# Math 361

Day 15

Inv. 1.9 – One Proportion Z-test, by hand and applet

# Performing a test of significance by hand

We'll develop some tools that will allow us to test  $H_0: \pi = (\text{some number})$  vs.  $H_a: \pi \neq (\text{some number})$ by hand

### Example: Is a coin fair?

Suppose we have a coin that we suspect of being biased. Let's test  $H_0: \pi = 0.5$ vs.  $H_a: \pi \neq 0.5$ where  $\pi$  is the probability of the coin landing "heads" Notice the distribution of the proportion of heads appears to have a specific form if n is large enough



Proportion of heads

#### Proportion of heads

0.24 0.30 0.38 0.42 0.48 0.54 0.60 0.68 0.72

# The Normal Distribution

Described by two parameters:

- mean
- standard deviation



Proportion of orange

The formula for the curve is messy...

## The Normal Approximation to the Binomial

Equations relating parameters:

• mean  $=\pi$ 

• Standard deviation = 
$$\sqrt{\frac{\pi(1-\pi)}{n}}$$

## The Normal Approximation to the Binomial

Equations relating parameters:

• mean  $=\pi$ 

• Standard deviation = 
$$\sqrt{\frac{\pi(1-\pi)}{n}}$$

# **But** the approximation only works well when $\pi n \ge 10$ and $(1-\pi)n \ge 10$

# Central Limit Theorem: CLT

The distribution of sample proportions (stemming from a binomial process) will be approximately normal

If the sample size is large relative to the value of  $\pi$ 

(that is,  $n\pi \ge 10$  and  $n(1-\pi) \ge 10$ )

Then



# Applying CLT...

Count # heads out of 20 coin tosses, then repeat 10,000 times...

Simulation-Based and	Exact One Proportion Inference
Probability of heads: 0.5 Number of tosses: 20 Number of repetitions: 9999	All Attempts(Last Repetition)
<ul><li>Animate</li><li>Draw Samples</li><li>Total = 10000</li></ul>	CEEEEEE
<ul> <li>Number of heads</li> <li>● Proportion of heads</li> <li>As extreme as ≥ Cou</li> </ul>	Tailed at Benefition a 12
<ul><li>Two-sided</li><li>Exact Binomial</li></ul>	Summary Stats
Normal Approximation           Reset	
_	Proportion of heads
t	he distribution of X = "proportion of heads" is $\approx$ Norm

# Suppose we observe 13 out of 20 tosses land heads...

- Sample proportion is 13/20=0.65
- Check "two-sided" box and "Normal Approximation"
- The p-value is 0.18, so fail to reject the null.

There is no evidence our coin is biased.

Drobobility of boods:	
Probability of heads. 0.5	
Number of tosses: 20	
Number of repetitions: 1000	
Animate	
Draw Samples	
Number of heads	
Proportion of heads	
As extreme as $\geq$ 0.65 Count	
	Summary Stats
🗹 Two-sided (between: 🔲 )	Mean = 0.500
Exact Binomial	SD = 0.112
Normal Approximation	
$p_{\rm V} = 0.1707 (7 - 1.24)$	
p-value = 0.1797 (2 = 1.54)	
Reset	
	0.10 0.20 0.30 0.40 0.50 0.80 0.70 0.80 0.90
	Proportion of beads

#### Must check conditions before applying CLT

Count # heads out of 2 coin tosses, then repeat 10,000 times... **Simulation-Based and Exact One Proportion Inference** Probability of heads: 0.5 Number of tosses: 2 All Attempts(Last Repetition) Number of repetitions: 10000 Animate Draw Samples Total = 20000 Heads(Last Repetition) = 1 Number of heads Proportion of heads As extreme as ≥ Count Tails(Last Repetition) = 1 Summary Stats Two-sided 808 Exact Binomial Normal Approximation 88 Reset 800 0.50 0 Proportion of heads

...the distribution of X = "proportion of heads" is not very Normal!

# Advantages to Normal Distribution

- Historically was more convenient for calculating probabilities (area under the curve)
  - Could standardize (z-score) and look up on a table
  - Especially useful for calculating power
- Still useful for some of today's "big data" problems
- Empirical rule (68/95/99.7)
  - the interval ( $\mu$   $\sigma$ ,  $\mu$  +  $\sigma$ ) should capture approximately 68% of the distribution.
  - the interval  $(\mu 2\sigma, \mu + 2\sigma)$  should capture approximately 95% of the distribution.



• the interval ( $\mu$  - 3 $\sigma$ ,  $\mu$  + 3 $\sigma$ ) should capture approximately 99.7% of the distribution.

The Empirical Rule allows us to perform a "two-sided" test by hand!

- Draw distribution using CLT
- Compute mean +2SD and mean 2SD
- Find rejection region using ER
- If the sample proportion is in rejection region, reject the null otherwise fail to reject the null

### Example: observed 13 "heads" in 20 tosses.

# One Sample *z*-test for proportions

- 1. Define parameter (process probability or population proportion)
- 2. State null and alternative hypotheses
- 3. Check whether CLT applies:  $n\pi$ ,  $n(1-\pi) \ge 10$
- 4. Calculate test statistic (*z*-score)
  - Interpretation: Number of SDs from null value of π
     OR calculate Mean + 2SD and Mean -2SD to find rejection region
- 5. Calculate p-value using normal distribution

OR check whether the sample proportion is within 2 SDs of the mean

6. State conclusions

#### Inv. 1.9 – Toy or Treat?

Try parts (a), (b), (c) and (d).

#### Inv. 1.9: toys or treat on Halloween?

a) **Obs. Units:** children **Variable:** Did a child choose the toy?

b) The parameter of interest is the proportion of all children who prefer toys to candy on Halloween ( $\pi$ )

c) Test  $H_0$ :  $\pi = 0.5$  vs.  $H_a$ :  $\pi \neq 0.5$ 

d) Of n=284 children, 135 chose the toy so  $\hat{p} = 0.475$ .

Could compute the p-value using simulations or Exact Binomial, but let's try applying the CLT and use the Normal Distribution instead (one sample proportion test)

#### Inv. 1.9 – by hand via Empirical Rule

(e) Check conditions  $n\pi \ge 10$  and  $n(1-\pi) \ge 10$ :

(f) Draw the normal distribution of  $\hat{p}$  assuming H<sub>0</sub> is true: mean=  $\pi$  and SD = $\sqrt{\pi(1-\pi)/n}$  and add our value of  $\hat{p}$ .

(i) z-score = how many SD's from the mean is our  $\hat{p}$ ? More than 2 is "extreme" by ER.

#### Inv. 1.9 – via applet

#### **Rossman/Chance Applet Collection**

#### **Theory-Based Inference**



## Investigation 1.9

- p-value  $\approx 0.40$
- Conclusions?