GROUP #1

1) (a) $x^3 + x^2y + 4y^2 = 6$

$3x^2 + (x^2 \cdot \frac{dy}{dx} + y \cdot 2x) + 8y \cdot \frac{dy}{dx} = 0$

$(x^2 + 8y) \cdot \frac{dy}{dx} = -3x^2 - 2xy$

$\boxed{\frac{dy}{dx} = \frac{-3x^2 - 2xy}{x^2 + 8y}}$

(b) $4 \cos x \cdot \sin y = 1$

$4 \cos x (\cos y \cdot \frac{dy}{dx}) + (-4 \sin x)(\sin y) = 0$

$4 \cos x (\cos y \cdot \frac{dy}{dx}) = 4 \sin x \sin y$

$\boxed{\frac{dy}{dx} = \frac{4 \sin x \sin y}{4 \cos x \cos y}}$

(c) $xy = \cot(x \cdot y)$

$x \cdot \frac{dy}{dx} + y = -\csc^2(x \cdot y) \cdot (x \cdot \frac{dy}{dx} + 1)$

$x \cdot \frac{dy}{dx} + y = -(\frac{dy}{dx} x + y) \csc^2(xy)$

$\Rightarrow x \frac{dy}{dx} + y = -(\frac{dy}{dx} + y) \csc^2(xy)$

$\csc^2 \frac{dy}{dx} (xy)$

$\left(\frac{\cancel{\csc^2 + x} + x}{\csc^2 + \cancel{x}}\right) \frac{dy}{dx} + (-y + y) - \csc(xy)$

$\csc^2 + x + x$

$\Rightarrow - \frac{\cancel{\csc^2}(xy)}{\cancel{\csc^2}(\cancel{y} + x)}$

$\Rightarrow \boxed{- \frac{y}{x} = \frac{dy}{dx}}$

#2

2. $x^2/9 + y^2/36 = 1$

$\frac{d}{dx}\left(x^2/9 + y^2/36\right) = \frac{d}{dx}(1)$

$\frac{2}{9}x + \frac{2y \cdot dy/dx}{36} = 0$

$\frac{2}{9}x + \frac{y \cdot dy/dx}{18} = 0$

$\frac{y \cdot dy/dx + 4x}{18} = 0$

$y \cdot dy/dx = -4x$

$dy/dx = -\frac{4x}{y} \rightarrow -4(-1)/4\sqrt{2} = 4/4\sqrt{2} = 1/\sqrt{2} = \sqrt{2}/2$

$y - (4\sqrt{2}) = \sqrt{2}/2\,(x - (-1))$

$y - 4\sqrt{2} = x\sqrt{2}/2 + \sqrt{2}/2$

$y = x\sqrt{2}/2 + 9\sqrt{2}/2$

7. $y = \sqrt{x}(e^{x^2})(x^2+1)^{10}/\cos x$ $\qquad$ $d/dx(e^{x^2}) = 2xe^{x^2}$

$\ln y = \ln\left(\sqrt{x}(e^{x^2})(x^2+1)^{10}/\cos x\right)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(\cos x)/\frac{d}{dx}\left(\sqrt{x}(e^{x^2})(x^2+1)^{10}\right)$

$dy/dx = y\left(-\sin x/\frac{d}{dx}(\sqrt{x})(e^{x^2})(x^2+1)^{10} + \frac{d}{dx}(e^{x^2})(\sqrt{x})(x^2+1)^{10} + \frac{d}{dx}((x^2+1)^{10})(\sqrt{x})(e^{x^2})\right)$

$dy/dx = y\left(-\sin x/\tfrac{1}{2}x^{-1/2}e^{x^2}(x^2+1)^{10} + 2xe^{x^2}\sqrt{x}(x^2+1)^{10} + 10(x^2+1)^9 2x\sqrt{x}e^{x^2}\right)$

