

Math 361

Day 10

The Binomial Test

Inv. 1.3 – Tim or Bob? Cont'd

Inv. 1.4 – Heart Transplants

Learning Objectives

1. Write **null** and **alternative hypotheses** for a research question.

2. Interpret a **p-value** in context

3. List the steps of a **Binomial Test**

Results of Tim/Bob Survey for (c)

Of 27 students, 23 chose Tim as on the left

More Practice

Example 2

A statistics class at Cal Poly collected data on a well-known campus legend.

Each student was asked to specify one of the four tires to answer in a situation where you have to make up which tire had recently been flat on your car. The prior conjecture is that a higher number than would be expected due to chance alone would pick the right front tire.

In this class, 24 of 54 students in class chose the right front tire (a tire identified in advance as being one that people tend to pick out of the four).

Population:

Sample:

Parameter:

Statistic:

Inv. 1.3 – parts (g) and (h)

Question: Do most people identify “Tim” as the guy on the left?

Parameter: let π be the proportion of people who pick “Tim” to be on the left.



Null Hypothesis:

$$\pi = 0.5$$

i.e. people are just guessing.

Alternative Hypothesis:

$$\pi > 0.5$$

i.e. the majority of people pick Tim to be on the left.

1. Write null and alternative hypotheses for a research question.

Practice

Example 1

Do voters make judgments about political candidates based on his/her facial appearance?

Researchers investigated this question in a study published in Science (Todorov, Mandisodka, Goren, & Hall, 2005).

Participants were shown pictures of two candidates and asked who has the more competent looking face. Researchers then predicted the winner to be the candidate whose face was judged to look more competent by most of the participants. For the 32 U.S. Senate races in 2004, this method predicted the winner correctly in 23 of them.

Parameter of interest:

Null Hypothesis:

Alternative Hypothesis:

More Practice

Example 2

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In this class, 24 of 54 students in class chose the right front tire (a tire identified in advance as being one that people tend to pick out of the four).

Parameter of interest:

Null Hypothesis:

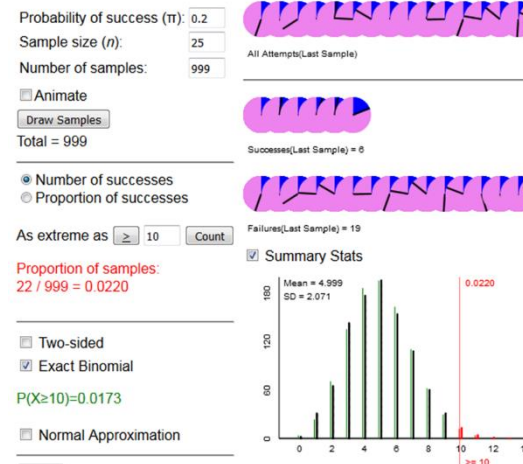
Alternative Hypothesis:

Definition of P-value

*In order to decide whether we believe the null hypothesis, we'll compute a **p-value**.*

A **p-value** is defined as the probability of seeing a **statistic** as extreme as ours if the *null model* is true.

Simulation-Based and Exact One Proportion Inference



We can use the exact formula or coin toss simulation from the One-Proportion Applet **IF** our data collection is a Binomial Random Process

Inv. 1.3 – part (j)

Rossman/Chance

Need to identify n and π
to compute the p-value
with the **exact Binomial**
probability formula

Simulation-Based and Exact One Proportion Inference

Probability of heads:
Number of tosses:
Number of repetitions:

Animate

Total = 0

Number of heads

Proportion of heads

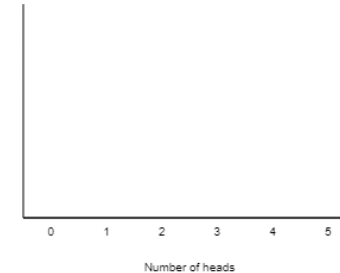
As extreme as

Two-sided

Exact Binomial

Normal Approximation

Summary Stats



Inv. 1.3: Interpretation of p-value

What does our study result,

the statistic, $\hat{p} = 0.85$

tells us about the

parameter under the null model, $\pi=0.5$?

- **P-value** = *probability of seeing 18 out of 20 students ($\hat{p} = 0.9$) chose Tim as the guy on the left if they were randomly guessing ($\pi=0.5$)*

Inv. 1.3: Using technology to compute the p-value either by simulation or math

Simulation-Based and Exact One Proportion Inference

Probability of heads: 0.5

Number of tosses: 20

Number of repetitions: 1000

Animate

Total = 1000

Number of heads

Proportion of heads

As extreme as 18

Proportion of repetitions:
 $1 / 1000 = 0.0010$

Two-sided

Exact Binomial

$P(X \geq 18) = 0.0002$

Normal Approximation



All Attempts (Last Repetition)

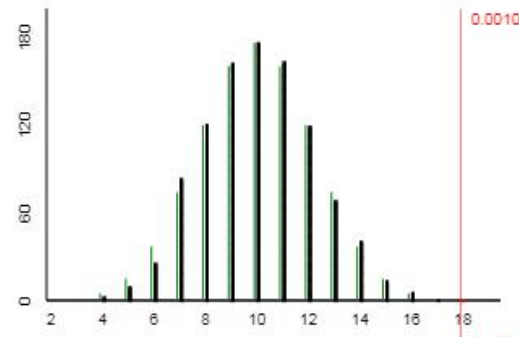


Heads (Last Repetition) = 13



Tails (Last Repetition) = 7

Summary Stats



Inv. 1.3: Drawing a conclusion from a p-value

What does our study result,

the statistic, $\hat{p} = 0.9$,

tells us about the

parameter under the null model, $\pi=0.5$?

There is strong evidence against the null model: students weren't just guessing.

Let $X = \#$ of students who put Tim on left out of 20.

Assuming X is a **binomial random variable**,

$$\text{p-value} = P(X \geq 18) = 0.0002$$

so it would be **very unlikely** to see 18 of 20 students ($\hat{p} = 0.9$) chose Tim on left **if** they were randomly guessing ($\pi=0.5$).

Let's generalize the steps in Inv. 1.3

A Test of Significance

Research question

Null Hypothesis: Nope: there's nothing going on

Alternative Hypothesis : Yes, something is going on

Collect data from a sample

Choose a random process that models the data collection well

Compute a p-value, *the probability of seeing results as extreme as the statistic if chance alone is at work*

If p-value is large,
there's no evidence against the **null hypothesis**.

If p-value is small,
there's evidence against **the null hypothesis**.

Binomial Test

Research question that involves parameter π from a Binomial Random Process

H_0 : $\pi =$ some number

H_a : $\pi \neq$ some number

Collect a binary variable from a sample of size n

Verify that the data collection is modelled well by a binomial process

Compute a binomial p-value, either through simulating a coin toss or the exact formula for a Binomial probability, assuming

$\pi =$ some number

If p-value is large,
there's no evidence against H_0 .
If p-value is small,
there's evidence against H_0 .

5. List the steps of a Binomial Test

Inv. 1.4

Try parts (a), (b), (c), (d), (e) and (f) in class.

Try parts (g)-(q) at home.