

kNN ok

kNN ☹️

$n \gg p$  or

$n \ll p$

can fit Linear Regression

Linear regression blows up

# Statistical Machine Learning

Linear, Ridge and LASSO Regression and kNN

A comparison when  $n \approx p$

Day 18

$n = \# \text{ examples}$

$p = \# \text{ predictors}$

# AI in the news...

“As part of an effort to combat the US’s growing prison population, the US attorney-general is required to develop an ‘evidence-based’ risk assessment system by July 2019 to help decide how long inmates remain incarcerated.”

FT, 27 April/28 April 2019

# Let's build a model to predict "Violent Crime"



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## Communities and Crime Unnormalized Data Set

Download: [Data Folder](#), [Data Set Description](#)

**Abstract:** Communities in the US. Data combines socio-economic data from the '90 Census, law enforcement data from the 1990 Law Enforcement Management and Admin Stats survey, and crime data from the 1995 FBI UCR

<b>Data Set Characteristics:</b>	Multivariate	<b>Number of Instances:</b>	2215	<b>Area:</b>	Social
<b>Attribute Characteristics:</b>	Real	<b>Number of Attributes:</b>	147	<b>Date Donated</b>	2011-03-02
<b>Associated Tasks:</b>	Regression	<b>Missing Values?</b>	Yes	<b>Number of Web Hits:</b>	121688

### Source:

- Creator: Michael Redmond (redmond 'at' lasalle.edu); Computer Science; La Salle University; Philadelphia, PA, 19141, USA
- culled from 1990 US Census, 1995 US FBI Uniform Crime Report, 1990 US Law Enforcement Management and Administrative Statistics Survey, available from ICPSR at U of Michigan.
- Donor: Michael Redmond (redmond 'at' lasalle.edu); Computer Science; La Salle University; Philadelphia, PA, 19141, USA

# Available predictors...

- 1 -- population: population for community: (numeric - expected to be integer)
- 2 -- householdsize: mean people per household (numeric - decimal)
- 3 -- racePctBlack: percentage of population that is african american (numeric - decimal)
- 4 -- racePctWhite: percentage of population that is caucasian (numeric - decimal)
- 5 -- racePctAsian: percentage of population that is of asian heritage (numeric - decimal)
- 6 -- racePctHisp: percentage of population that is of hispanic heritage (numeric - decimal)
- 7 -- agePct12t21: percentage of population that is 12-21 in age (numeric - decimal)
- 8 -- agePct12t29: percentage of population that is 12-29 in age (numeric - decimal)
- 9 -- agePct16t24: percentage of population that is 16-24 in age (numeric - decimal)
- 10 -- agePct65up: percentage of population that is 65 and over in age (numeric - decimal)
- 11 -- numbUrban: number of people living in areas classified as urban (numeric - expected to be integer)
- 12 -- pctUrban: percentage of people living in areas classified as urban (numeric - decimal)
- 13 -- medIncome: median household income (numeric - may be integer)
- 14 -- pctWWage: percentage of households with wage or salary income in 1989 (numeric - decimal)
- 15 -- pctWFarmSelf: percentage of households with farm or self employment income in 1989 (numeric - decimal)
- 16 -- pctWInvInc: percentage of households with investment / rent income in 1989 (numeric - decimal)
- 17 -- pctWSocSec: percentage of households with social security income in 1989 (numeric - decimal)
- 18 -- pctWPubAsst: percentage of households with public assistance income in 1989 (numeric - decimal)

...

