Math 361

Day 8

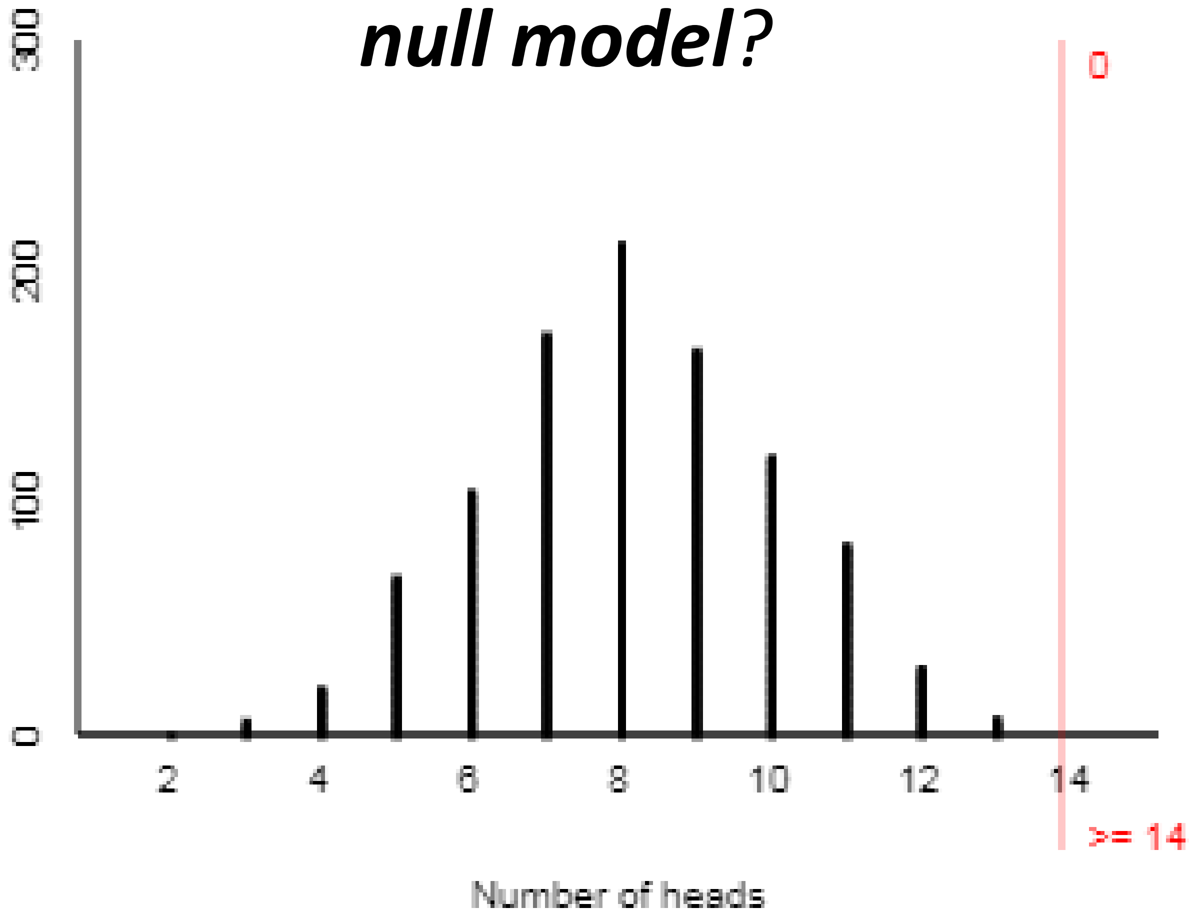
Binomial Random Variables – pages 27 and 28

Inv. 1.2 - Do you have ESP?

Review

<u>Inv. 1.1:</u> Friend or Foe

Is a particular study result consistent with the ***null model****?*



Learning Objectives

1. List the 4 characteristics of a **Binomial Random Process**

2. Specify the parameters for the **Binomial Random Process** associated with a **null model**.

3. Determine whether a given random process is a **Binomial Random Process**.

4. Compute a **p-value** using the **Binomial** probability formula.

5. Decide whether to reject a **null model** based on a **p-value**.

Inv. 1.1 Friend or Foe?

In Inv. 1.1, we were interested in the random process of **babies choosing between two toys**.

In order to determine whether the study result was consistent with the null model explanation we **simulated** another random process…

Inv. 1.1 Friend or Foe?

In Inv. 1.1, we were interested in the random process of **babies choosing between two toys**.

In order to determine whether the study result was consistent with the null model explanation we **simulated** another random process…

coin tosses

Two tools for calculating probabilities

Simulation of the random process (e.g. coin tosses)

or

Exact mathematical calculation

- write out the sample space
- calculate an appropriate random variable X
- compute P(X=x) using equally-likely outcomes

Let's generalize the **exact** method for a coin toss

$P(X=k)$

Goal: Find a formula for P(X≥k) where X is the number of heads in **n** coin tosses.

