Quiz 7 Friday
Ch 5. 40, 44

# Math 361

Least Squares Regression Line – Inv. 5.8 and 5.9

# Prediction

So far, we've

- *described* a dataset through graphs and numerical summaries,
- *tested* whether a parameter is a value ($H_0$ vs. $H_a$), and
- *estimated* a parameter (95% confidence interval)

Today, we'll

- *predict* the value of a quantitative variable based on the value of a second quantitative variable.

# Inv. 5.8: Footlength vs. Height

Given the length of a person’s **foot**, *predict* their **height**.

Collect data:

# Inv. 5.8: Footlength vs. Height

Given the length of a person's **foot**, *predict* their **height**.

Collect data:

*Observational units:* 20 statistics students

*Explanatory variable*: foot length in cm

*Response variable*: height in inches

# Inv. 5.8: Footlength vs. Height

*Direction?*

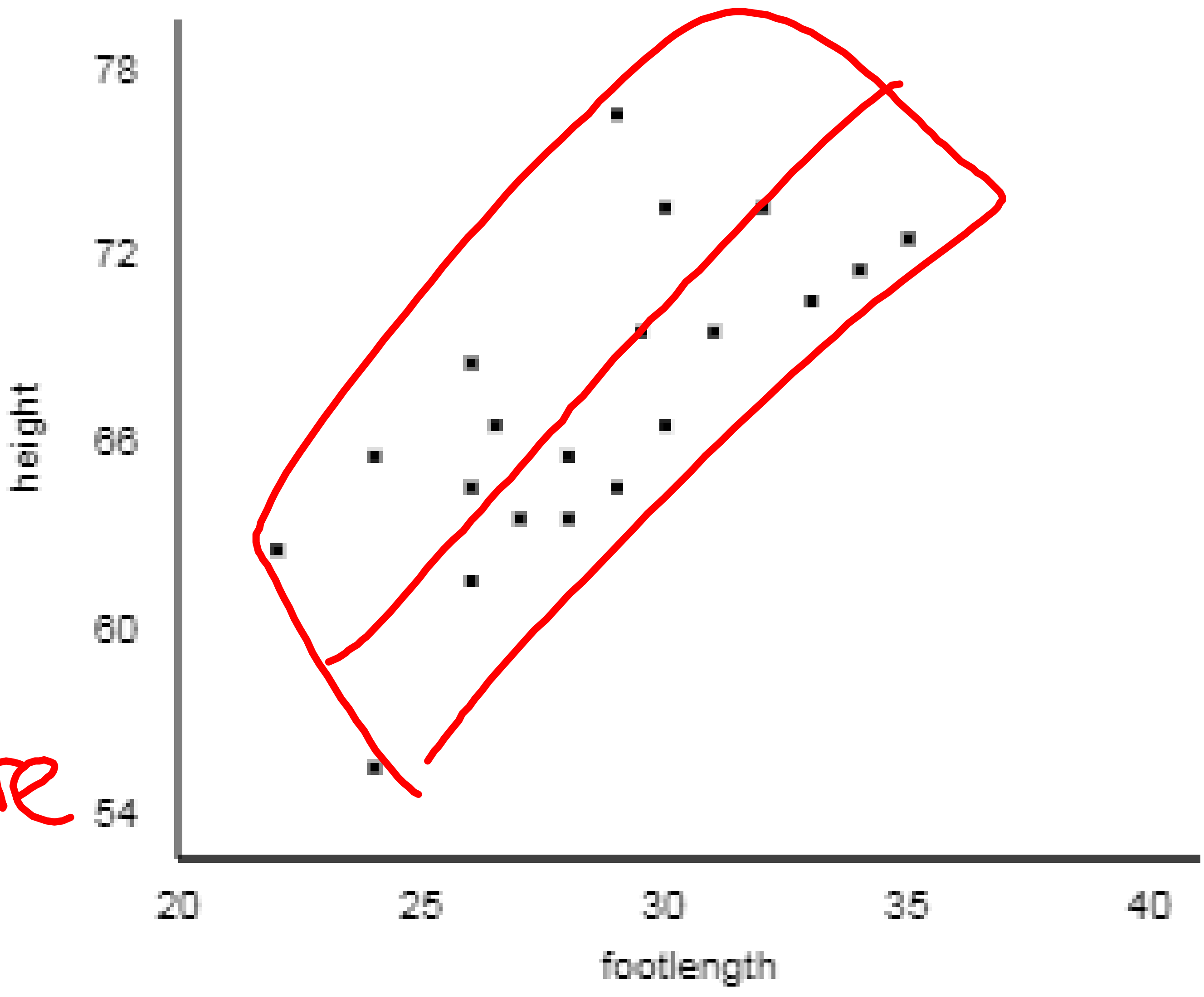
+

*Linear?*

yes

*Strength?*

moderate