- 106 -- PolicReqPerOffic: total requests for police per police officer (numeric - decimal)
- 107 -- PolicPerPop: police officers per 100K population (numeric - decimal)
- 108 -- RacialMatchCommPol: a measure of the racial match between the community and the police force.
- 109 -- PctPolicWhite: percent of police that are caucasian (numeric - decimal)
- 110 -- PctPolicBlack: percent of police that are african american (numeric - decimal)
- 111 -- PctPolicHisp: percent of police that are hispanic (numeric - decimal)
- 112 -- PctPolicAsian: percent of police that are asian (numeric - decimal)
- 113 -- PctPolicMinor: percent of police that are minority of any kind (numeric - decimal)
- 114 -- OfficAssgnDrugUnits: number of officers assigned to special drug units (numeric - expected to be integer)
- 115 -- NumKindsDrugsSeiz: number of different kinds of drugs seized (numeric - expected to be integer)
- 116 -- PolicAveOTWorked: police average overtime worked (numeric - decimal)
- 117 -- LandArea: land area in square miles (numeric - decimal)
- 118 -- PopDens: population density in persons per square mile (numeric - decimal)
- 119 -- PctUsePubTrans: percent of people using public transit for commuting (numeric - decimal)
- 120 -- PolicCars: number of police cars (numeric - expected to be integer)
- 121 -- PolicOperBudg: police operating budget (numeric - may be integer)
- 122 -- LemasPctPolicOnPatr: percent of sworn full time police officers on patrol (numeric - decimal)
- 123 -- LemasGangUnitDeploy: gang unit deployed (numeric - integer - but really nominal - 0 means NO, 10 means YES,
- 124 -- LemasPctOfficDrugUn: percent of officers assigned to drug units (numeric - decimal)
- 125 -- PolicBudgPerPop: police operating budget per population (numeric - decimal)

# The Crime dataset

There are  $p=125$  quantitative pieces of information available for  $n = 2215$  communities to predict the number of violent crimes per 100,000.

Let

$Y$  = number of violent crimes per 100,000 people

$\mathbf{X} = (1, X_1, \dots, X_{125})$

True Relationship:  $Y = f(\mathbf{X}) + \varepsilon$

# Which methods are appropriate to try?

- Linear regression ✓
- kNN regression ✓
- kNN classification ✗
- Logistic regression ✗
- LDA ✗
- QDA ✗
- Ridge regression ✓

There are  $p=125$  quantitative pieces of information available for  $n = 2215$  communities to predict the number of violent crimes per 100,000.

Let

$Y$  = number of violent crimes per 100,000 people

$\mathbf{X} = (1, X_1, \dots, X_{125})$

# Training/selection/assessment

Unfortunately, there are a lot of “?” in the dataset...

When I remove communities with at least one piece of missing information, there are only  $n=319$  communities left.

I'll omit the assessment step and just train and select a model to suggest to the Attorney-general.

I'll use 3-fold cross-validation, means that models will be trained on about 200 communities. So  $(n = 200) \approx (p=125)$



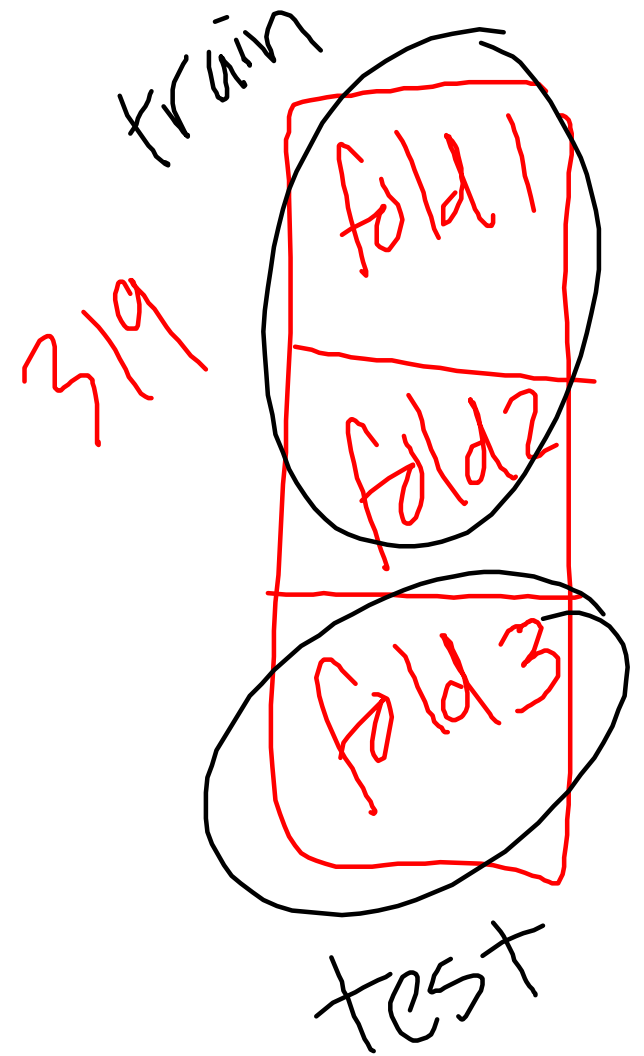
# Selection by MAE

I'll select the model with the best cross-validated  
Mean Absolute Error (MAE)

$$MAE = \frac{1}{m} \sum_{i=1}^m |y_i - \hat{y}_i|$$

all  $m$  values in test set

*This is an easy measure to interpret – it's just the average error of the model*

