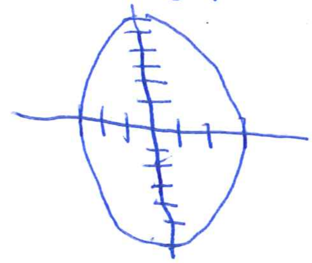
Practice Problems for Exam 3

1. Find $\frac{dy}{dx}$ by implicit differentiation:
   a) $x^3 + x^2 y + 4y^2 = 6$
   b) $4\cos x \cdot \sin y = 1$
   c) $xy = \cot(x \cdot y)$

2. The graph of $x^2/9 + y^2/36 = 1$ is an ellipse



Find the equation of the line tangent to the ellipse at $(-1, 4\sqrt{2})$ and add it to the graph.

3. Derive the following rules using implicit differentiation and right triangles:
   a) $\frac{d}{dx}[\sin^{-1}(x)] = \frac{1}{\sqrt{1-x^2}}$
   b) $\frac{d}{dx}[\cos^{-1}(x)] = -\frac{1}{\sqrt{1-x^2}}$
   c) $\frac{d}{dx}[\tan^{-1}(x)] = \frac{1}{1+x^2}$
   d) $\frac{d}{dx}[\csc^{-1}(x)] = \frac{-1}{x\sqrt{x^2-1}}$
   e) $\frac{d}{dx}[\sec^{-1}(x)] = \frac{1}{x\sqrt{x^2-1}}$
   f) $\frac{d}{dx}[\cot^{-1}(x)] = -\frac{1}{1+x^2}$

4. Use the rules in 3. to find the derivative of:
   a) $f(x) = \cos^{-1}(x^2)$
   b) $g(x) = x^2 \cot^{-1}(3x)$

5. Use implicit differentiation and the fact that $\frac{d}{dx}[e^x] = e^x$ to prove that $\frac{d}{dx}[\ln x] = \frac{1}{x}$ if $x > 0$

6. Find $\frac{dy}{dx}$ for a) $y = \ln\sqrt{x}$, b) $y = x\log_{10} x$, c) $y = 10 \cdot 2^x$

7. Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = \frac{\sqrt{x}(e^{x^2})(x^2+1)^{10}}{\cos x}$.

8. Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = x^{\sin x}$

9. If a snowball melts so that its surface area decreases at a rate of $1\ cm^3/min$, find the rate at which the diameter decreases when the diameter is 10 cm.

10. Gravel is being dumped at a rate of $30\ ft^3/min$ and it's coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always the same. How fast is the height of the pile increasing when the pile is 10 ft. high?

11. For each function, find both local and absolute max or min values (if they exist) on the interval. Also state the intervals where the function is increasing, decreasing, concave up or concave down.

    a) $f(x) = 3x^2 - 12x + 5$ over $[0,3]$

    b) $f(x) = x^{1/3} - x^{-2/3}$ over $[-5,-1]$

    c) $f(x) = x \ln x$ over $(0,\infty)$

12. A box with a square base and open top must have a volume of $32{,}000\ cm^3$. Find the dimensions that minimize the amount of material used.

13. Suppose a man can row at 6km/h and run at 8km/h. Where should he land to go from A to B as quickly as possible?