Let $X = \#$ heads in n coin tosses.

Want to make a formula for $P(X=k)$

$P(\text{"heads" in 1 toss}) = \pi$

$0 \le \pi \le 1$

$\underline{n=1}$ $P(X=k) = \pi^k \cdot (1-\pi)^{1-k}$

$P(X=1) = \pi$

$P(X=0) = 1-\pi$

$n=2$

$X=k$	$P(X=k)$
$k=2$	$\pi^{2}(1-\pi)^{0}$
$k=1$	$2\pi^{1}(1-\pi)^{1}$
$k=0$	$(1-\pi)^{2}\pi^{0}$

$$P(X=k) = \underline{\quad}\,\pi^{k}(1-\pi)^{2-k}$$

$n = 4$

$X = k$	$P(X = k)$
4	$1\,\pi^4$
3	$4\,\pi^3(1-\pi)$
2	$6\,\pi^2(1-\pi)^2$
1	$4\,\pi(1-\pi)^3$
0	$1\,(1-\pi)^3$

Cachlator ~~calculator~~

n $\boxed{nCr}$ r=k

$\binom{4}{2}$ 4 $\boxed{nCr}$ 2 = 6

$\binom{2}{2}$ 2 $\boxed{nCr}$ 2 = 1

Binomial Random Process

Assume:

1. trial is a success or failure
2. P(success) = π is the same for all trials
3. n trials
4. trials are independent

Ex 3 let $X = \#$ of 3's in $n=5$ rolls of a die.

success $\to$ $\pi = 1/6$

$$P(X=1) = \binom{5}{1}\left(\frac{1}{6}\right)^1\left(1-\frac{1}{6}\right)^{5-1}$$

exactly 1 "3" in 5 rolls

$$= 5\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4 \approx 0.401$$

$n=1$

Sample space	X	Prob.
H	1	π
T	0	$1-\pi$

$n = 2$

Sample space	X	Prob
HH	2	π^2
HT	1	$\pi(1-\pi)$
TH	1	$(1-\pi)\pi$
TT	0	$(1-\pi)^2$

$n = 3$

$X = k$	$P(X = k)$
3	π^3
2	$3\pi^2(1-\pi)$
1	$3\pi(1-\pi)^2$
0	$(1-\pi)^3$

"magic" number

$$\binom{n}{k} = \boxed{nCk} = \frac{n!}{k!(n-k)!}$$

$$\boxed{nCr}$$

where $X! = x \cdot (x-1)(x-2) \cdots (2)(1)$

Let X = # successes in n trials then

$$P(X=k) = \binom{n}{k} \pi^k (1-\pi)^{n-k}$$

$\boxed{nCr}$ in most calculators

$$P(X \geq k)$$
$$= P(X=k) + P(X=k+1) + \cdots + P(X=n)$$

calculate in applet