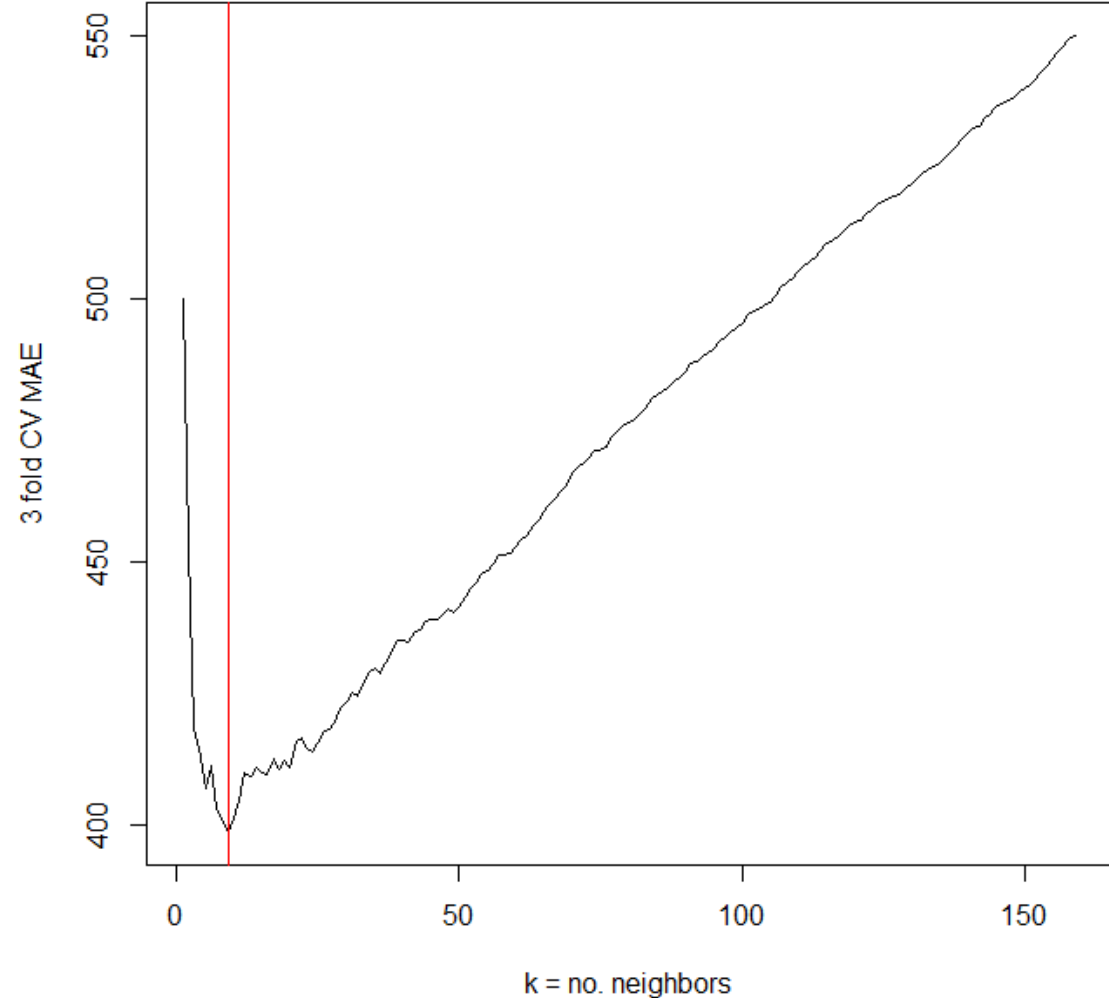


repeat

# kNN Regression

3-fold cross-validated MAE: 398.7 at k=9

*The model is off, on average, by 399 violent crimes per 100,000 people.*

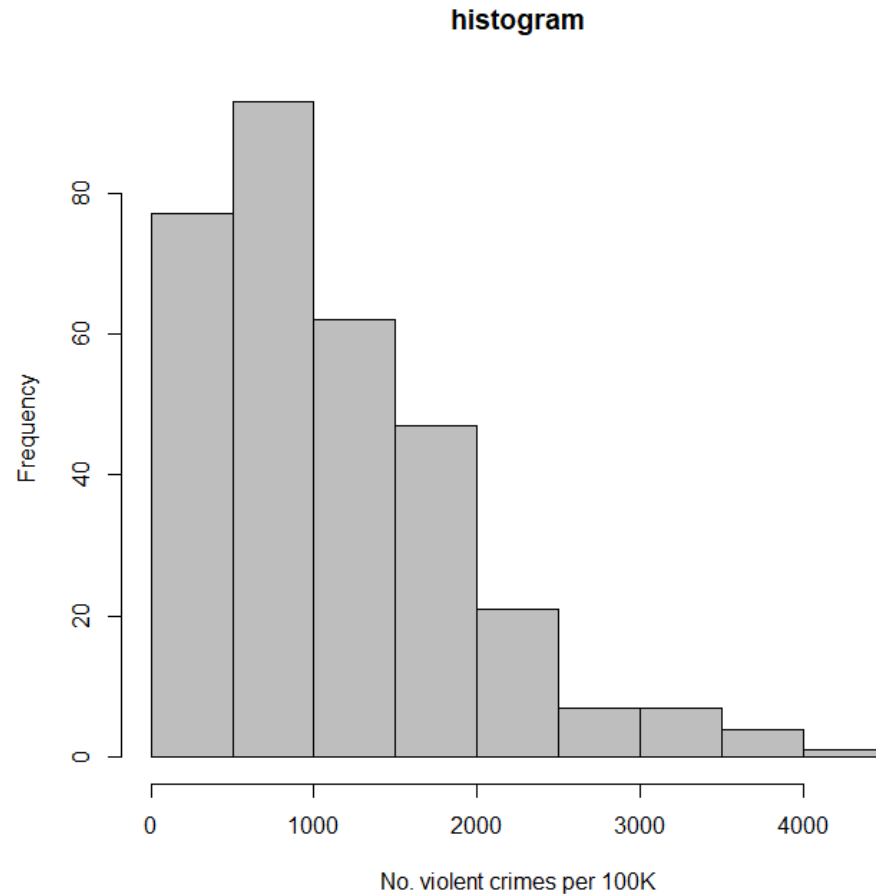


# Linear Regression

3-fold cross-validated MAE: 666.9