$dy/dx = \left(\sqrt{x}(e^{x^2})(x^2+1)^{10}/\cos x\right)\left(-\sin x/x^{-1/2}/2\, e^{x^2}(x^2+1)^{10} + 2xe^{x^2}\sqrt{x}(x^2+1)^{10} + 10(x^2+1)^9 2x\sqrt{x}e^{x^2}\right)$

Group 3 Casey Nick matt Aaron

3a. $\frac{d}{dx}\left[\sin^{-1}(x)\right] \stackrel{?}{=} \frac{1}{\sqrt{1-x^2}}$

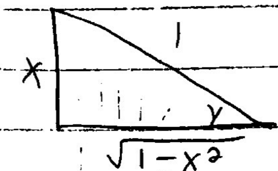
$a^2+b^2=c^2$

$a^2+x^2=1^2$

$a^2=1-x^2$

$a=\sqrt{1-x^2}$

$y=\sin^{-1}(x)$



$x=\sin(y)$

$1=\cos y \frac{dy}{dx}$

$\frac{dy}{dx}=\frac{1}{\cos y}$

$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$

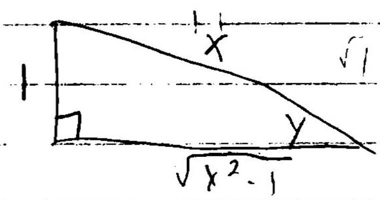
3d. $\frac{d}{dy}\left[\csc^{-1}(x)\right] \stackrel{?}{=} \frac{-1}{x\sqrt{x^2-1}} = \frac{1}{\sin^{-1}(x)}$

$y=\csc^{-1}(x)$

$x=\csc y=\frac{1}{\sin y}$

$1=\csc^2 y \cot y \frac{dy}{dx}$

$\frac{dy}{dx}=-\frac{1}{\csc y \cot y}$



$a^2+b^2=c^2$

$a^2+1^2=x^2$

$a^2=x^2-1$

$=\sqrt{x^2-1}$

$\frac{dy}{dx}=-\frac{1}{x\left(\sqrt{x^2-1}\right)}$

$\cot y=\frac{\cos}{\sin}=\frac{\frac{\sqrt{x^2-1}}{x}}{1/x}$

$\csc x=\frac{1}{\sin x} \rightarrow \frac{\sin x\cdot 0-1\cdot\cos x}{\sin^2 x}$

$\frac{d}{dx}=-\csc x\cot x$

Group 3

b) $\frac{d}{dx}[\cos^{-1}(x)] = \frac{1}{\sqrt{1-x^2}}$

$\cos y = x$

$-\sin y \cdot \frac{dy}{dx} = 1$

$a^2 + b^2 = c^2$

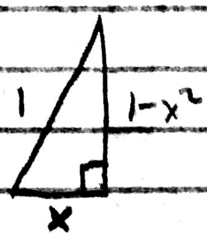
$x^2 + b^2 = 1^2$

$x^2 + b^2 = 1$

$b^2 = 1 - x^2$

$\sqrt{b^2} = \sqrt{1-x^2}$

$b = \sqrt{1-x^2}$



Group 3 Casey Matt
Nick
Aaron

3. ~~c)~~ $\cancel{\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}}$ ~~Know: tan~~

$\cancel{y = \tan^{-1}(x)}$

$\cancel{\tan(y)\frac{dy}{dx} = x = 1}$

$\cancel{\tan(y)\frac{dy}{dx} = 1}$

$\cancel{\frac{dy}{dx} = \frac{1}{\tan(y)}}$

c) $\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}$ Know: $\tan(y) = \frac{x}{1}$ $\frac{opp}{adj}$

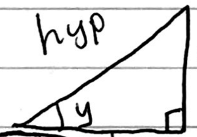
Want: $\sec^2(y) = \left(\frac{hyp}{adj}\right)^2$

$y = \tan^{-1}(x)$

$\tan(y) = x$

$\sec^2(y)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2(y)} = \frac{1}{\left(\frac{\sqrt{x^2+1}}{1}\right)^2} = \frac{1}{x^2+1} = \boxed{\frac{1}{1+x^2}}$



