

Answers to Practice Problems for exam #2, MATH 251

1. Compute the derivative of each function.

a) $f(x) = 3x - 7$ 3

b) $y = 2x^5 - 4x^3 + 5x + 10$ $10x^4 - 12x^2 + 5$

c) $g(x) = 5e^x$ $5e^x$

d) $h(x) = \frac{x}{1+e^x}$

$[(1+e^x) - xe^x] / (1+e^x)^2$

e) $y = e^x(1 + \tan x)$

$e^x \sec^2 x + e^x(1 + \tan x) - \cos x / (2\sqrt{1 + \sin x})$

f) $f(x) = \sqrt{1 + \sin x}$

g) $P(x) = (e^x + x^2)^{100}$

$100(e^x + 2x)^{99} (e^x + 2)$

h) $g(x) = e^{-x^2/2}$

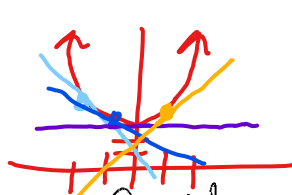
$(-x) e^{-x^2/2}$

i) $y = \sqrt[3]{1 - 2x^5}$

$-10x^4 / (3\sqrt[3]{(1 - 2x^5)^2})$

2. Let $f(x) = x^2 + 3$.

a) Graph $f(x)$



b) Find the equations of the lines tangent to $f(x)$ at $x = -2, x = -1, x = 0$ and $x = 1$.

c) Add the graphs of your tangent lines in b) to the graph from a)

3. Is there a function $f(x)$ so that $f(x) = 2f'(x)$? If so, give a function and prove it satisfies the equation. yes: $f(x) = e^{x/2}$

4. Let $f(x) = \sin(x)$. Find all values of n so that

$\frac{d^n}{dx^n} [\sin(x)] = \sin(x)$, that is, $f^{(n)}(x) = f(x)$.

$n = 4, 8, 12, 16, \dots$

5. Prove each of the following statements using only the facts that $\frac{d}{dx}[\sin x] = \cos x$ and $\frac{d}{dx}[\cos x] = -\sin x$ with rules for combining derivatives.

a) $\frac{d}{dx}[\tan x] = \sec^2 x$

b) $\frac{d}{dx}[\sec x] = \sec x \tan x$

c) $\frac{d}{dx}[\csc x] = -\csc x \cot x$

d) $\frac{d}{dx}[\cot x] = -\csc^2 x$

Use
 $\tan x = \frac{\sin x}{\cos x}$

$\sec x = \frac{1}{\cos x}$

$\csc x = \frac{1}{\sin x}$

$\cot x = \frac{\cos x}{\sin x}$

6. Take the derivative of both sides of the identity $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ to prove that the following identity is true: Use chain rule
 $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$

7. The height of a model rocket with an initial launch velocity of 120 ft/sec is given by $s(t) = -16t^2 + 120t + 0.5$ t seconds after launch.

a) What are the units of $s(t)$? $s'(t)$? $s''(t)$? $s^{(3)}(t)$?

b) What is the instantaneous velocity, 2 seconds after launch? 56 ft/s instantaneous acceleration? 32 ft/s^2

c) When is the rocket speeding up? slowing down?
always

8. Bill is on a trip. His distance from home as a function of time is below. Sketch graphs of his instantaneous velocity and acceleration.