*The model is off, on average, by 667 violent crimes per 100,000 people.*

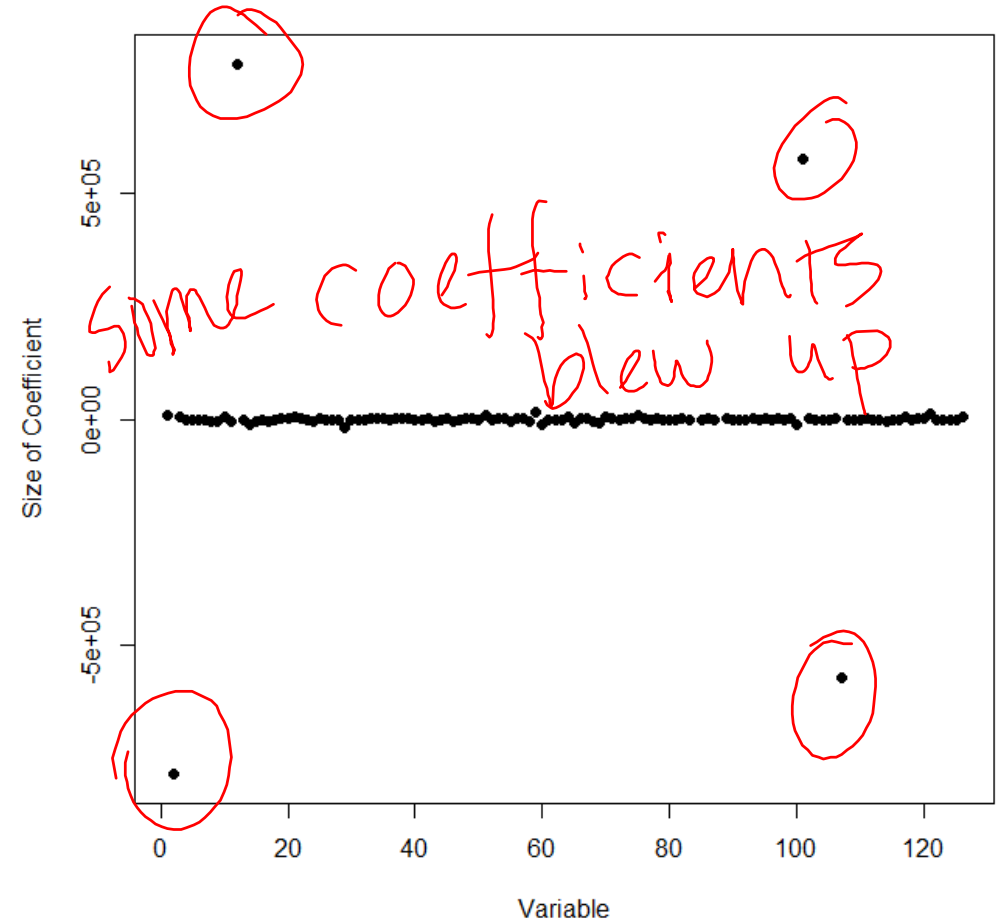
Some perspective:



# What's happening with Linear Regression?

Coefficients: (2 not defined because of singularities)

