Probability Examples:

1. In one of Dr. O's statistics classes, the following table was created using information from 11 students:

	US states visited:	
# of Siblings:	≥20	<20
≤ 2	2	4
> 2	0	5

a. Find the probability that a randomly selected student has visited at least 20 states.
b. Find the probability that a randomly selected student has more than 2 siblings.
c. Find the probability that a randomly selected student has visited at least 20 states given that they have more than 2 siblings.
d. Are the events that a student has visited at least 20 states and has more than 2 siblings independent? Support your answer with a probability calculation.

$2/11 = 0.\overline{18}$ a)

$5/11$ b)

c) $\frac{0}{5} = 0$

d) not independent

$P(A) \neq P(A \mid B)$

OR $P(A \text{ and } B) \neq P(A) \cdot P(B)$

2. Suppose a screening test for a particular disease is 95% accurate, that is, the test is positive for 95% of people with the disease and is negative for 95% of people without the disease. Suppose that it is known that approximately 0.1% of Americans have the disease. If a random American is given the test and it comes back positive, what is the probability that he or she has the disease? Show your work, using either a tree diagram or Bayes rule.

3. Suppose a six-sided die is rolled 5 times, and that each roll is independent of the other rolls.
   a. Assuming that each of the six sides is equally likely, what is the probability of getting the side with 3 dots exactly one time in the 5 rolls?
   b. Assuming that each of the six sides is equally likely, what is the probability of getting the side with 3 dots at least one time in the 5 rolls? Hint: use the complement rule.
   c. Suppose you saw the side with 3 dots come up 4 times in 5 rolls of a die. Do you think that the die is biased towards the side with 3 dots? Support your answer with a probability calculation.

D = event have disease

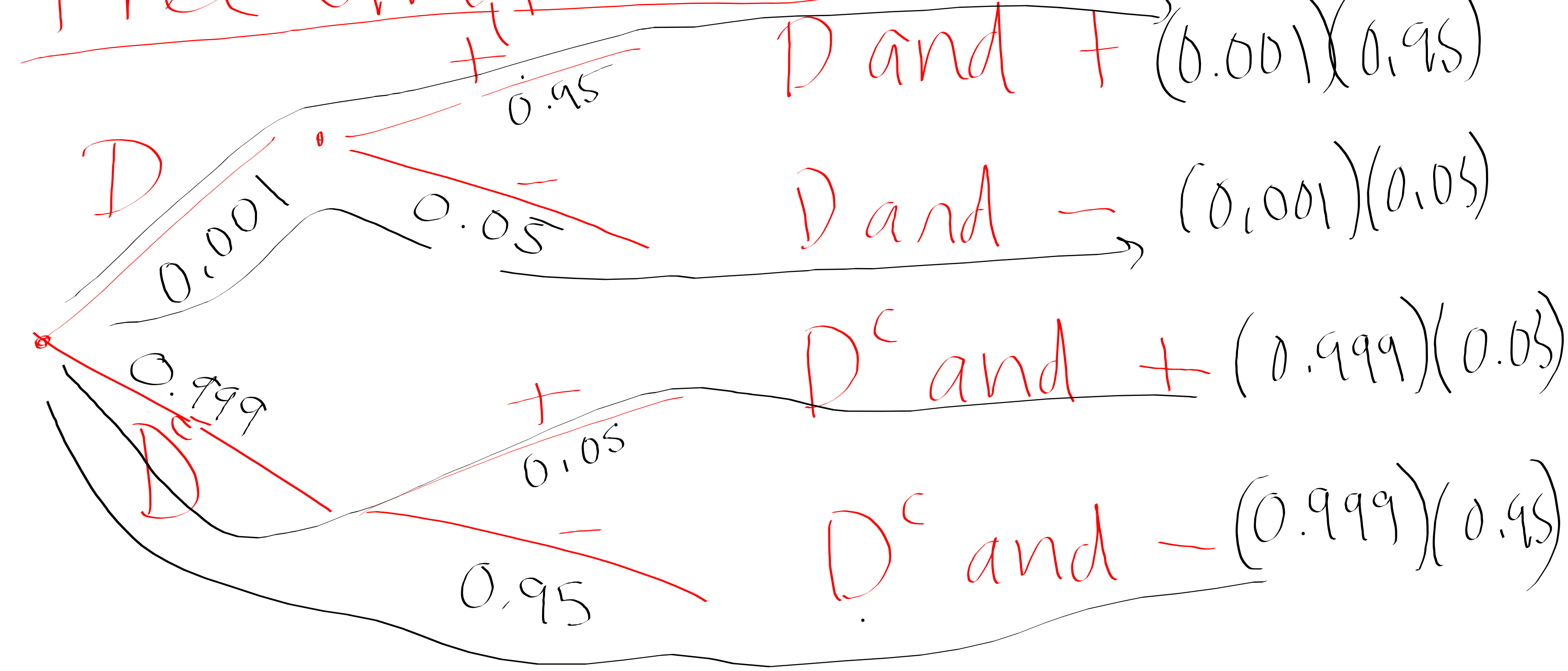
D^c = event doesn't have disease

$+$ = positive test

$-$ = negative test

Want $P(D \mid +)$

Tree diagram



A = event visited ≥ 20

B = event has > 2

$$P(A \mid B) = \frac{\text{size of } A \text{ and } B}{\text{size of } B}$$

$$= \frac{P(A \text{ and } B)}{P(B)}$$

<u>independent</u>:

$$P(A \mid B) = P(A)$$

A = ≥ 20 states, B = ≥ 2 siblings

$$\frac{0}{5} \neq \frac{2}{11}$$

so A and B are <u>not</u> independent

Ex: 2

Bayes Rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Know

$$P(+\mid D) = 0.95$$

$$P(-\mid D^c) = 0.95$$

$$P(D) = 0.001$$

$$P(D \mid +) = \frac{P(D \text{ and } +)}{P(+)}$$

$$= \frac{(0.001)(0.95)}{(0.001)(0.95) + (0.999)(0.05)}$$

$$= 0.018$$