have  $\frac{\partial f}{\partial x} = \frac{2z}{\sqrt{x^2+z^2}}$ , so that  $\frac{\partial^2 f}{\partial x \partial y} = 0$ . But we also have that  $\frac{\partial f}{\partial y} = \frac{2y}{\sqrt{x^2+z^2}}$ , so that  $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{-2xy}{(x^2+z^2)^{3/2}}$ . These two are not equal, which is a contradiction. Therefore  $\vec{F}$  cannot be a gradient vector field.

(b)  $\vec{F}$  is a gradient vector field. By integrating the components of the vector field  $\vec{F}$ , we can see that  $f(x, y, z) = 2x^2 + 2y^2 + 2z^2$  is a potential function for  $\vec{F}$ .

(c)  $\vec{F}$  is not a gradient vector field. If  $\vec{F}$  were a gradient vector field, then we would have  $\frac{\partial f}{\partial x} = x\sqrt{3x^2+y^2+z^2}$ , so that  $\frac{\partial^2 f}{\partial x \partial y} = xy(3x^2+y^2+z^2)^{-1/2}$ . But we also have that  $\frac{\partial f}{\partial y} = y\sqrt{3x^2+y^2+z^2}$ , so that  $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 3xy(3x^2+y^2+z^2)^{-1/2}$ . These two are not equal, which is a contradiction. Therefore  $\vec{F}$  cannot be a gradient vector field.

(d)  $\vec{F}$  is not a gradient vector field. If  $\vec{F}$  were a gradient vector field, then we would have  $\frac{\partial f}{\partial y} = 4z$ , so that  $\frac{\partial^2 f}{\partial y \partial z} = 4$ . But we also have that  $\frac{\partial f}{\partial z} = 0$ , so that  $\frac{\partial^2 f}{\partial z \partial y} = \frac{\partial^2 f}{\partial y \partial z} = 0$ . These two are not equal, which is a contradiction. Therefore  $\vec{F}$  cannot be a gradient vector field.

Correct Answers:

- not a gradient vector field
- a gradient vector field
- not a gradient vector field
- not a gradient vector field



1. (1 point) Library/Michigan/Chap18Sec4/Q23.pg

Let  $\vec{F} = 14xe^y\vec{i} + 7x^2e^y\vec{j}$  and  $\vec{G} = 14(x-y)\vec{i} + 7(x+y)\vec{j}$ . Let  $C$  be the path consisting of lines from  $(0,0)$  to  $(3,0)$  to  $(3,2)$  to  $(0,0)$ . Find each of the following integrals exactly:

- (a)  $\int_C \vec{F} \cdot d\vec{r} =$  \_\_\_\_\_  
 (b)  $\int_C \vec{G} \cdot d\vec{r} =$  \_\_\_\_\_

2. (1 point) Library/AlfredUniv/anton8e/chapter16/16\_4/prob9.p

g Find the work done by the vector field  $\langle 4x + yx, x^2 + 6 \rangle$  on a particle moving along the boundary of the rectangle  $0 \leq x \leq 2, 0 \leq y \leq 2$  in the counterclockwise direction.

(The force is measured in newtons, length in meters, work in joules=(newton-meters).)

W = \_\_\_\_\_ joules

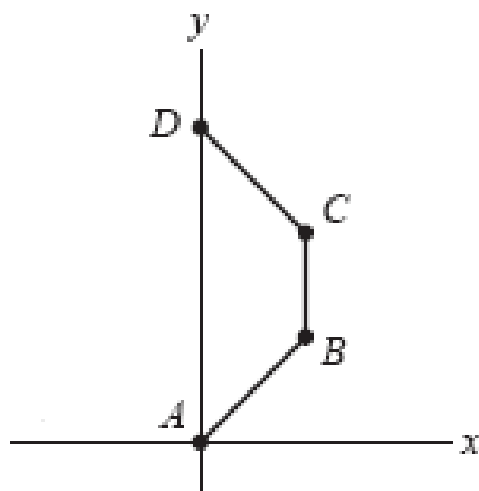
3. (1 point) Library/Michigan/Chap18Sec4/Q20.pg

Let  $\vec{F} = 3(x+y)\vec{i} + \sin(y)\vec{j}$ . Find the line integral of  $\vec{F}$  around the perimeter of the rectangle with corners  $(5,0), (5,3), (-1,3), (-1,0)$ , traversed in that order.

line integral = \_\_\_\_\_

4. (1 point) Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/17\_Fundamental\_Theorems\_of\_Vector\_Analysis/17.1\_Greens\_Theorem/17.1.13.pg

Evaluate  $I = \int_C (\sin x + 2y) dx + (3x + y) dy$  for the nonclosed path  $ABCD$  in the figure.