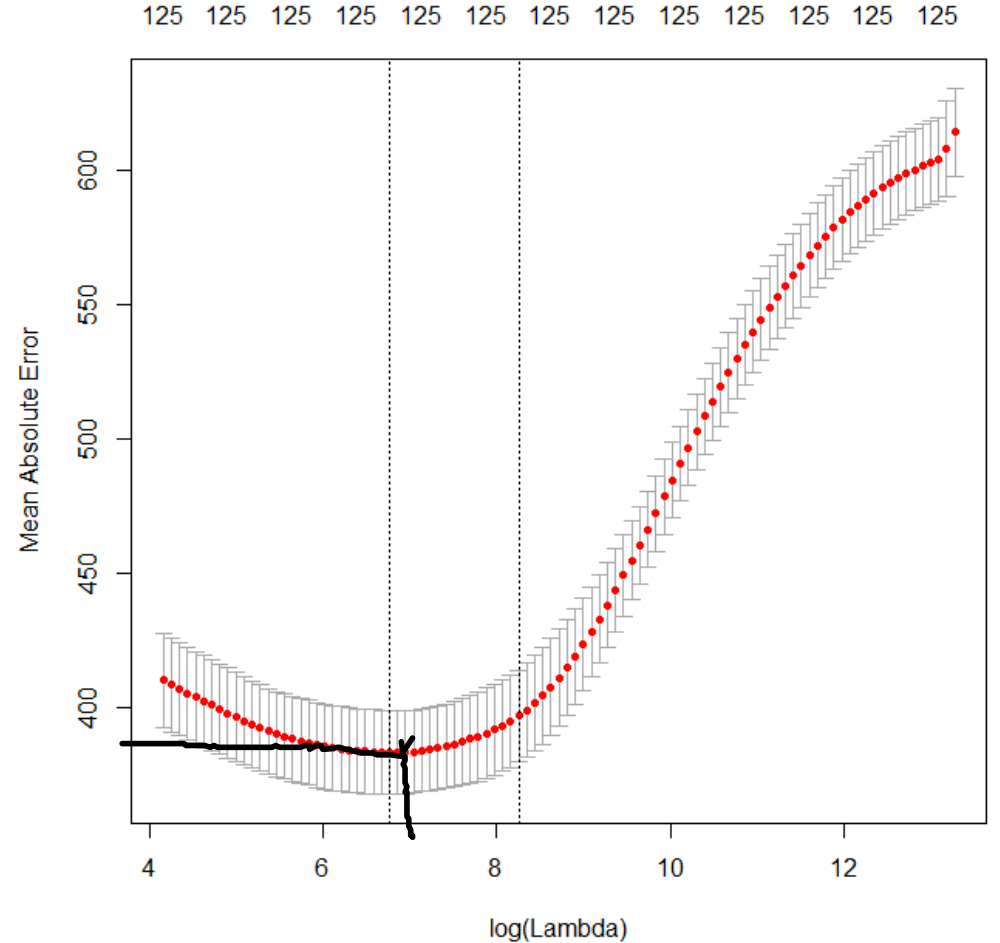
	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	8783.18	4855.33	2.017	0.0451 *
V6	-786386.38	351548.85	-2.237	0.0264 *
V7	6531.90	3870.36	1.688	0.0931 .
V8	-511.28	1611.55	-0.317	0.7514
V9	-319.70	1535.42	-0.208	0.8353
V10	84.62	1358.78	0.062	0.9404
V11	-360.91	1261.88	-0.286	0.7752
V12	-3608.42	4618.87	-0.781	0.4356
V13	-4675.59	4633.36	-1.009	0.3142
V14	4622.65	7321.53	0.631	0.5285
V15	-3919.81	3979.34	-0.985	0.3258
V16	787758.12	351498.75	2.241	0.0261 *
V17	-2117.90	952.00	-2.225	0.0272 *
V18	-12056.30	5431.52	-2.220	0.0276 *
V19	-5666.41	2627.07	-2.157	0.0322 *
V20	-2678.91	1010.73	-2.650	0.0087 **
V21	-3220.86	1288.51	-2.500	0.0133 *
V22	-1497.63	2752.04	-0.544	0.5819
V23	660.73	1421.50	0.465	0.6426
V24	850.01	948.38	0.896	0.3712
V25	5069.53	5104.68	0.993	0.3217
V26	2284.08	3741.31	0.611	0.5422
V27	-898.81	2791.56	-0.322	0.7478
V28	-4018.97	4053.86	-0.991	0.3227
V29	1350.29	1695.56	0.796	0.4268
V30	304.12	921.56	0.330	0.7418
V31	-716.43	1066.91	-0.671	0.5027
V32	-26.39	694.67	-0.038	0.9697
V33	-19731.01	7314.75	-2.697	0.0076 **
V34	-1229.27	1730.22	-0.710	0.4783
V35	-1504.30	1779.27	-0.845	0.3989
V36	-1448.93	1897.00	-0.764	0.4459
V37	2504.51	1602.29	1.563	0.1197