$a^2 + b^2 = c^2$

$x^2 + 1^2 = c^2$

$x^2 + 1 = c^2$

$\sqrt{x^2+1} = c$

f) $\frac{d}{dx}\left[\cot^{-1}(x)\right] = -\frac{1}{1+x^2}$ Know: $\cot(y) = \frac{x}{1}$ $\frac{adj}{opp}$

Want: $-\csc^2(y)$

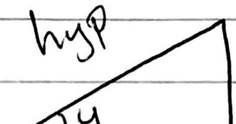
$-\left(\frac{hyp}{opp}\right)^2 = -\left(\frac{\sqrt{1+x^2}}{1}\right)^2 =$ [illegible]

$y = \cot^{-1}(x)$

$\cot(y) = x$

$-\csc^2(y)\frac{dy}{dx} = \cancel{3}\,1$

$\frac{dy}{dx} = \frac{1}{-\csc^2(y)} = \boxed{-\frac{1}{1+x^2}}$



$a^2 + b^2 = c^2$

$1^2 + x^2 = c^2$

$1 + x^2 = c^2$

$\sqrt{1+x^2} = c$

group 4

4) A) $ACOS(x^2) \rightarrow$ chain rule $\overset{f'(g)\cdot g'}{}$ $\dfrac{-1}{\sqrt{1-(x^2)^2+1}} \cdot 2X$

$\frac{d}{dx}(A\cos) = \frac{-1}{\sqrt{1-x}}$ $\frac{d}{dx}(x^2) = 2x$

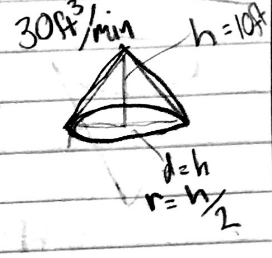
P) $X^2 \cdot ACot(3x) \rightarrow$ chain rule $\overset{f'(g)\cdot g'}{}$ $\rightarrow$ product rule $\overset{f'\cdot g + f\cdot g'}{}$ $\rightarrow ACot(3x)\cdot 2x + \frac{1}{3x^2+1}\cdot 3 \cdot x^2$

$\frac{d}{dx}(ACot) = \frac{1}{x^2+1}$ $\frac{d}{dx}(3x) = 3$

Group 5

5) $\frac{d}{dx}[e^x] = e^x$ to prove $\frac{d}{dx}[\ln x] = \frac{1}{x}$ if $x > 0$

$$y = \ln x$$
$$e^y = e^{\ln x}$$
$$e^y = x$$
$$\frac{d}{dx}[e^y] = \frac{d}{dx}[x]$$
$$e^y \cdot \frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{e^y} = \boxed{\frac{1}{x}}$$



10.) $V = \pi\left(\frac{h}{2}\right)^2 \frac{h}{3}$

$$V = \pi \frac{h^2}{4} \cdot \frac{h}{3}$$
$$V = \frac{\pi}{12} \cdot h^3$$
$$\frac{dV}{dt} = \frac{\pi}{12}(3h^2) \cdot \frac{dh}{dt}$$
$$30\frac{ft^3}{min} = \frac{\pi}{12}(300) \cdot \frac{dh}{dt}$$
$$\frac{1}{10} = \frac{\pi}{\cancel{12}} \cdot \frac{dh}{dt}$$
$$10 / \frac{\pi}{12} = \frac{dh}{dt}$$
$$\boxed{\frac{12}{10\pi} = \frac{dh}{dt}}$$

Group #6 Colleen Rayburn

6. find $\frac{dy}{dx}$ for

$\boxed{\sqrt{x} = x^{\frac{1}{2}}}$

a.) $y = \ln\sqrt{x}$ $\rightarrow$ $\boxed{\text{Rule: } \frac{d}{dx}[\ln x] = \frac{1}{x} \text{ if } x>0}$

chain rule $y = \ln\sqrt{x} \xrightarrow{\text{apply Rule}} \frac{1}{\sqrt{x}} \cdot \frac{d}{dx} = \frac{x^{-\frac{1}{2}}}{2} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{2x}$

b.) $y = x\log_{10}x$ $\boxed{\text{Rule: } \frac{d}{dx}[\log_b x] = \frac{1}{x\ln b} \quad x>0}$

