Math 361

Day 4

Random Babies – Inv. B cont’d

Last time – Random Babies simulation

- we mimicked the process of randomly returning 4 babies to their mothers by shuffling and then dealing out 4 slips of colored paper.
- Our class results in part (d) were

Number of matches	0	1	2	3	4
Count	5	14	7	0	1
Proportion	5/27=0.185	14/27=0.519	7/27=0.259	0/27=0	1/27=0.037

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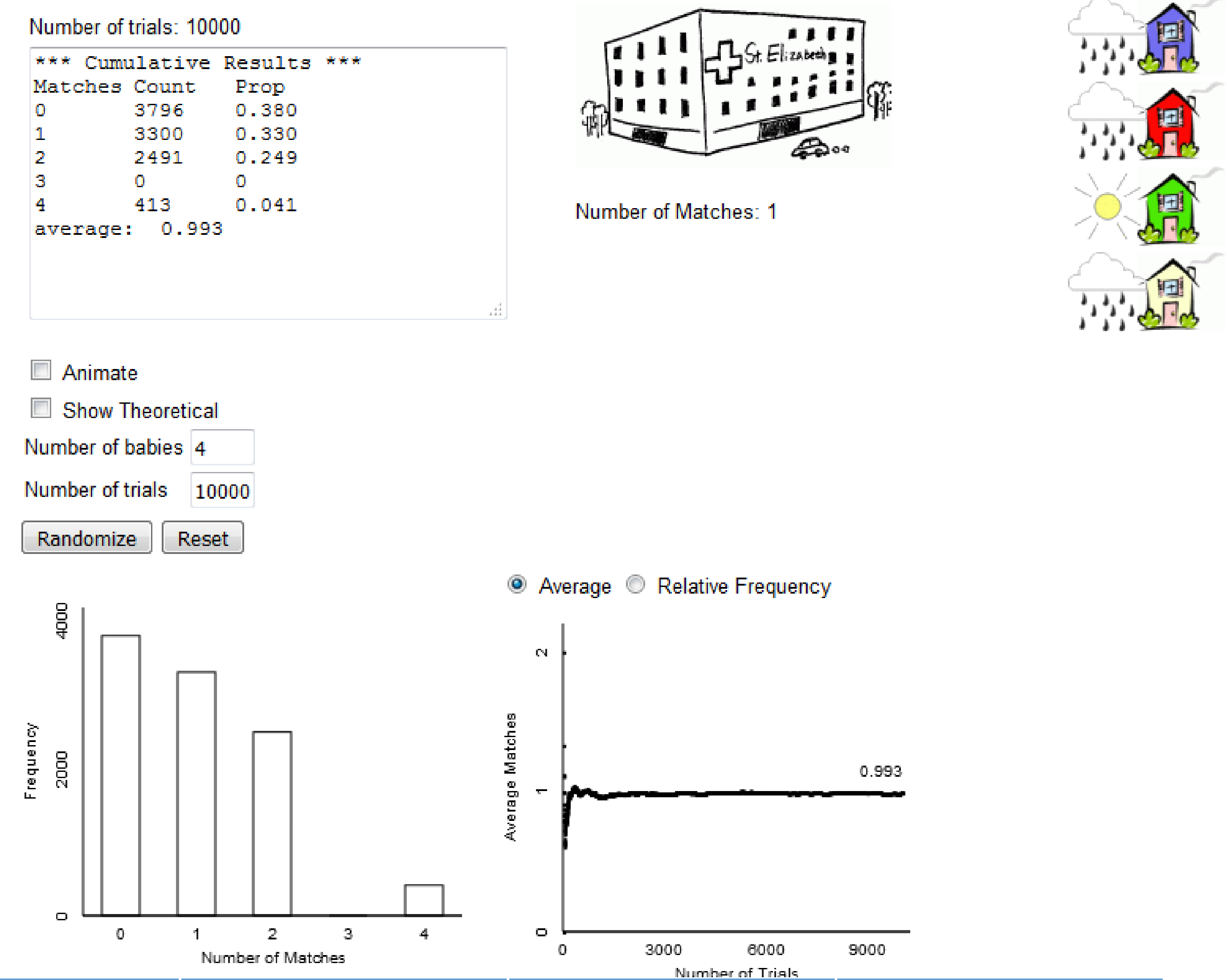
- Part (f). The probability of at least one correct match is

0.519+0.259+0+0.037=0.815

According to our simulation of 27 repetitions of the process of randomly returning 4 babies, there is a 81.5% chance that at least one mother will get her own baby.

Inv. B parts (h) and (j)

We can improve our estimates of the probabilities of the numbers of matches by performing more simulations