# Ridge Regression

Best cross-validated MAE:  
383.5 at  $\lambda = 865.3445$

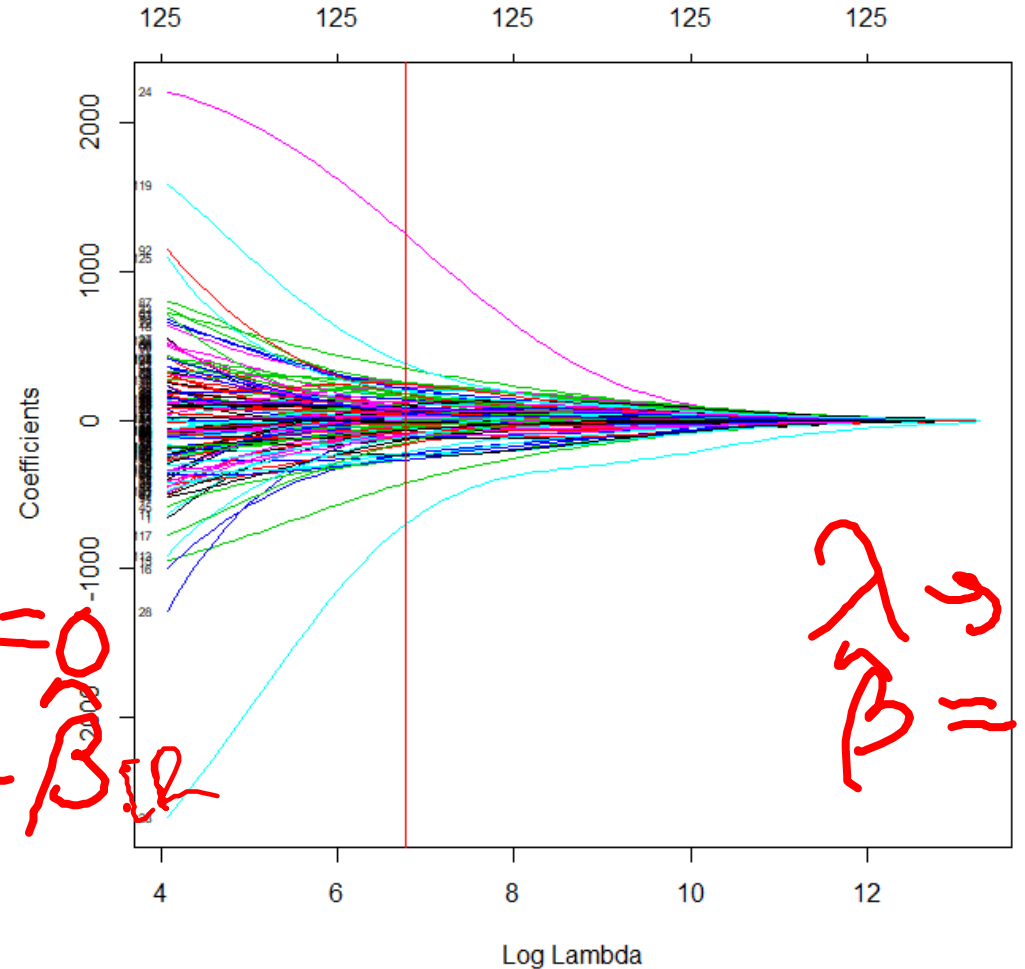
*The model is off, on average, by  
384 violent crimes per 100,000 people*



# Coefficients of "Best" Ridge Regression Model

var    coef    var    coef    var    coef

(Intercept)	1655.5299886	X43	68.1816928	X86	-21.7420117
X1	-25.9406933	X44	-268.6734087	X87	153.9386967
X2	31.6771839	X45	-270.0369352	X88	-12.2074142
X3	253.6215204	X46	-266.7784123	X89	94.9898535
X4	-241.1060424	X47	-241.9013078	X90	0.5753564
X5	-7.7106474	X48	79.1263784	X91	-135.9476513
X6	-0.8404995	X49	48.6251480	X92	171.7000971
X7	13.0517883	X50	37.7958524	X93	67.6431714
X8	-163.1090232	X51	345.1184421	X94	59.1452610
X9	-117.4990281	X52	58.7145850	X95	-101.4125248
X10	14.7159633	X53	-41.1922829	X96	61.3908462
X11	-25.4783550	X54	-32.2638457	X97	70.3107946
X12	26.9740744	X55	33.8815962	X98	60.0060485
X13	-117.7290150	X56	109.9933423	X99	-26.4912937
X14	-114.4387752	X57	-7.8824384	X100	-54.5273248
X15	-425.5980356	X58	13.1565185	X101	-63.8771521
X16	-234.0855010	X59	20.7717562	X102	-100.6357076
X17	-26.2195114	X60	38.4569656	X103	52.7116951