$P(\text{heads}) = \pi$

$0 \leq \pi \leq 1$

# of ways of choosing r things from n things where order doesn't matter

$$= \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

nCr or nCk

$$n! = n(n-1)(n-2)\cdots 1$$

Factorial

What is a **Binomial Random Process?**

Trials

- Two outcomes possible per trial: success or failure
- Independent trials: one trial's result does not affect another's
- P(success) = π
- There are a fixed number of trials, n

1. List the 4 characteristics of a **Binomial Random Process**

What is a **Binomial Random Process?**

Trials

- Two outcomes possible per trial: success or failure
- Independent trials: one trial's result does not affect another's
- P(success) = π
- There are a fixed number of trials, n

The probability of success may be any number between 0 and 1: the key is that it must be the ***same*** number for each trial

1. List the 4 ch **cess**

2. Specify the **parameters** for the Binomial Random Process associated with a null model.

The **parameters** of a Binomial Random Process are

- n, the number of trials, and
- π, the probability of success in single trial.

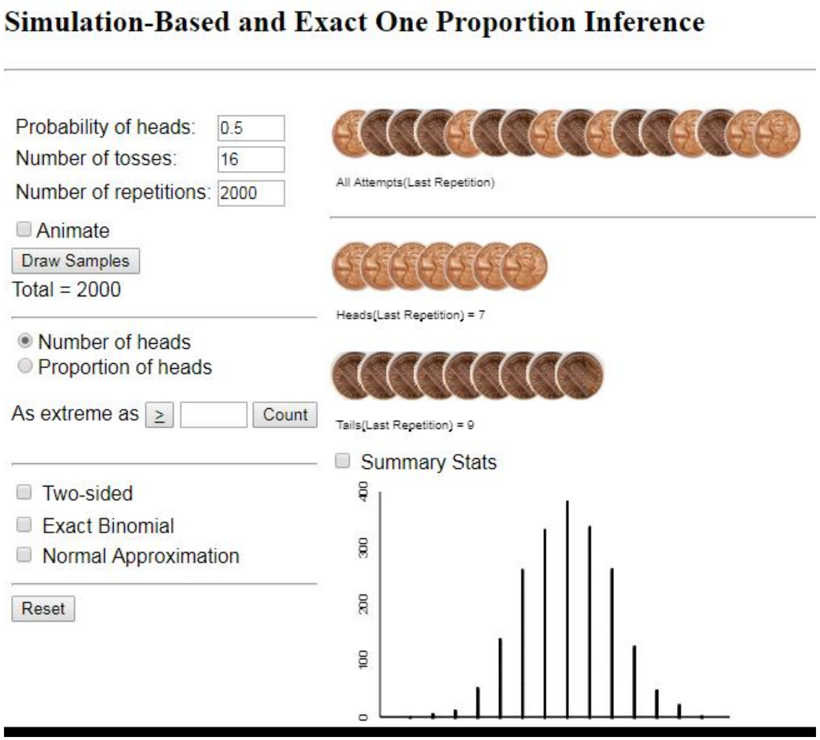
Example: the **parameters** of the random process of babies choosing between two toys are **$n=16$** and **$\pi=0.5$**

Why bother with all this jargon?

If our data collection method matches a **Binomial Random Process**

and we identify the parameters under the null model explanation,

then we know how to calculate a probability to decide whether our data is consistent with the null model



Inv. 1.2: Do you have ESP?

Research Question: Do I have ESP?

Collect Data:
https://psychicscience.org/esp3.aspx

Inv. 1.2: Do you have ESP?

Research Question: Do I have ESP?

Collect Data:
https://psychicscience.org/esp3.aspx

Let's say I got 10 matching cards out of 25 tries.

Do you believe I have ESP or not?

3. Determine whether a given random process is a **Binomial Random Process**.

Assume I'm just guessing and let C be the number of matches in 25 cards.

Try **part (a)** on page 30.

Is C a ***binomial random variable?***

Binomial Random Process

- Two outcomes possible per trial: success or failure
- One trial's result does not affect another's
- P(success) = π
- There are a fixed number of trials, n

Inv. 1.2: Do you have ESP?

Part (a) Let C be the number of correct matches in 25 trials.

*Justify that C is a **binomial random variable**:*

Trial = one guess of 5 possible cards

- *Each guess is correct or not*
- *Assume previous results don't influence your current guess*
- *Assume there is an underlying probability of choosing the "correct" card*
- *There are a fixed number of guesses, 25.*