$y = \underset{u}{x}\,\underset{v}{\log_{10}x}$

$u \cdot v' + u' \cdot v$

$u' = 1 \quad v' = \frac{1}{x\ln 10} = x\left[\frac{1}{x\ln 10}\right] + 1 \cdot \log_{10}x =$

$\frac{\cancel{x}}{\cancel{x}\ln 10} + \log_{10}x$

$= \frac{1}{\ln 10} + \log_{10}x$

c.) $y = 10(2^x)$ $\boxed{\text{Rule: } \frac{d}{dx}[b^x] = b^x\ln b \text{ if } b>0 \text{ \& } b\neq 1}$

$10 \cdot \frac{d}{dx}2^x$

$\boxed{u \cdot v' + u' \cdot v}$

$10(2^x\ln 2)$

$u = 10 \quad v = 2^x$

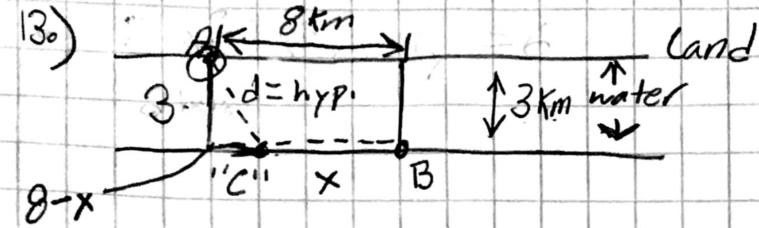
$u' = 0 \quad v' = 2^x\ln 2 = (10)\cdot(2^x\ln 2) + (0)(2^x)$

$= (10)\cdot(2^x\ln 2)$

Exam #3 Review

Group #1

13a)



Known

rowing rate $\frac{dw}{dt} = 6 \frac{Km}{hr}$

running rate $\frac{dL}{dt} = 8 \frac{Km}{hr}$

rate = $\frac{dist}{time}$

want

"C" Landing point

rate = $AC \frac{Km}{hr} + \frac{CB}{hr}$

$\frac{\sqrt{(8-.485)^2+9}}{6} \frac{Km}{hr} + \frac{.485}{8} \frac{Km}{hr}$

Answer $9.41 \frac{Km}{hr} + .06 \frac{Km}{hr} = 9.47 \frac{Km}{hr}$

Quickest time is $9.47 \frac{Km}{hr}$ @ Landing spot .485

$\frac{\sqrt{(8-x)^2+9} = d}{6\ Km/hr} + \frac{x}{8\ Km/hr}$

$\frac{\sqrt{(8-x)(8-x)+9}}{6\ Km/hr} - \frac{x}{8\ Km/hr} = d$

$\frac{\sqrt{64-8x-8x-x^2}\ \cancel{Km}}{6\ \cancel{Km}/hr} - \frac{x\ \cancel{Km}}{8\ \cancel{Km}/hr} = d$

$\frac{1}{6}hr \cdot (64-16x-x^2)^{1/2} - \frac{1}{8}hr \cdot x = d$

$\frac{1}{6}hr \cdot \frac{1}{2}(64-16x-x^2)^{-1/2} \cdot (-16-2x) - \frac{1}{8}hr = 0$

$\frac{1}{6}hr \cdot \frac{\frac{1}{2}(-16-2x)}{(64-16x-x^2)} - \frac{1}{8}hr = 0$

