Math 361

Day 15

Inv. 1.9 – One Proportion Z-test, by hand and applet

Performing a test of significance **by hand**

We'll develop some tools that will allow us to test

$$H_0: \pi = \text{(some number)}$$

vs.

$$H_a: \pi \neq \text{(some number)}$$

by hand

Example: Is a coin fair?

Suppose we have a coin that we suspect of being biased. Let's test

$$H_0: \pi = 0.5$$

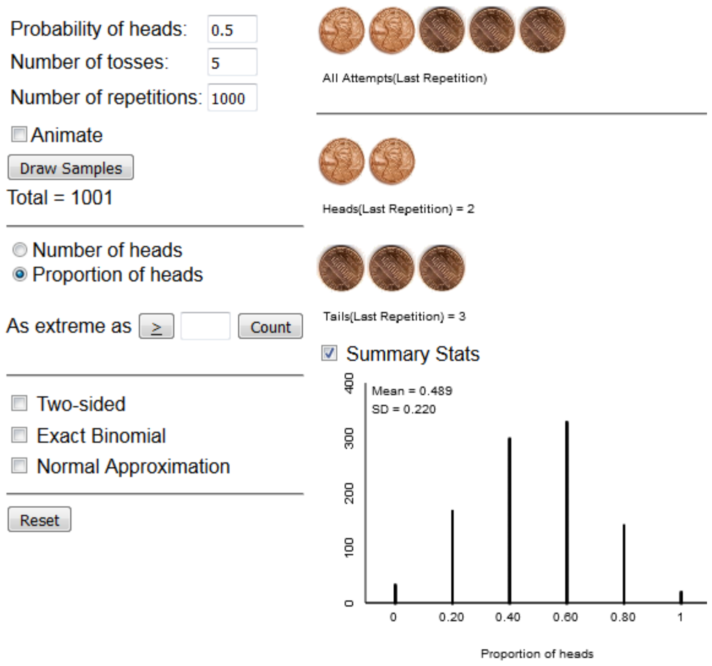
vs.

$$H_a: \pi \neq 0.5$$

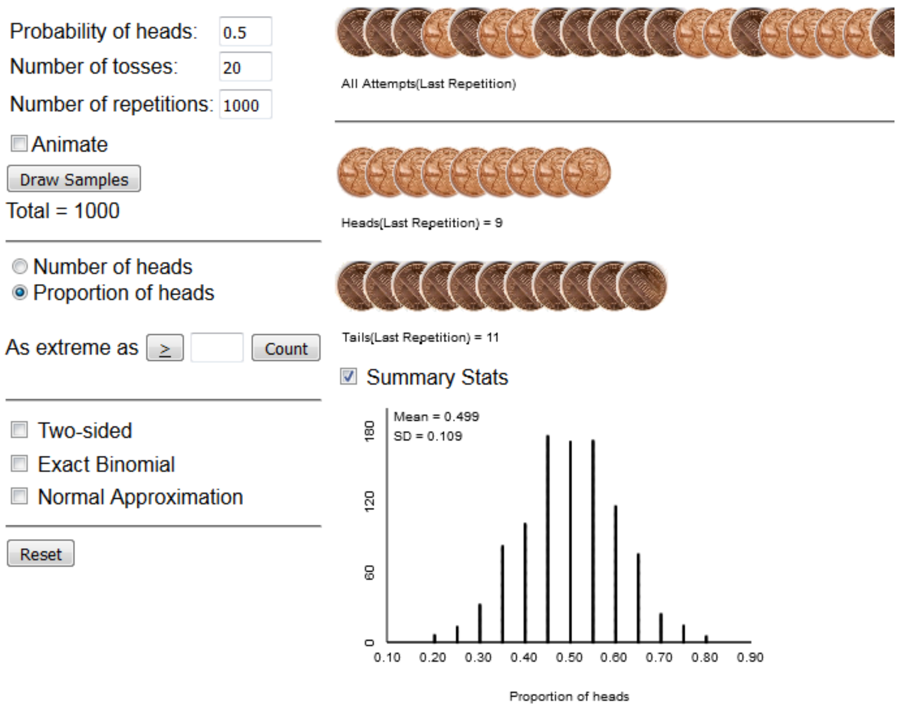
where π is the probability of the coin landing "heads"

Notice the distribution of the proportion of heads appears to have a specific form if n is large enough