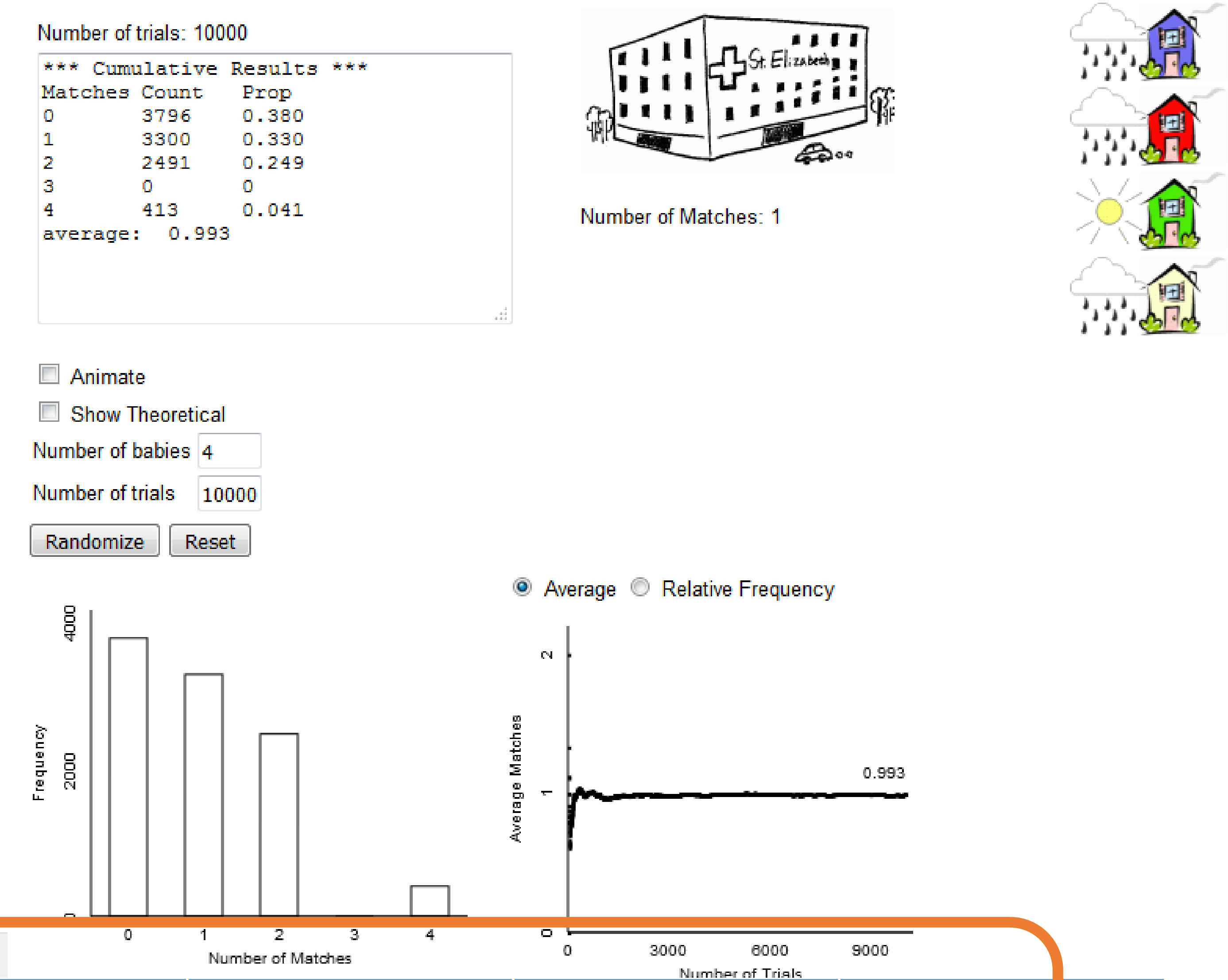


Number of matches	0	1	2	3	4
Proportion	0.38	0.33	0.249	0	0.041

Inv. B parts (j) and (k)

With 10,000 trials (*simulations*) of returning 4 babies to their mothers, we estimate the probability of at least one match to be

0.33+0.249+0+0.041=**0.62**



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Random Processes

Definition: An ongoing process whose outcomes have some uncertainty

Example: randomly returning 4 babies to their mothers: this process might return in 0, 1, 2 or 4 correct matches, each with some probability.

Example: tossing a coin: each toss results in “heads” or “tails” with some probability.

Probability

Definition: the *probability* of a random event is the long-run proportion of times that the event would occur if the random process were repeated over and over under identical conditions.

Example: The probability of a "heads" is 0.5 if a fair coin is repeatedly tossed.

Two ways of analyzing a random process

We can compute the probability of a certain outcome of a random process by either

- *Simulating* the process a large number of times, then computing the proportion of times the event occurred

OR

- Assuming a model for the process and *using exact mathematical calculations*.

Learning Objectives – Inv. B, Day 4

Today, we'll learn how to use exact mathematical calculations to analyze a random process

1. Write out the **sample space** associated with a **random process**

2. Compute the value of a **random variable** for a particular outcome

3. Calculate **probabilities** using **random variables** and the assumption of *equally likely outcomes*.

2. Calculate the **expected value** of a **random variable**

Some terminology and a principle

Sample space – a list of all possible outcomes of a random process

function

Random variable – a map between the sample space of a random process and a set of numbers

Principle of equally-likely outcomes – if all **n** outcomes in the sample space are equally likely to occur, the probability of a particular outcome occurring is **1/n**.

Example: coin toss

- Carry out an exact analysis to compute the probability of at least one heads in 3 tosses of a fair coin.
- Compute the expected number of heads in 3 tosses of a fair coin.

$$X \geq 1$$

1. Write out the **sample space** associated with a **random process**

Sample space

$X=3$	HHH	TTT	$X=0$
$X=2$	HHT	TTH	$X=1$
$X=2$	HTH	THT	$X=1$
$X=2$	THH	HTT	$X=1$

2. Compute the value of a **random variable** for a particular outcome

Let X be the number of heads in 3 coin tosses.

3. Calculate **probabilities** using **random variables** and the assumption of *equally likely outcomes*.

$$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3)$$

↑ probability

$$= 3/8 + 3/8 + 1/8 = 7/8$$

X	0	1	2	3
	1	3	3	1
$P(X=x)$	$1/8$	$3/8$	$3/8$	$1/8$

2. Calculate the **expected value** of a **random variable**

Random Babies - Inv. B cont’d

Generally, we’ll analyze random processes either by simulating the process a large number of times OR by performing exact mathematical calculations.

Try **parts n, o, p, q, r, s, t and u** to see the exact mathematical calculation of the probability of at least one mother receiving the correct baby.

Compare with your answer from the simulation (0.62)