$A = (0,0), B = (3,3), C = (3,6), D = (0,9)$   
 $I =$  \_\_\_\_\_

5. (1 point) Library/Rochester/setVectorCalculus2/ur\_vc\_12\_6.p

g Let  $C$  be the positively oriented circle  $x^2 + y^2 = 1$ . Use Green's Theorem to evaluate the line integral  $\int_C 3y dx + 14x dy$ .

6. (1 point) Library/FortLewis/Calc3/18-2-Line-integrals-parametrized/HGM4-18-2-08b-Line-integrals-parametrized.pg

Suppose  $\vec{F}(x,y) = \langle 2y, -\sin(y) \rangle$ . Find the circulation of  $\vec{F}$  around a circle of radius 3 centered at the origin and oriented counterclockwise.

Circulation = \_\_\_\_\_

7. (1 point) Library/Dartmouth/setMTWCh4S4/problem\_5.pg

Let  $F(x,y,z) = (-7xz^2, -8xyz, -xy^3z)$  be a vector field and  $f(x,y,z) = x^3y^2z$ .

- $\nabla f = ($  \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_).  
 $\nabla \times F = ($  \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_).  
 $F \times \nabla f = ($  \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_).  
 $F \cdot \nabla f =$  \_\_\_\_\_.

8. (1 point) Library/FortLewis/Calc3/20-3-Curl/HGM4-20-3-08-Curl.pg

Compute the curl of the vector field  $\vec{F} = \langle x^2, y^2, z^3 \rangle$ .

$\text{curl}(\vec{F}(x,y,z)) =$  \_\_\_\_\_

What is the curl at the point  $(5,4,-3)$ ?

$\text{curl}(\vec{F}(5,4,-3)) =$  \_\_\_\_\_

Is this vector field irrotational or not?

- Choose
- irrotational
- not irrotational
- cannot be determined

9. (1 point) Library/FortLewis/Calc3/20-1-Idea-of-divergence/HGM4-20-1-22-Divergence.pg

Let  $f(x,y) = axy + ax^2y + y^3$ .

(a)  $\text{div}(\text{grad}(f)) =$  \_\_\_\_\_

(b) Find  $a$  so that  $\text{div}(\text{grad}(f)) = 0$  for all  $x,y$ .

$a =$  \_\_\_\_\_

10. (1 point) Library/272/setStewart16\_5/ur\_vc\_12\_9.pg

I.

Let  $\mathbf{F} = 3xi + 3yj + 3zk$ . Compute the divergence and the curl.

- A.  $\text{div } \mathbf{F} =$  \_\_\_\_\_  
 B.  $\text{curl } \mathbf{F} =$  \_\_\_\_\_  $\mathbf{i} +$  \_\_\_\_\_  $\mathbf{j} +$  \_\_\_\_\_  $\mathbf{k}$

II.

Let  $\mathbf{F} = (2xy, 8y, 10z)$ .

The curl of  $\mathbf{F} = (\underline{\hspace{2cm}}, \underline{\hspace{2cm}}, \underline{\hspace{2cm}})$ .

Is there a function  $f$  such that  $\mathbf{F} = \nabla f$ ?        (yes/no)

**11. (1 point)** Library/Hope/Multi2/11-07-Scalar-surface-integrals/Surface-integral-06.pg

Evaluate the surface integral  $\iint_H 6y \, dA$  where  $H$  is the helicoid (i.e., spiral ramp) given by the vector parametric equation

$$\vec{r}(u, v) = \langle u \cos v, u \sin v, v \rangle, \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 7\pi.$$

$$\iint_H 6y \, dA = \underline{\hspace{2cm}}$$

Find the surface integral  $\iint_{\sigma} f(x, y, z) \, dS$  where  $f = (x^2 + y^2)z$

and  $\sigma$  is the sphere  $x^2 + y^2 + z^2 = 4$  above  $z = 1$ .