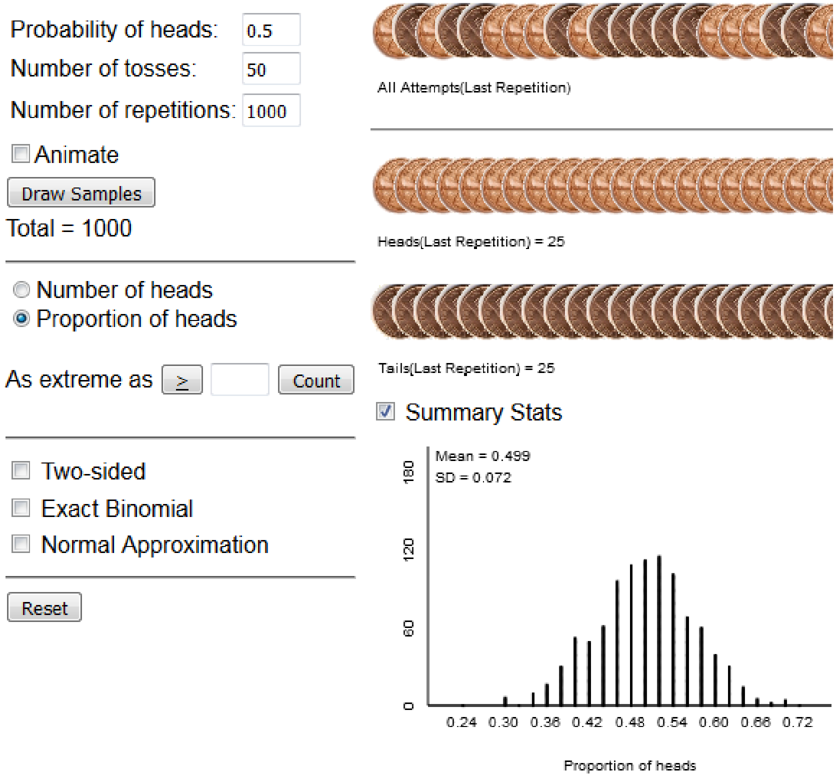
n =5



n=20

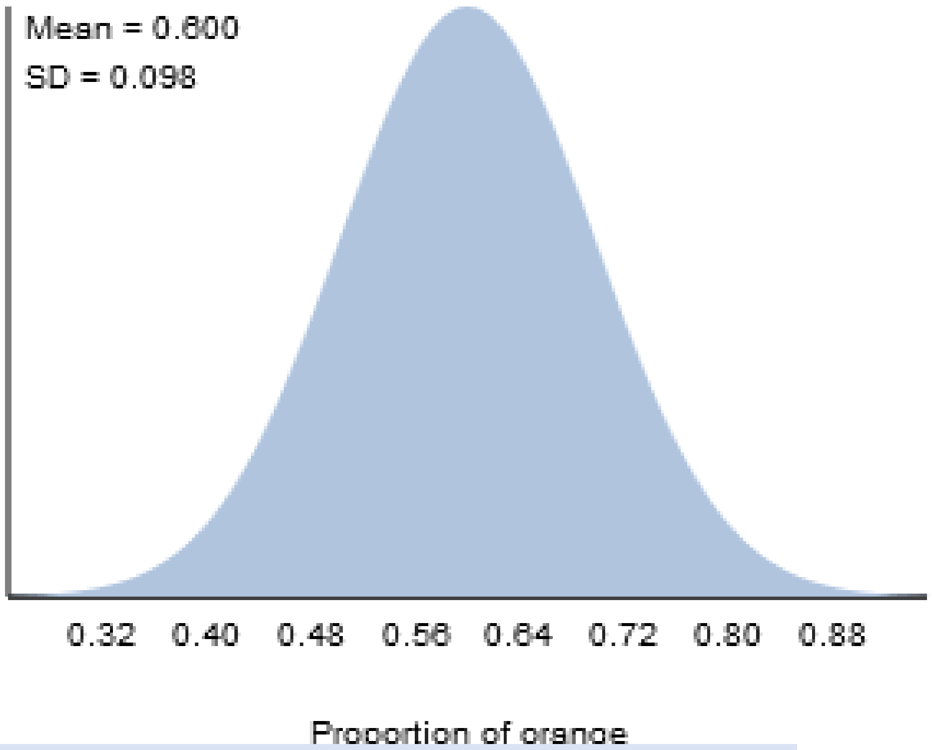


n=50



The Normal Distribution

Described by two parameters:
- mean
- standard deviation



The formula for the curve is messy…

The Normal Approximation to the Binomial

Equations relating parameters:

- mean $= \pi$
- Standard deviation $= \sqrt{\frac{\pi(1-\pi)}{n}}$

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But the approximation only works well when

$\pi n \geq 10$ and $(1-\pi)n \geq 10$

Central Limit Theorem: CLT

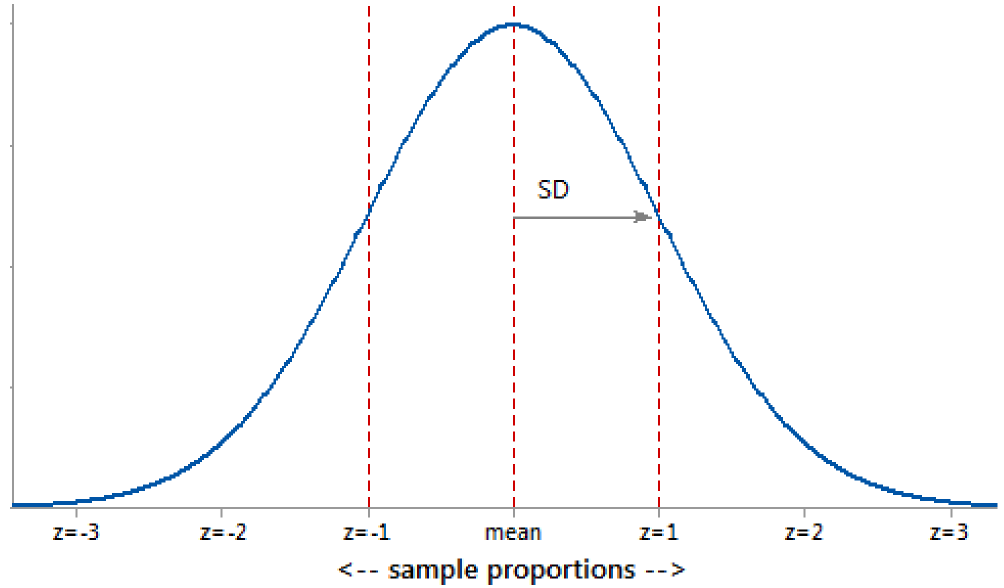
The distribution of sample proportions (stemming from a binomial process) will be approximately normal

If the sample size is large relative to the value of π

(that is, $n\pi \geq 10$ and $n(1-\pi) \geq 10$)