$\frac{1}{6}hr \cdot \frac{-8-1x}{64-16x-x^2} - \frac{1}{8}hr = 0$

Can't be negitive

Undefined @ $x = \cancel{-16.48}$

$x = .485$

$f' = 0$ @ $\cancel{x=-8}$

$1 \cdot -8 - 1x - 1 = 0$

$1 \cdot -8 - x = 1$

$-8 - x = 0$

$-x = 8$

$x = -8$

or

$f' =$ und

$g' = 0$

$-x^2 - 16x + 64 = 0$

$\frac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2(a)}$

$\frac{-(-16) \pm \sqrt{-16^2 - 4(-1)(64)}}{2(-1)}$

$\frac{16 \pm \sqrt{32 + 256}}{-2}$

$\frac{16 \pm 16.97}{-2}$

$x = -16.48 \quad x = .485$

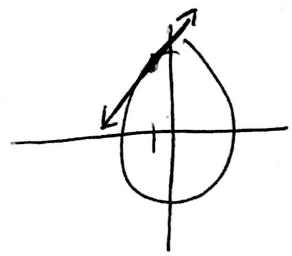
Answers to Practice Problems 3

1. a. $\frac{dy}{dx} = \frac{-(3x^2+2xy)}{(x^2+8y)}$

1. b. $\frac{dy}{dx} = \tan x \tan y$

1. c. $\frac{dy}{dx} = -\frac{y}{x}$

2. Equation: $(y-4\sqrt{2}) = \frac{1}{4\sqrt{2}}(x+1)$

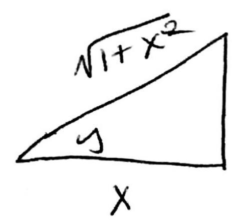


3. f) Let $y = \cot^{-1}(x)$

then $\cot(y) = x$

$\frac{d}{dx}[\cot(y)] = 1$

$-\csc^2(y) \cdot \frac{dy}{dx} = 1$



$\cot y = \frac{x}{1}$

$\frac{dy}{dx} = -\sin^2(y) = -\left(\frac{1}{\sqrt{1+x^2}}\right)^2$

$= -\left(\frac{1}{1+x^2}\right)$

4. a) $f'(x) = -\frac{2x}{\sqrt{1-x^4}}$

b) $g'(x) = \frac{3x^2}{1+9x^2} + 2x\cot^{-1}(3x)$

5. $\left.\begin{array}{l} y = \ln x \\ \frac{dy}{dx} = ? \end{array}\right\}$

$e^y = e^{\ln x}$

$e^y = x$

$\frac{d}{dx}[e^y] = \frac{d}{dx}[x]$

$e^y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$

6. a) $\frac{dy}{dx} = \frac{1}{2x}$

b) $\frac{dy}{dx} = \frac{1}{\ln 10} + \log_{10} x$

c) $\frac{dy}{dx} = 10 \cdot 2^x \ln(2)$.

7. $\frac{dy}{dx} = \left[\frac{1}{2x} + 2x + \frac{20x}{x^2+1} + \frac{\sin x}{\cos x}\right]\left[\frac{\sqrt{x}\,(e^{x^2})(x^2+1)^{10}}{\cos x}\right]$

8. $\frac{dy}{dx} = \left[\frac{\sin x}{x} + \cos x \ln y\right]\left[x^{\sin x}\right]$

9. $\frac{dD}{dt} = -\frac{1}{20\pi}$ cm/min

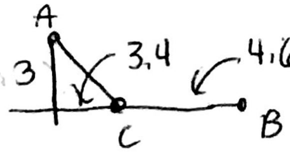
10. $\frac{dh}{dt} = \frac{6}{5\pi}$ ft/min

11.a. f(x) has a local min of -7 at x = 2, which is also it's global min. There is a global max of 5 at x = 0. f(x) is increasing ~~before~~ after 2, decreasing before 2. f(x) is always concave up.

11.b. Local max of -1.9 at x = -2, also absolute.
Absolute min of -2.05 at x = -5
increasing (-5, -2), decreasing (-2, -1)
concave down (-5, -1)

11.c. Absolute and local min of $-e^{-1}$ at $x = e^{-1}$
~~Decr~~ Decreasing $(0, e^{-1})$, increasing (e^{-1}, ∞)
concave up $(0, \infty)$.
no global (absolute) or local max.

12. Base is 40 cm by 40 cm, height is 80 cm.

13. 

Row across the river to a point 4.6 km west of B.