X18	244.8169960	X61	5.8848302	X104	42.9524209
X19	-126.2979337	X62	24.5868176	X105	157.5064347
X20	-85.0308688	X63	100.8602217	X106	-54.5175791
X21	25.4030863	X64	39.9275534	X107	-249.4361533
X22	212.6749419	X65	-15.2593080	X108	-92.3897942
X23	-703.3025639	X66	82.7231759	X109	100.6166765
X24	1251.5794359	X67	-58.8026489	X110	-87.8502968
X25	79.5174122	X68	-24.1535080	X111	180.1292554
X26	56.9404134	X69	159.0554848	X112	37.1604905
X27	141.0275606	X70	214.9933593	X113	-109.4079050
X28	-54.1535938	X71	-59.6524941	X114	31.1973694
X29	97.1378066	X72	69.7405702	X115	-68.5743070
X30	-56.4716972	X73	-239.6287457	X116	240.2279076
X31	22.8392993	X74	-38.3487533	X117	-152.2577582
X32	-9.4608207	X75	242.5791949	X118	83.0179093
X33	255.2446486	X76	-21.5011064	X119	382.8650393
X34	-113.7162632	X77	24.1582147	X120	74.2974657
X35	-216.4232295	X78	119.8836865	X121	-13.4444366
X36	96.9022541	X79	91.1979114	X122	48.0421912
X37	11.1448310	X80	-51.8435693	X123	-54.1244668
X38	-3.0859963	X81	-34.7838020	X124	52.0602389
X39	217.1387325	X82	-20.0464827	X125	193.4160188
X40	139.2293310	X83	8.6270720		
X41	201.2565753	X84	-96.3995678		
X42	218.5047427	X85	-19.6537283		