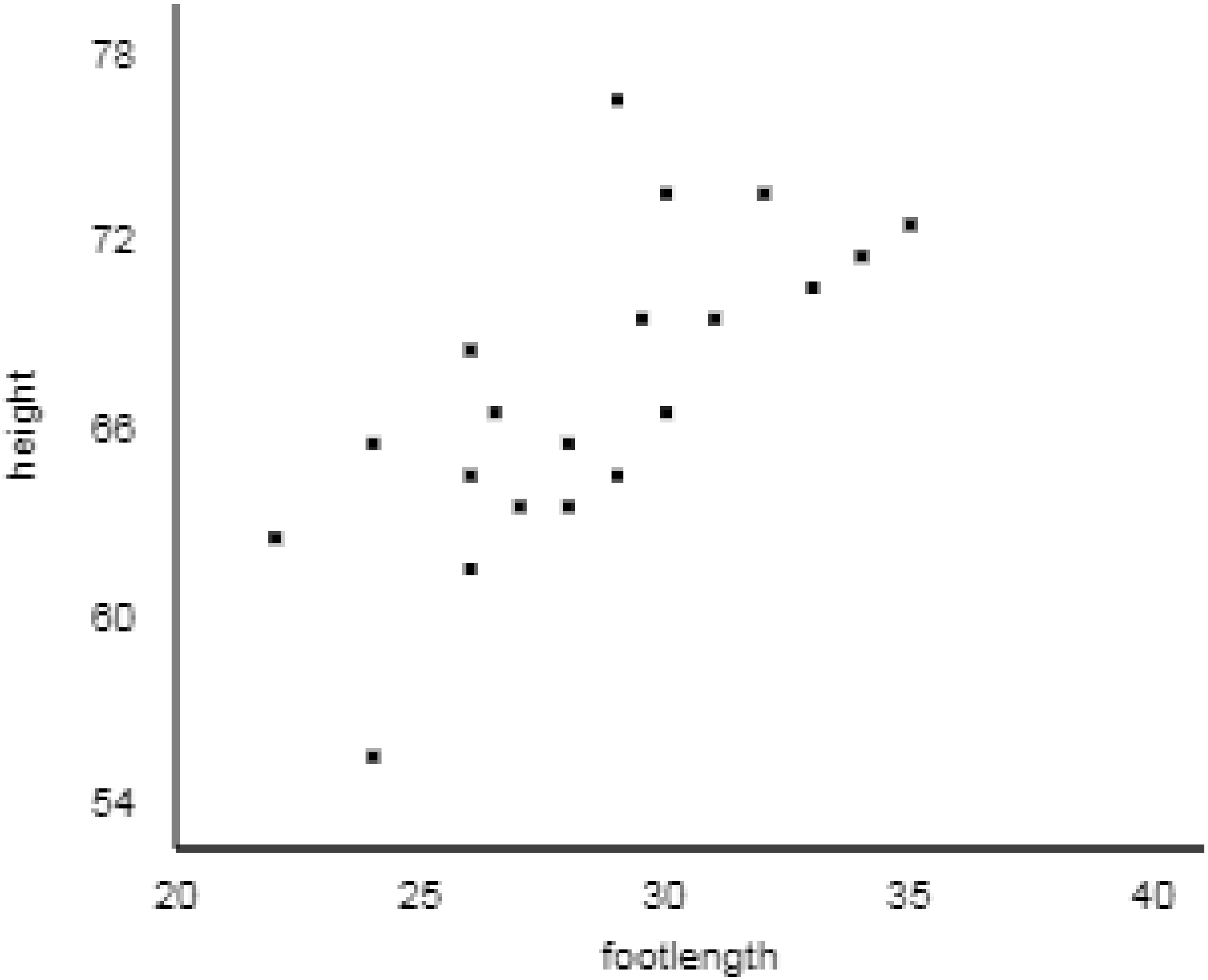


# Inv. 5.8: Footlength vs. Height

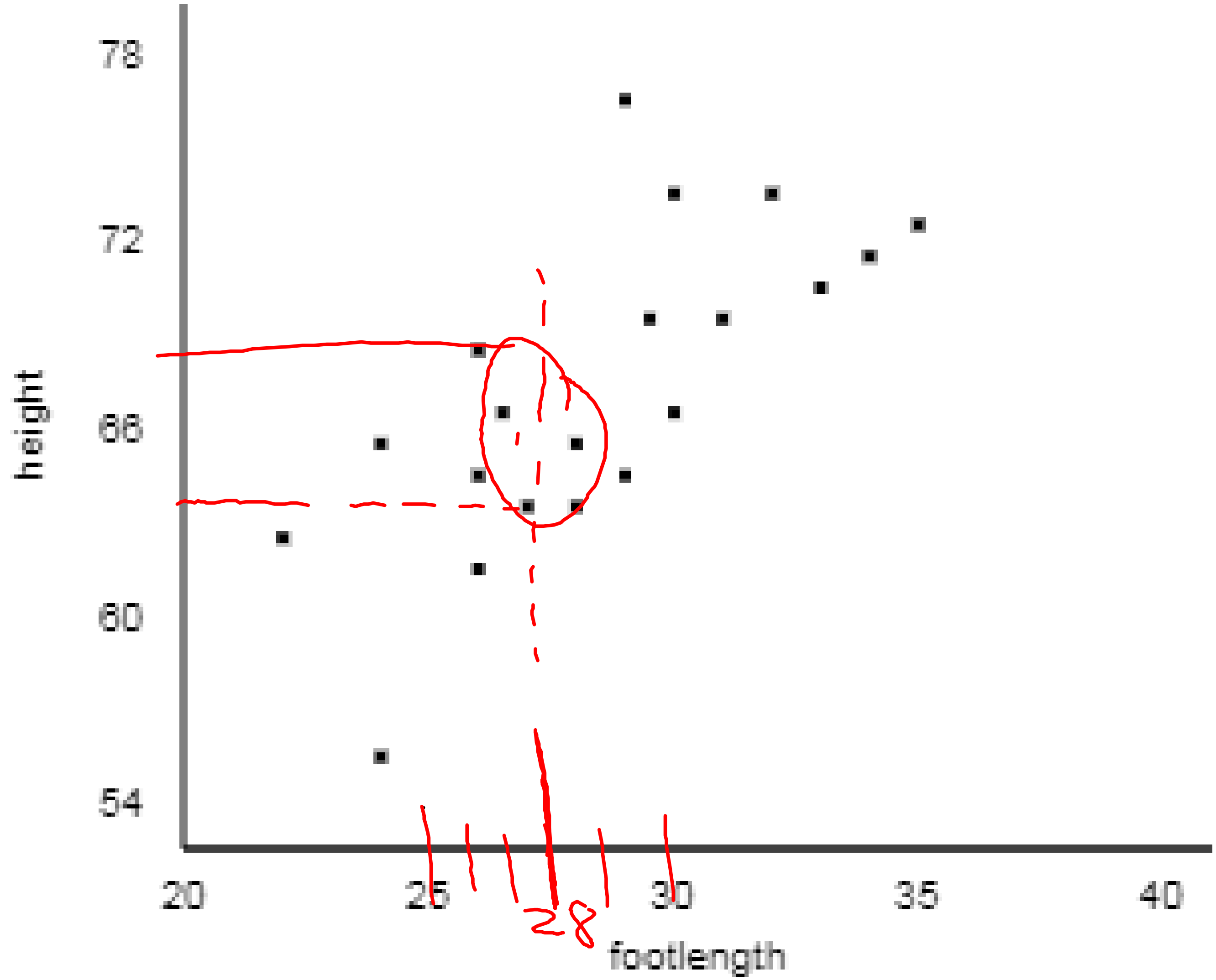
*Direction?*
*positive*

*Linear?*
*Yes*

*Strength?*
$r$ =0.711

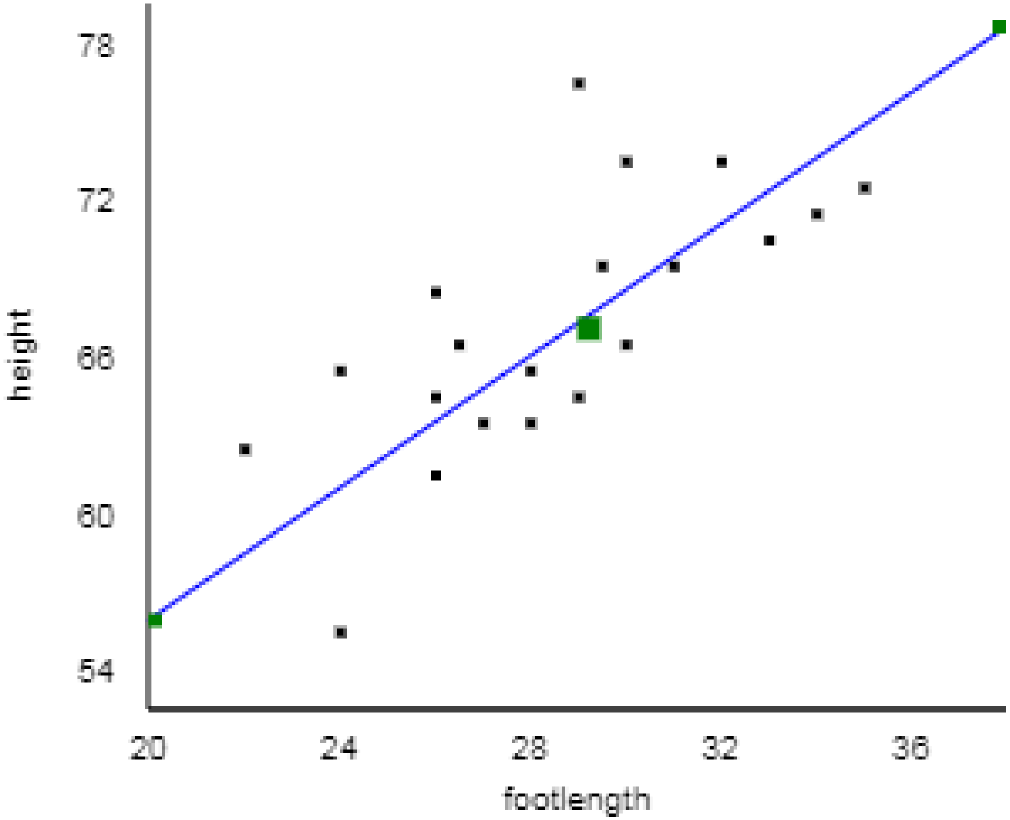


Suppose you come across a footprint that is 28 cm long. How tall do you predict the maker was?