Parameterize the surface integral

$$\begin{aligned} \iint_{\sigma} f(x, y, z) \, dS &= \iint_R f(x(\theta, \phi), y(\theta, \phi), z(\theta, \phi)) \left\| \frac{\partial \mathbf{r}}{\partial \theta} \times \frac{\partial \mathbf{r}}{\partial \phi} \right\| \, dA \\ &= \int \int \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}} \left\| \underline{\hspace{1cm}} \right\| \, d\theta \, d\phi \end{aligned}$$

**Note:** For  $\theta$  type theta and for  $\phi$  type phi.

**13. (1 point)** Library/Rochester/setVectorCalculus3/ur\_vc\_13\_2.pg

Find the surface area of the part of the sphere  $x^2 + y^2 + z^2 = 4$  that lies above the cone  $z = \sqrt{x^2 + y^2}$

**14. (1 point)** Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/16\_Line\_and\_Surface\_Integrals/16.4\_Parametrized\_Surfaces\_and\_Surface\_Integrals/16.4.19.pg

Calculate  $\iint_S f(x, y, z) \, dS$  For

$$x^2 + y^2 = 4, \quad 0 \leq z \leq 7; \quad f(x, y, z) = e^{-z}$$

$$\iint_S f(x, y, z) \, dS = \underline{\hspace{2cm}}$$

**15. (1 point)** Library/FortLewis/Calc3/19-1-Idea-of-flux/HGM4-19-1-12-Idea-of-flux.pg

(a) Calculate the flux of the vector field  $\vec{F}(x, y, z) = 6\vec{i} - 6\vec{k}$  through a sphere of radius 4 centered at the origin, oriented outward.

Flux =                     

(b) Calculate the flux of the vector field  $\vec{F}(x, y, z) = \vec{i} - 8\vec{j} + 5\vec{k}$  through a cube of side length 4 with sides parallel to the axes, oriented outward.

Flux =                     

**16. (1 point)** Library/FortLewis/Calc3/19-1-Idea-of-flux/HGM4-19-1-21-Idea-of-flux.pg

Calculate the flux of the vector field  $\vec{F}(x, y, z) = 3\vec{i} - 2\vec{j} + 5\vec{k}$  through a square of side length 5 lying in the plane  $4x + 4y + 2z = 1$ , oriented away from the origin.

Flux =                     

**17. (1 point)** Library/FortLewis/Calc3/19-2-Flux-integrals/HGM4-19-2-09-Flux-integrals.pg

(a) Set up a double integral for calculating the flux of the vector field  $\vec{F}(x, y, z) = -2xz\vec{i} - 2yz\vec{j} + z^2\vec{k}$  through the part of the cone  $z = \sqrt{x^2 + y^2}$  for  $0 \leq z \leq 4$ , oriented upward.

$$\text{Flux} = \iint_{\text{Disk}} \underline{\hspace{2cm}} \, dx \, dy$$

(b) Evaluate the integral.

$$\text{Flux} = \iint_S \vec{F} \cdot d\vec{A} = \underline{\hspace{2cm}}$$

**18. (1 point)** Library/FortLewis/Calc3/19-2-Flux-integrals/HGM4-19-2-03-Flux-integrals.pg

(a) Set up a double integral for calculating the flux of the vector field  $\vec{F}(\vec{r}) = \vec{r}$ , where  $\vec{r} = \langle x, y, z \rangle$ , through the part of the upward oriented surface  $z = 3(x^2 + y^2)$  that lies above the disk  $x^2 + y^2 \leq 4$ .

$$\text{Flux} = \iint_{\text{Disk}} \underline{\hspace{2cm}} \, dx \, dy$$

(b) Evaluate the integral.

$$\text{Flux} = \iint_S \vec{F} \cdot d\vec{A} = \underline{\hspace{2cm}}$$

**19. (1 point)** Library/Michigan/Chap19Sec2/Q05.pg

Compute the flux of the vector field  $\vec{F}(x, y, z) = x\vec{j} + 4y\vec{k}$  through the surface  $S$ , where  $S$  is the part of the surface  $z = -y + 5$  above the rectangle  $0 \leq x \leq 2, 0 \leq y \leq 2$ , oriented upward.

flux =                     

**20. (1 point)** Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/16\_Line\_and\_Surface\_Integrals/16.5\_Surface\_Integrals\_of\_Vector\_Fields/16.5.7.pg

Compute the surface integral over the given oriented surface:

$\mathbf{F} = \langle 0, 3, x^2 \rangle$ , hemisphere  $x^2 + y^2 + z^2 = 64, z \geq 0$ , outward-pointing normal

Answer:                     

**21. (1 point)** Library/Michigan/Chap20Sec4/Q13.pg

Find  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is a circle of radius 1 in the plane  $x + y + z = 2$ , centered at  $(2, 4, -4)$  and oriented clockwise when viewed from the origin, if  $\vec{F} = 5y\vec{i} - x\vec{j} + 3(y - x)\vec{k}$

$$\int_C \vec{F} \cdot d\vec{r} = \underline{\hspace{2cm}}$$

22. (1 point) Library/Rochester/setVectorCalculus3/ur\_vc\_13\_8.

pg

Use Stoke's Theorem to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + 3(x^2 + y^2)\mathbf{k}$  and  $C$  is the boundary of the part of the paraboloid where  $z = 64 - x^2 - y^2$  which lies above the  $xy$ -plane and  $C$  is oriented counterclockwise when viewed from above.

