Math 251 - Practice Problems for Exam 1

Name: ______________________________

You should know at least the following formulas:

$x^a x^b = x^{a+b}$	$a^2 - b^2 = (a-b)(a+b)$	$v_{av} = \frac{s(t_1)-s(t_0)}{t_1-t_0}$
$x^{-a} = \frac{1}{x^a}$	$a^3 - b^3 = (a-b)(a^2+ab+b^2)$	$v_{inst} = \lim\limits_{t\to t_0} \dfrac{s(t)-s(t_0)}{t-t_0}$
$(x^a)^b = x^{ab}$	$a^3 + b^3 = (a+b)(a^2-ab+b^2)$	$\lim\limits_{x\to a} mx+b = ma+b$
$\dfrac{dy}{dx} = f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$		
If $g(x) \le f(x) \le h(x)$ for x near a and $\lim\limits_{x\to a} g(x) = \lim\limits_{x\to a} h(x) = L$ then $\lim\limits_{x\to a} f(x) = L$.		
If f is continuous in $[a,b]$ and L is such that $f(a) < L < f(b)$ then there exists a c in (a,b) so that $f(c) = L$.		

1. The position of a object at t seconds after takeoff is given by $s(t) = 3\sin(t)$. Find the average velocity from $t = 0$ to $t = 1.5$ seconds.

2. The height of a small pinecone t seconds after being thrown up is given by $s(t) = -16t^2 + 48t + 5$. Create and use a table with at least 3 average velocities to guess the instantaneous velocity of the pinecone at time $t = 2.5$ seconds.

3. Make a table with at least three points and use it to estimate the following limit:

$$\lim_{x\to 0^+} x^x$$

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4. Sketch the graph of a function $f(x)$ satisfying all of the following properties:
   - $\lim_{x\to\infty} f(x) = 4$
   - $\lim_{x\to 2^+} f(x) = -\infty$
   - $\lim_{x\to 2^-} f(x) = \infty$
   - $f(0) = 0$
   - $\lim_{x\to -3^+} f(x) = -\infty$
   - $\lim_{x\to -3^-} f(x) = -\infty$

5. Compute the following limits exactly or state DNE (does not exist):
   (a) $\lim_{x\to 1} \frac{1}{x^3-1}$
   (b) $\lim_{x\to 4} \frac{4-x}{2-\sqrt{x}}$
   (c) $\lim_{x\to\infty} \frac{4x^4+5x^{100}}{3x^{101}+2}$
   (d) $\lim_{x\to 2^-} \frac{1}{x-2}$
   (e) $\lim_{x\to 4} \frac{(x+2)(1-x)}{(x-4)^2(x-3)^2}$
   (f) $\lim_{x\to 5} \frac{x-5}{x^3-125}$

6. Suppose

$$g(x) = \begin{cases} \frac{x^2-16}{x-4} & \text{if } x \neq 4 \\ 16 & \text{if } x = 4 \end{cases}.$$

Is $g(x)$ continuous at $x = 4$? Why or why not?

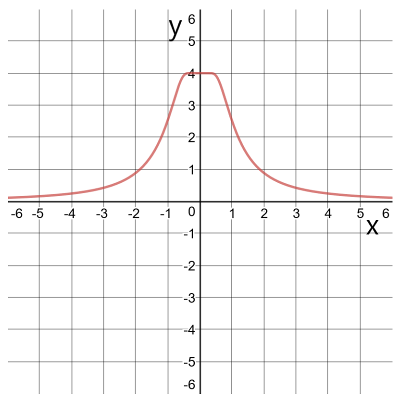
7. Find an interval that contains a solution to $x^5 + 7x + 5 = 0$. Give a complete argument based on the Intermediate Value Theorem for why the interval you found must contain a solution.

8. Use the limit laws to compute $\lim_{x\to 2} \sqrt{\frac{x^2+x+3}{3x-2}}$. Show each step in the calculation.

9. Find $\lim_{x\to -\infty} \frac{\sin(x)}{x^4}$ and give a complete argument based on the Squeeze Theorem for why your answer is correct.

10. Use the limit definition of derivative to compute $f'(3)$ for $f(x) = \sqrt{x-2}$. Answers obtained by methods other than the limit definition will receive no credit.

11. The graph of the function $y = f(x)$ is shown below. Use it to answer the following questions.



(a) Give a value for x at which $f'(x)$ is approximately 0 on the graph.

(b) Estimate $f'(2)$ from the graph.

(c) Use the graph to rank the following quantities, from smallest to largest: $f'(-2)$, $f'(0)$, $f'(1)$, $f'(3)$, $f'(5)$.