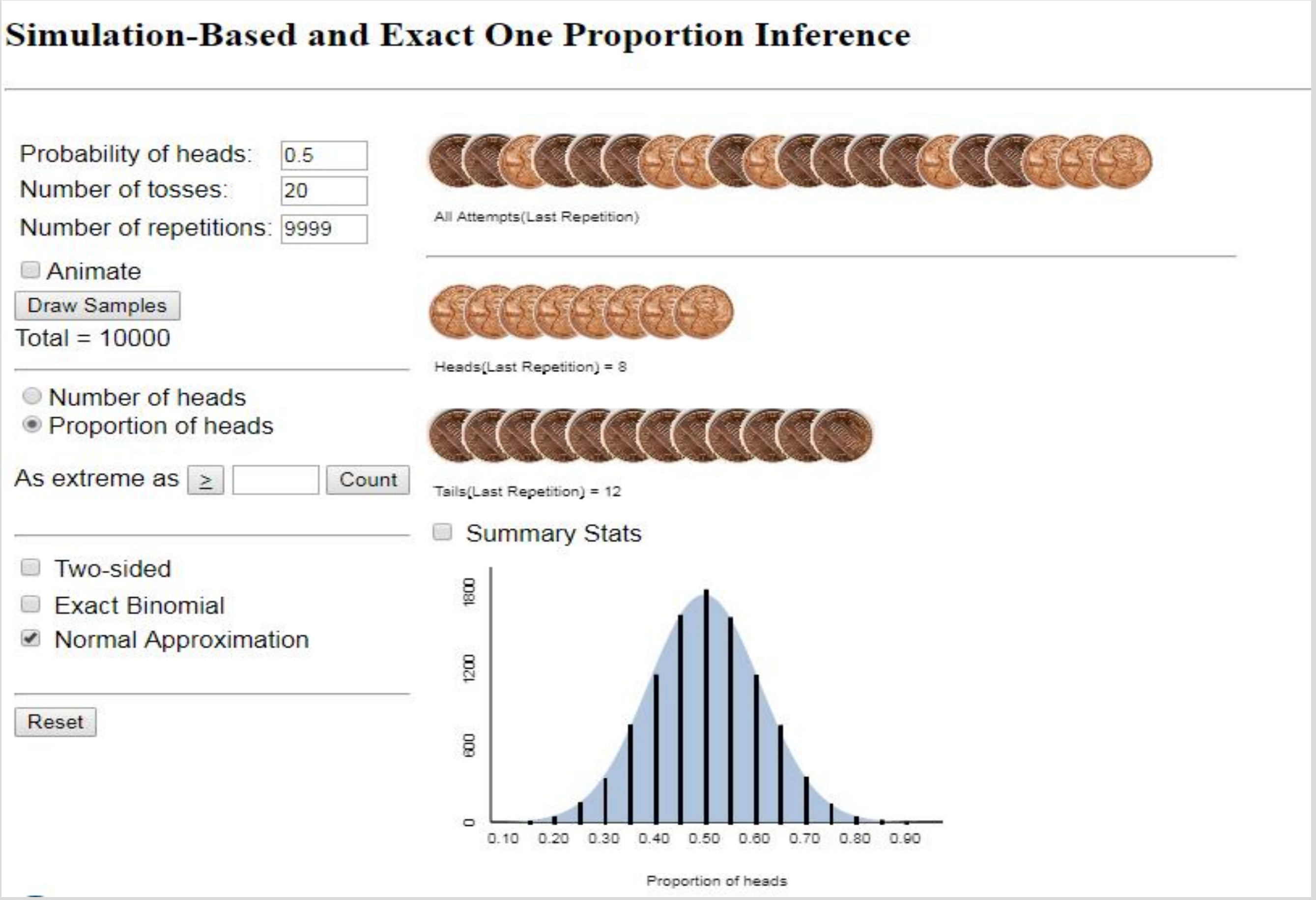
Then

- the mean ≈ π
- and SD ≈ $\sqrt{\pi(1-\pi)/n}$



Applying CLT…

Count # heads out of 20 coin tosses, then repeat 10,000 times…



…the distribution of X = “proportion of heads” is ≈ Normal!

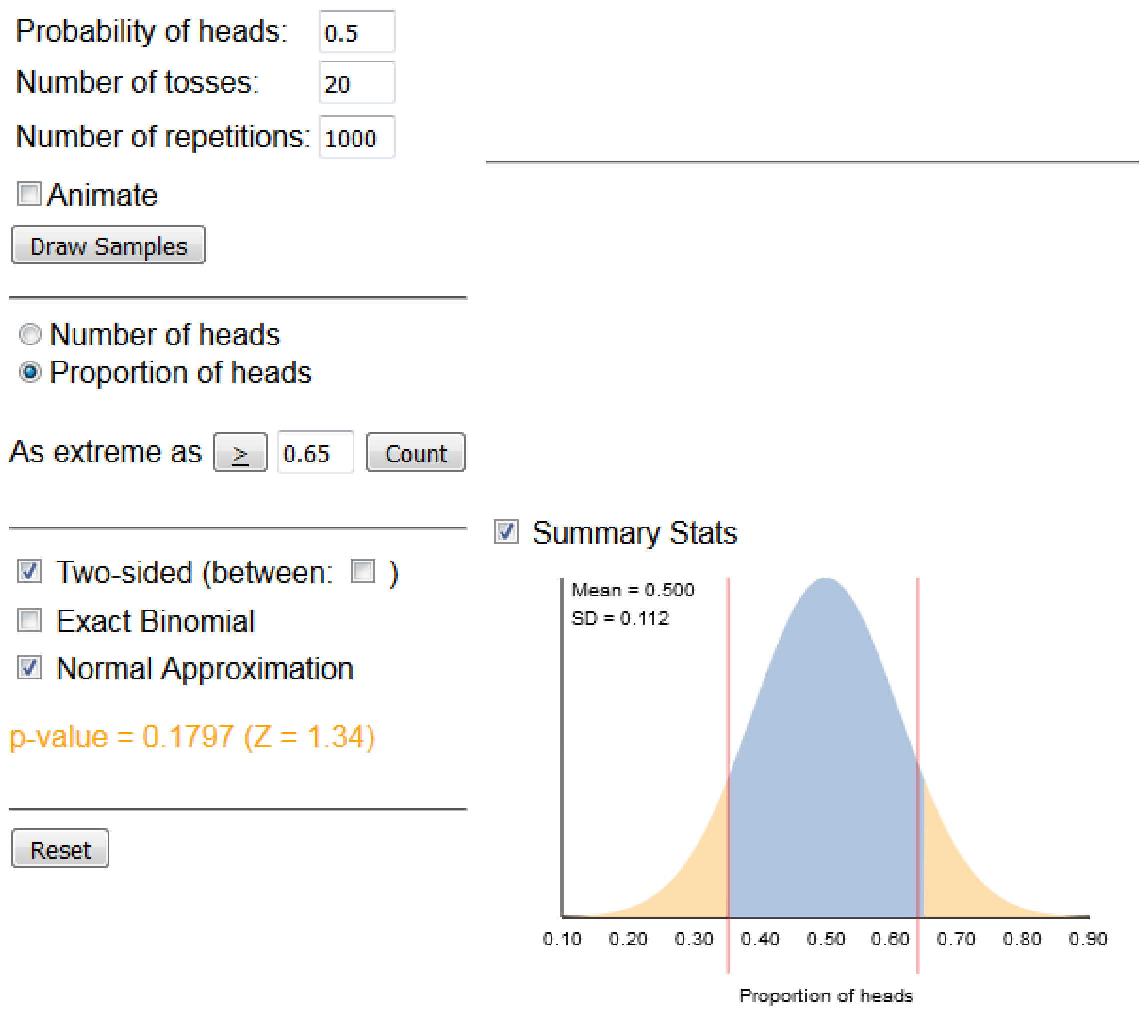
Suppose we observe 13 out of 20 tosses land heads...

