Notation: $P(A) = 0.3$

probability of $\underbrace{\text{event } A}_{\text{a set of outcomes}}$ is 0.3

Axioms Let S be a sample space.

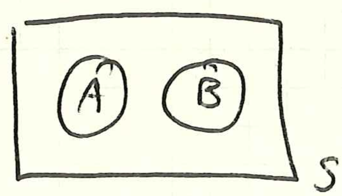
1. $P(S) = 1$
2. $0 \le P(A) \le 1$ for any event A
3. If A and B are disjoint events then $P(A) + P(B) = P(A \text{ or } B)$

Addition Rule (disjoint)

Def: Two events are disjoint if they can't both happen at the same time

Ex: Can't get heads + tails in 1 coin toss.

Venn diagram:



A and B don't overlap if disjoint

Ex: Find probability of rolling a "5" or "6".

$P(\text{"5" or "6"}) = P(\text{"5"}) + P(\text{"6"})$ ← {can't get both a 5 & 6

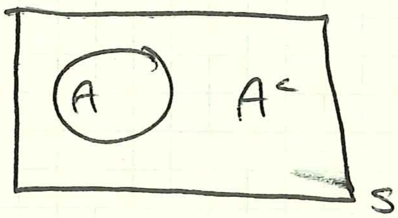
$= 1/6 + 1/6$ ← {equally likely outcomes

$= 1/3$

Complement Rule

Let A^c be everything in the sample space S that is not in A.

Then $P(A^c) = 1 - P(A)$



Ex: Find the probability of rolling less than a "6".

$$P(\text{"}<6\text{"}) = 1 - P(\text{"}=6\text{"}) = 1 - 1/6 = 5/6$$

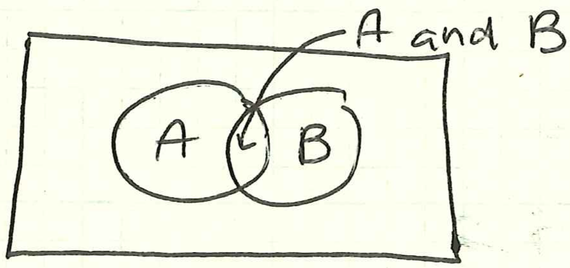
$$S = \{\underbrace{1, 2, 3, 4, 5}_{<6}, 6\}$$

$$\boxed{5/6}$$

General Addition Rule

Let A and B be two events in S.

$$P(A \text{ or } B) = P(A) + P(A) - P(A \text{ and } B)$$



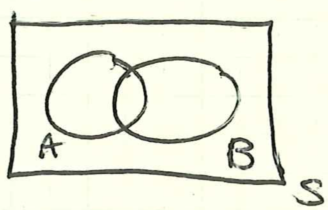
don't double count this!

Conditional Probabilities

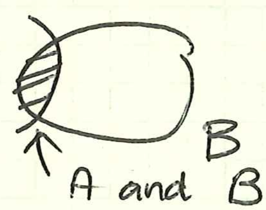
$P(A \mid B)$ Notation

probability of A given that B already happened.

Formula $\quad P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$



B already happened
→ B is the new S



Ex: Find the probability of rolling a "1" given that the roll was odd.

$S = \{1, 2, 3, 4, 5, 6\}$

B = event roll was odd: $P(B) = 3/6$

A = event roll was 1: $P(A) = 1/6$

$P(A \text{ and } B) = 1/6$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{1/6}{3/6} = \frac{1}{3}$$

$\boxed{\frac{1}{3}}$

<u>Def</u>: Two events A and B are <u>independent</u> if $P(A|B) = P(A)$, that is, knowledge of B doesn't change probability of A happening.

Algebra: if $P(A|B) = P(A)$ then

$$P(A) = \frac{P(A \text{ and } B)}{P(B)}$$

<u>Multiplication Rule for independent events</u>

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

if A is independent of B.

<u>Bayes Rule</u>

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$= \frac{P(A) \cdot P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}$$

~~EX: What is the probability of at least~~