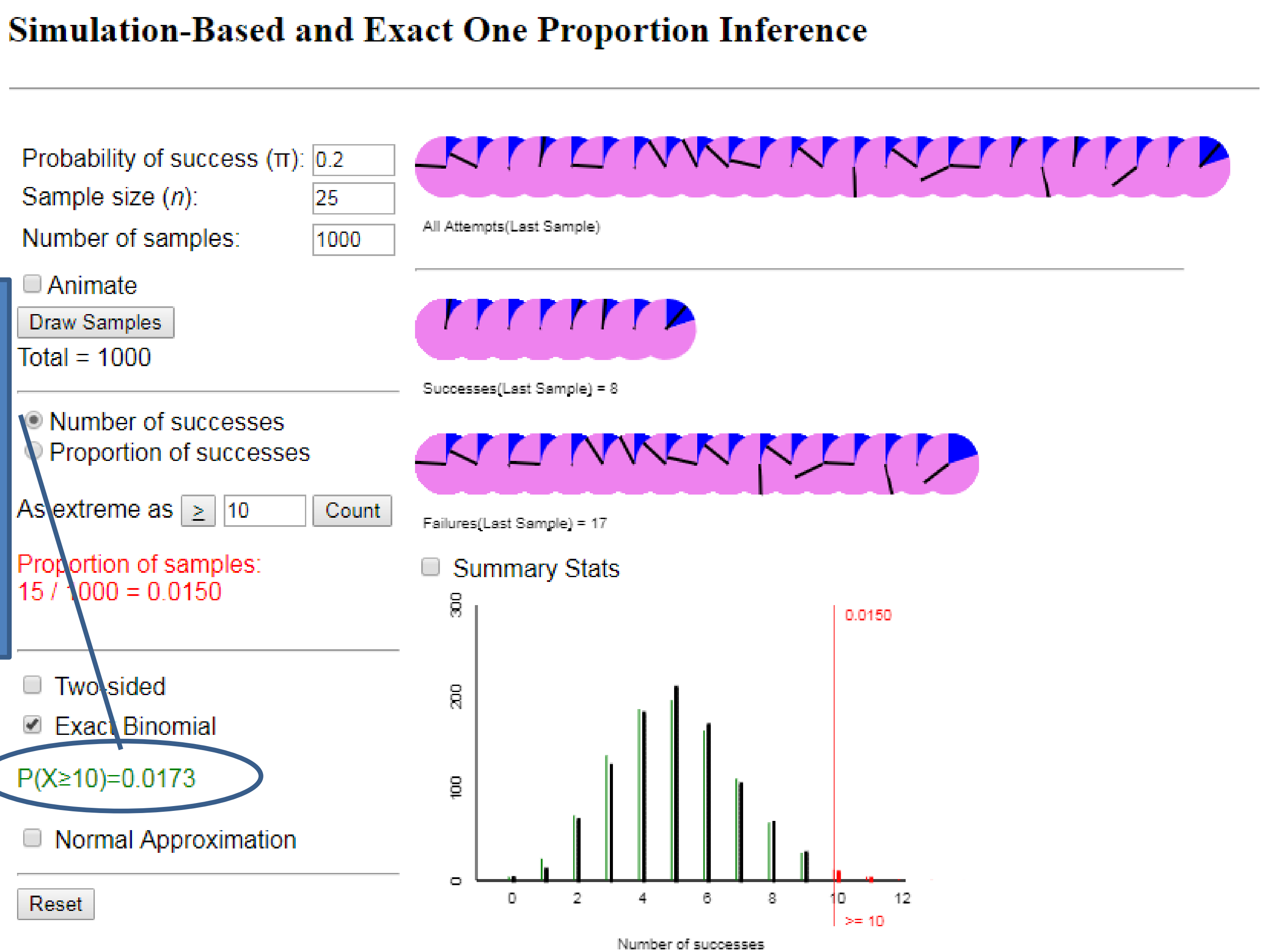
*Identify the **parameters**, assuming the subject is guessing:*

$n = 25$, $\pi = 1/5 = 0.20$

Inv. 1.2: Do you have ESP?

Part (c) Using technology, what is the probability that a guessing subject would get 10 or more correct?

Inv. 1.2, part (c)



This is the **exact** probability of my result (10 cards) assuming I'm just guessing

Do I have ESP?

Assuming I am just guessing, you'd expect to see me get 10 or more matches in 25 cards only 2% of the time…

Do I have ESP?

Assuming I am just guessing, you'd expect to see me get 10 or more matches in 25 cards only 2% of the time…

…so there's evidence **against** the null model that says I'm just guessing!

5. Decide whether to reject a **null model** based on a **p-value**.

More Practice

Is X a **binomial random variable**? If so, find the parameters n and π. If not, state which of the 4 characteristics the random process does not have.

1. Suppose a fair die is rolled three times. Let X be the number of 6's in the three rolls.

2. Suppose four babies are randomly returned to their mothers. Let X be the number of correct matches.

3. Suppose a student is guessing on a multiple choice exam. The exam has 10 questions, with 4 choices each. Let X be the number of questions the student gets right.

Solutions

1. Suppose a fair die is rolled three times. Let X be the number of 2's in the three rolls. **Yes, $n=3$, $\pi=1/6$**

2. Suppose four babies are randomly returned to their mothers. Let X be the number of correct matches.
No: a trial = one pairing and the trials are not independent. Also, the probability of success (a correct match) is not the same between trials.

3. Suppose a student is guessing on a multiple choice exam. The exam has 10 questions, with 4 choices each. Let X be the number of questions the student gets right. **Yes, as long as the student's selected answer on one question does not affect his or her answer on another question (e.g. the student is really randomly choosing, on each question not just picking "c")**