Sample proportion is 13/20=0.65

Check “two-sided” box and “Normal Approximation”

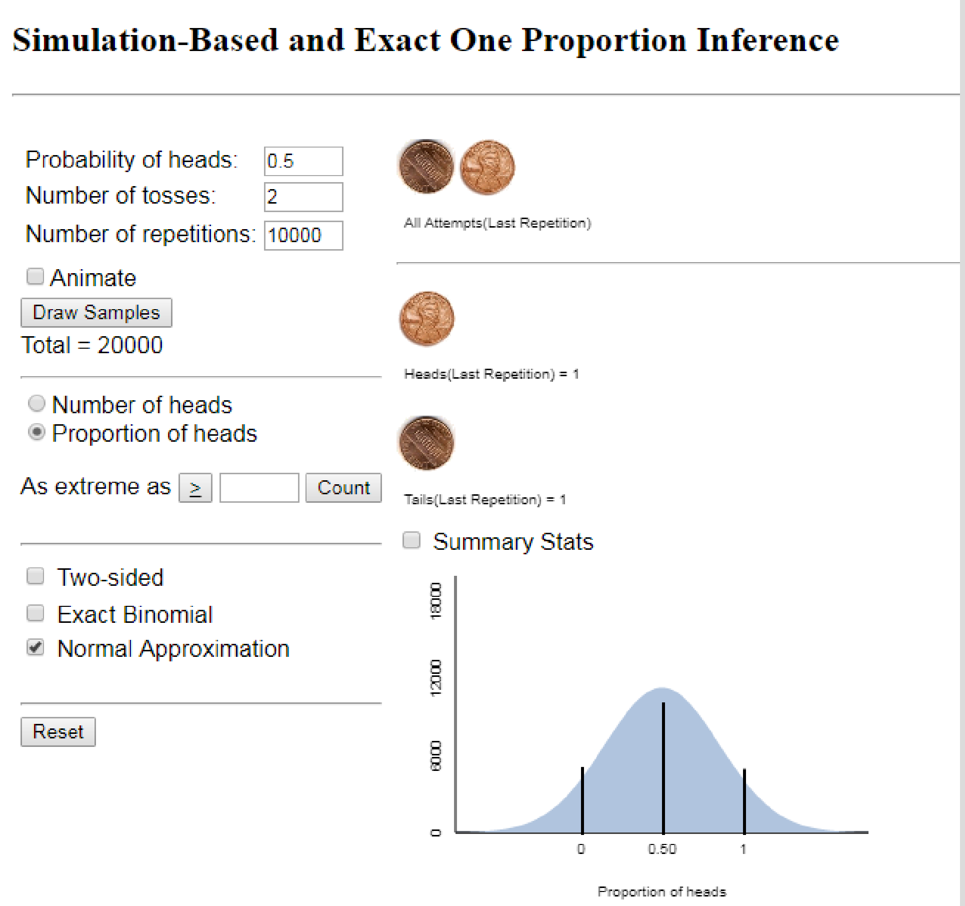
The p-value is 0.18, so fail to reject the null.