23. (1 point) Library/Rochester/setVectorCalculus3/ur\_vc\_13\_7.

pg

Use Stokes' theorem to evaluate  $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$  where  $\mathbf{F}(x, y, z) = -3yz\mathbf{i} + 3xz\mathbf{j} + 2(x^2 + y^2)z\mathbf{k}$  and  $S$  is the part of the paraboloid  $z = x^2 + y^2$  that lies inside the cylinder  $x^2 + y^2 = 1$ , oriented upward.

24. (1 point) Library/FortLewis/Calc3/20-2-Divergence-theorem/HGM5-20-2-15-Divergence-theorem.pg

Use the divergence theorem to calculate the flux of the vector field  $\vec{F}(x, y, z) = -4xy\vec{i} - 4yz\vec{j} + 5xz\vec{k}$  through the sphere  $S$  of radius 5 centered at the origin and oriented outward.

$$\iint_S \vec{F} \cdot d\vec{A} = \underline{\hspace{2cm}}$$

25. (1 point) Library/FortLewis/Calc3/20-2-Divergence-theorem/HGM5-20-2-16-Divergence-theorem.pg

Use the divergence theorem to calculate the flux of the vector field  $\vec{F}(x, y, z) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$  out of the closed, outward-oriented surface  $S$  bounding the solid  $x^2 + y^2 \leq 16$ ,  $0 \leq z \leq 6$ .

$$\iint_S \vec{F} \cdot d\vec{A} = \underline{\hspace{2cm}}$$

26. (1 point) Library/UBC/calculusStewart/divergence10.pg

Use the Divergence Theorem to calculate the flux of  $\mathbf{F}$  across  $S$ , where  $\mathbf{F} = zi + yj + z\mathbf{k}$  and  $S$  is the surface of the tetrahedron

enclosed by the coordinate planes and the plane

$$\frac{x}{2} + \frac{y}{5} + z = 1$$

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \underline{\hspace{2cm}}$$

27. (1 point) Library/Michigan/Chap20Sec2/Q17.pg

The region  $W$  lies between the spheres  $x^2 + y^2 + z^2 = 9$  and  $x^2 + y^2 + z^2 = 16$  and within the cone  $z = \sqrt{x^2 + y^2}$  with  $z \geq 0$ ; its boundary is the closed surface,  $S$ , oriented outward. Find the flux of  $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$  out of  $S$ .

$$\text{flux} = \underline{\hspace{2cm}}$$

28. (1 point) Library/WHFreeman/Rogawski\_Calculus\_Early\_Transcendentals\_Second\_Edition/17\_Fundamental\_Theorems\_of\_Vector\_Analysis/17.3\_Divergence\_Theorem/17.3.9.pg

Verify the Divergence Theorem for the vector field and region:

$\mathbf{F} = \langle 7x, 9z, 3y \rangle$  and the region  $x^2 + y^2 \leq 1$ ,  $0 \leq z \leq 2$

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \underline{\hspace{2cm}}$$

$$\iiint_{\mathcal{R}} \text{div}(\mathbf{F}) dV = \underline{\hspace{2cm}}$$

29. (1 point) Library/Rochester/setVectorCalculus3/ur\_vc\_13\_9.

pg

Use the divergence theorem to find the outward flux of the vector field  $\mathbf{F}(x, y, z) = 3x^2\mathbf{i} + 3y^2\mathbf{j} + 4z^2\mathbf{k}$  across the boundary of the rectangular prism:  $0 \leq x \leq 3$ ,  $0 \leq y \leq 4$ ,  $0 \leq z \leq 2$ .

30. (1 point) Library/Rochester/setVectorCalculus2/ur\_vc\_12\_13

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Let  $\mathbf{F} = 1x\mathbf{i} + 4y\mathbf{j}$  and let  $\mathbf{n}$  be the outward unit normal vector to the positively oriented circle  $x^2 + y^2 = 25$ . Compute the flux integral  $\int_C \mathbf{F} \cdot \mathbf{n} ds$ .

$$\underline{\hspace{2cm}}$$