# Prediction from a line

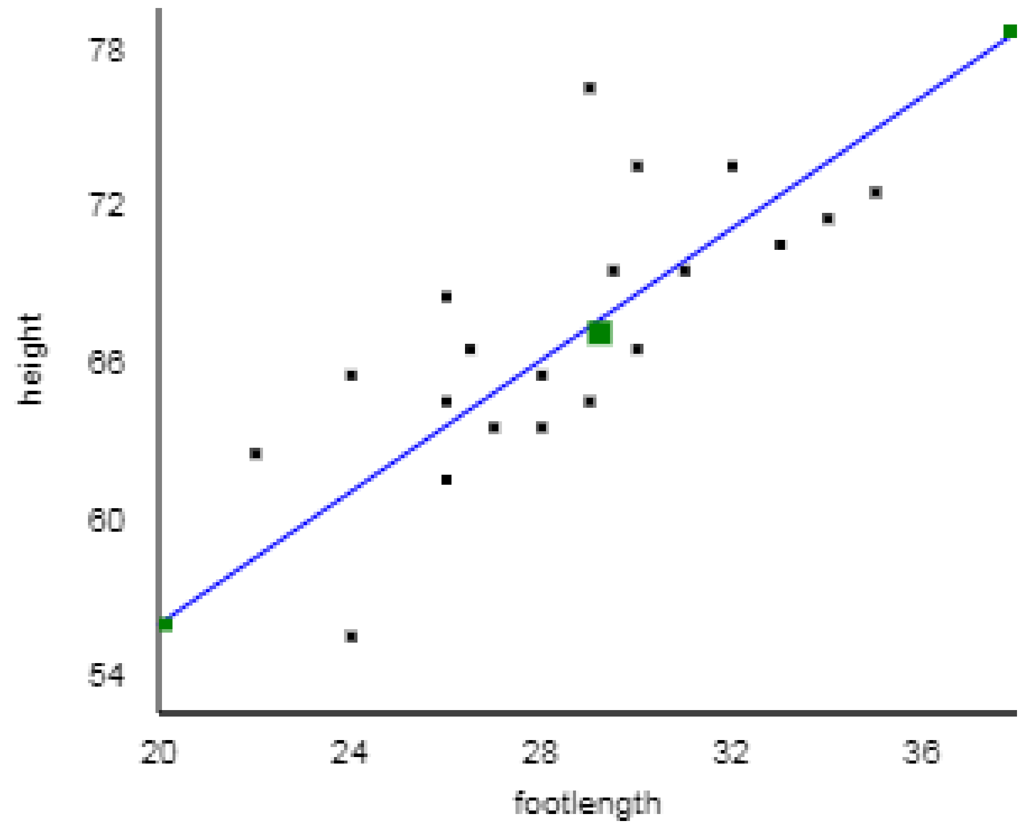
By drawing a **line** through the points, I can consistently predict the value of the response variable for a given value of the explanatory variable.



# Prediction from a line

By agreeing on ***how to draw*** a **line** through the points, **we** can consistently predict the value of the response variable for a given value of the explanatory variable.

Idea: Choose the line that minimizes the distances from the points to the prediction line



# In HW 9, you'll be asked to find the equation of the line you would use.

## Analyzing Two Quantitative Variables

---

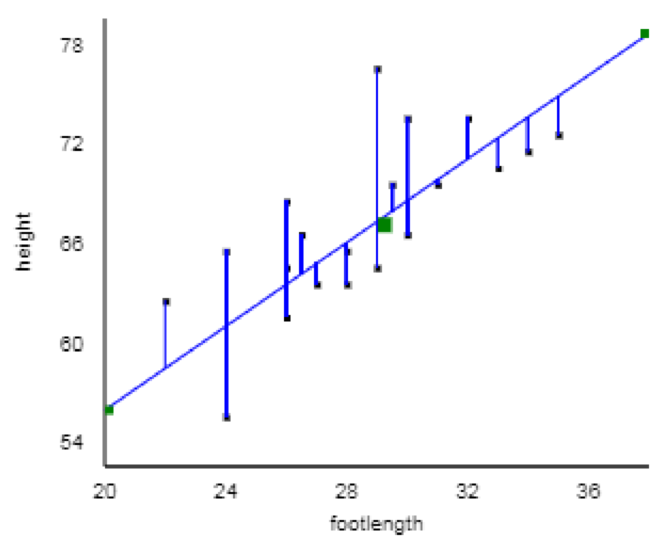
Sample data:

(Explanatory, Response)

| | |
|---|---|
| 26.0 | 62 |
| 26.5 | 67 |
| 28.0 | 66 |
| 28.0 | 64 |
| 26.0 | 69 |
| 35.0 | 73 |
| 30.0 | 74 |
| 31.0 | 70 |
| 29.0 | 65 |
| 34.0 | 72 |

Use Data  Revert  Clear

n = 20




Show Movable Line: ☑

height^ = 31.18 + 1.26 x footlength

Show Residuals: ☑

SAE=58.24

Show Squared Residuals: ☐

Show Regression Line: ☐

# Some terminology for choosing the "best" line

line

A **residual** is the difference between the *predicted* value and the *observed* value

point

distance

The *sum of the absolute residuals* is denoted **SAE**

The sum of squared residuals is denoted **SSE**

# Choosing the “best” line

- Could choose the line that minimizes either SAE or SSE.

- Historically, people have chosen the line that minimizes SSE because it is possible to compute without computers: this line is called the “**least squares line**” or “**regression line**”

# The Least Squares Line

## Analyzing Two Quantitative Variables

Sample data:

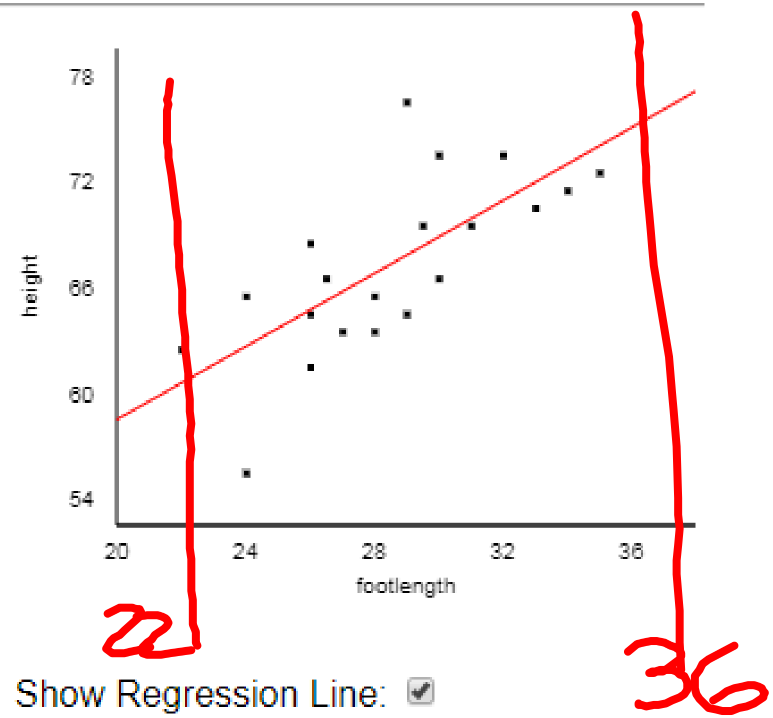
(Explanatory, Response)

| | |
|---|---|
| 26.0 | 62 |
| 26.5 | 67 |
| 28.0 | 66 |
| 28.0 | 64 |
| 26.0 | 69 |
| 35.0 | 73 |
| 30.0 | 74 |
| 31.0 | 70 |
| 29.0 | 65 |
| 34.0 | 72 |

Use Data Revert Clear

n = 20

Show Movable Line:



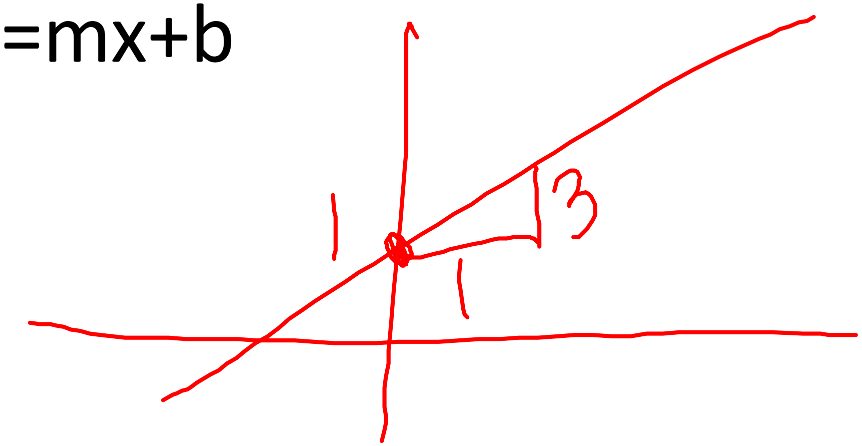

Show Regression Line:

*height*^ = 38.30 + 1.03 x *footlength*

# Equation of a line

Recall the equation of a line from algebra:

$$y=mx+b$$




<u>Example:</u> $y=3x+1$

What does the 3 mean?

For an increase in x of 1 increses y by 3

What does the 1 mean?

y=1 when x=0

# Equation of a line

Recall the equation of a line from algebra:

$$y=mx+b$$

Example: $y=3x+1$

What does the 3 mean?

*If x increases by 1 unit then y increases by 3 units*

What does the 1 mean?

*When $x = 0$, $y = 1$.*

# Equation of a line

Recall the equation of a line from algebra:

$$y=mx+b$$

Example: $y=-3x+1$

What does the -3 mean?

*If x increases by 1 unit then y decreases by 3 units*

# Equation of a "least squares line"

$$\hat{y} = b_0 + b_1 x$$

$\cancel{y = mx + b}$

Here,

- $\hat{y}$ is the **predicted value** of the response variable when the value of the explanatory variable is $x$

- $b_1$ is the **regression slope**,

- $b_0$ is the **regression intercept**

# The “least squares line” from Inv. 5.8

$$\widehat{height} = 38.1 + 1.03 footlength$$

Here,

- $\widehat{height}$ is the **predicted** height for a given footlength
- 1.03 is the **regression slope**,
- 38.1 is the **regression intercept**

predicted height when footlength = 0

# Interpreting the "least squares line" from Inv. 5.8

$$\widehat{height} = 38.3 + 1.03 footlength$$

<u>Slope:</u>

x

*If the footlength increases by 1 cm then the predicted height increases by 1.03 inches.*

y

<u>Intercept:</u>

*The predicted height is 38.3 inches when the footlength is 0.*

# Using the "least squares line" from Inv. 5.8

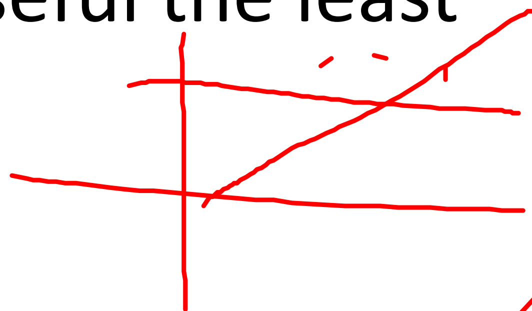
$$\widehat{height} = 38.3 + 1.03 footlength$$

Predict the height of someone whose footlength is 28 cm:

$$\widehat{height} = 38.3 + 1.03(28) = 67.14$$

# Coefficient of determination, $R^2$

- Provides a measure of how useful the least squares line is



$R^2$ = percent error of the SSE of prediction line $\bar{y}$ and the SSE of the least squares line

where SSE is the sum of the squared residuals

# Interpretation of $R^2$ in Inv. 5.8

$R^2$ = percent of variability of the response variable y that is ***explained*** by the least squares line with the explanatory variable x

$R^2$ =50.6% so…

$r = 0.711$

$r^2 = (0.711)^2 = 0.506$

The least square line with footlength ***explains*** 50.6% of the variability in height.

# Interpretation of $R^2$

$R^2$ = percent of variability of the response variable y that is explained by the least squares line with the explanatory variable x

Notes:

- $R^2$ is always between 0% and 100%
- $R^2 = r^2$, where r is the correlation coefficient.