There is no evidence our coin is biased.



Must check conditions before applying CLT

Count # heads out of 2 coin tosses, then repeat 10,000 times…

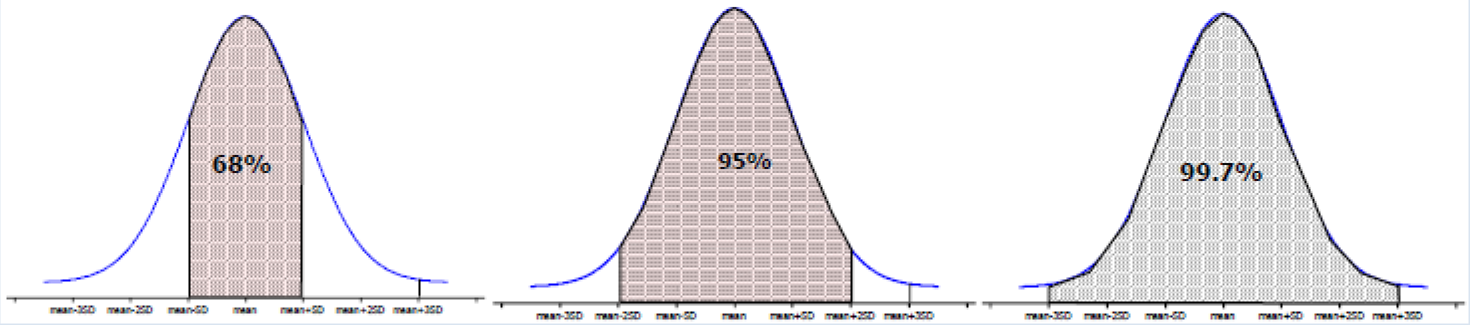


…the distribution of X = "proportion of heads" is not very Normal!

Advantages to Normal Distribution

- Historically was more convenient for calculating probabilities (area under the curve)
  - Could standardize (z-score) and look up on a table
  - Especially useful for calculating power
- Still useful for some of today's "big data" problems
- Empirical rule (68/95/99.7)

- the interval ($\mu - \sigma$, $\mu + \sigma$) should capture approximately 68% of the distribution.
- the interval ($\mu - 2\sigma$, $\mu + 2\sigma$) should capture approximately 95% of the distribution.
- the interval ($\mu - 3\sigma$, $\mu + 3\sigma$) should capture approximately 99.7% of the distribution.



The Empirical Rule allows us to perform a “two-sided” test by hand!

- Draw distribution using CLT
- Compute **mean +2SD** and **mean – 2SD**
- Find rejection region using ER
- If the sample proportion is in rejection region,
  reject the null otherwise fail to reject the null

Example: observed 13 “heads” in 20 tosses.

One Sample *z*-test for proportions

1. Define parameter (process probability or population proportion)
2. State null and alternative hypotheses
3. Check whether CLT applies: $n\pi$, $n(1-\pi) \geq 10$
4. Calculate test statistic (*z*-score)
    - Interpretation: Number of SDs from null value of π

    OR calculate Mean + 2SD and Mean -2SD to find rejection region
5. Calculate p-value using normal distribution

OR check whether the sample proportion is within 2 SDs of the mean

6. State conclusions

Inv. 1.9 – Toy or Treat?

Try parts (a), (b), (c) and (d).

Inv. 1.9: toys or treat on Halloween?

a) **Obs. Units:** children

Variable: Did a child choose the toy?

b) The parameter of interest is the proportion of all children who prefer toys to candy on Halloween (π)

c) Test H_0: $\pi = 0.5$ vs. H_a: $\pi \neq 0.5$

d) Of n=284 children, 135 chose the toy so $\hat{p} = 0.475$.

Could compute the p-value using simulations or Exact Binomial, but let's try applying the CLT and use the Normal Distribution instead (one sample proportion test)

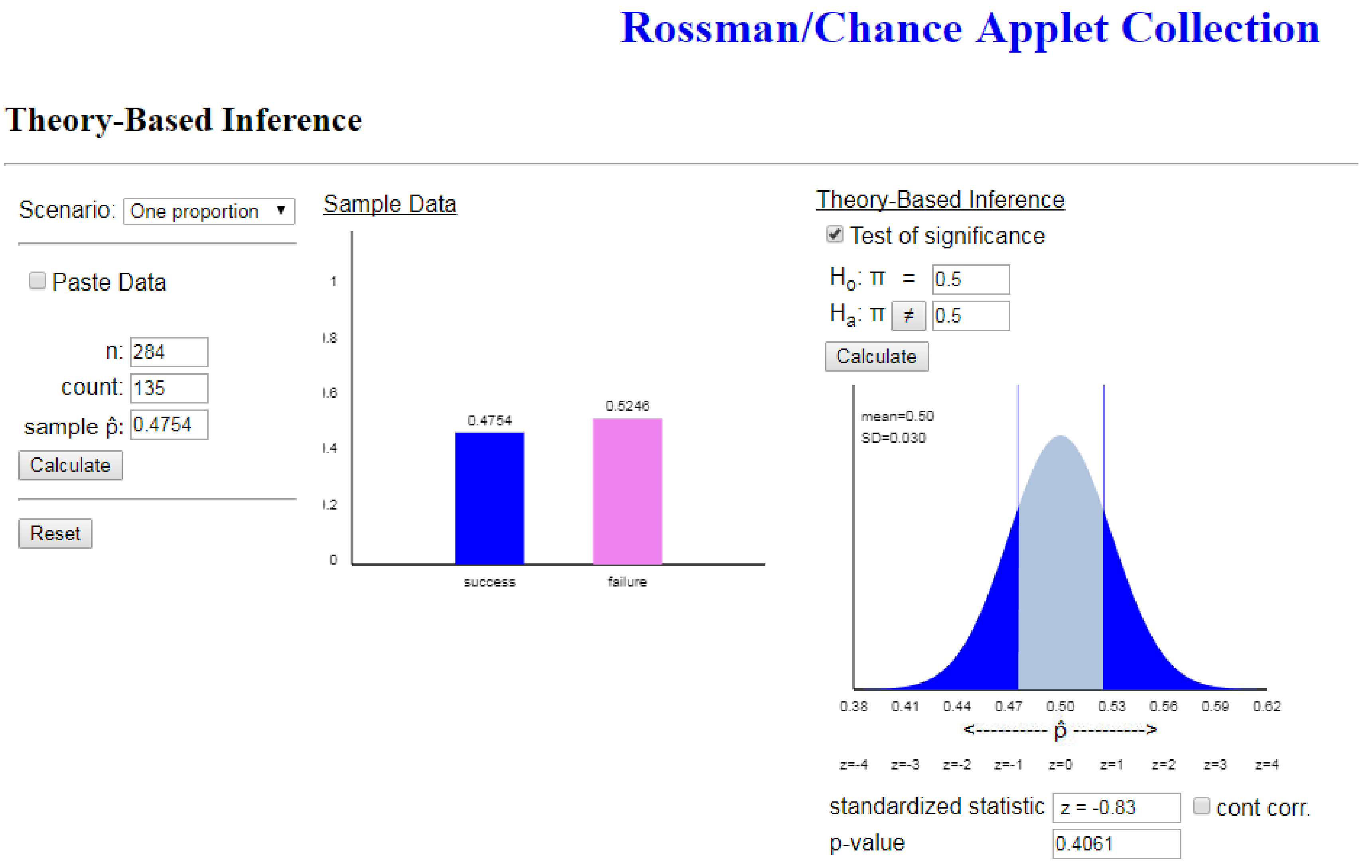
Inv. 1.9 – by hand via Empirical Rule

(e) Check conditions $n\pi \geq 10$ and $n(1-\pi) \geq 10$:

(f) Draw the normal distribution of $\hat{p}$ assuming H_0 is true:
mean= π and SD = $\sqrt{\pi(1-\pi)/n}$ and add our value of $\hat{p}$.

(i) z-score = how many SD's from the mean is our $\hat{p}$? More than 2 is "extreme" by ER.

Inv. 1.9 – via applet



Investigation 1.9

- p-value ≈ 0.40
- Conclusions?