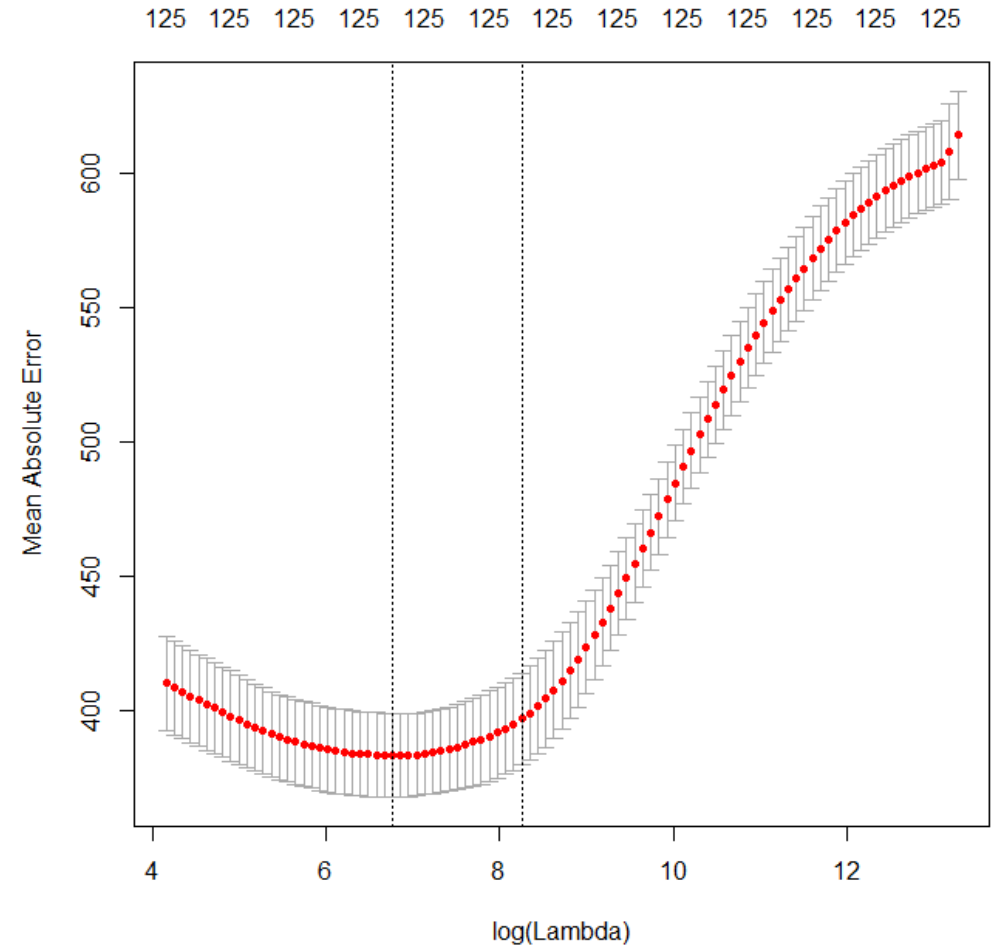
$\hat{\lambda} = 0$   
 $\hat{\beta} = \beta_{OLS}$

$\lambda \rightarrow \infty$   
 $\hat{\beta} = 0$

# LASSO

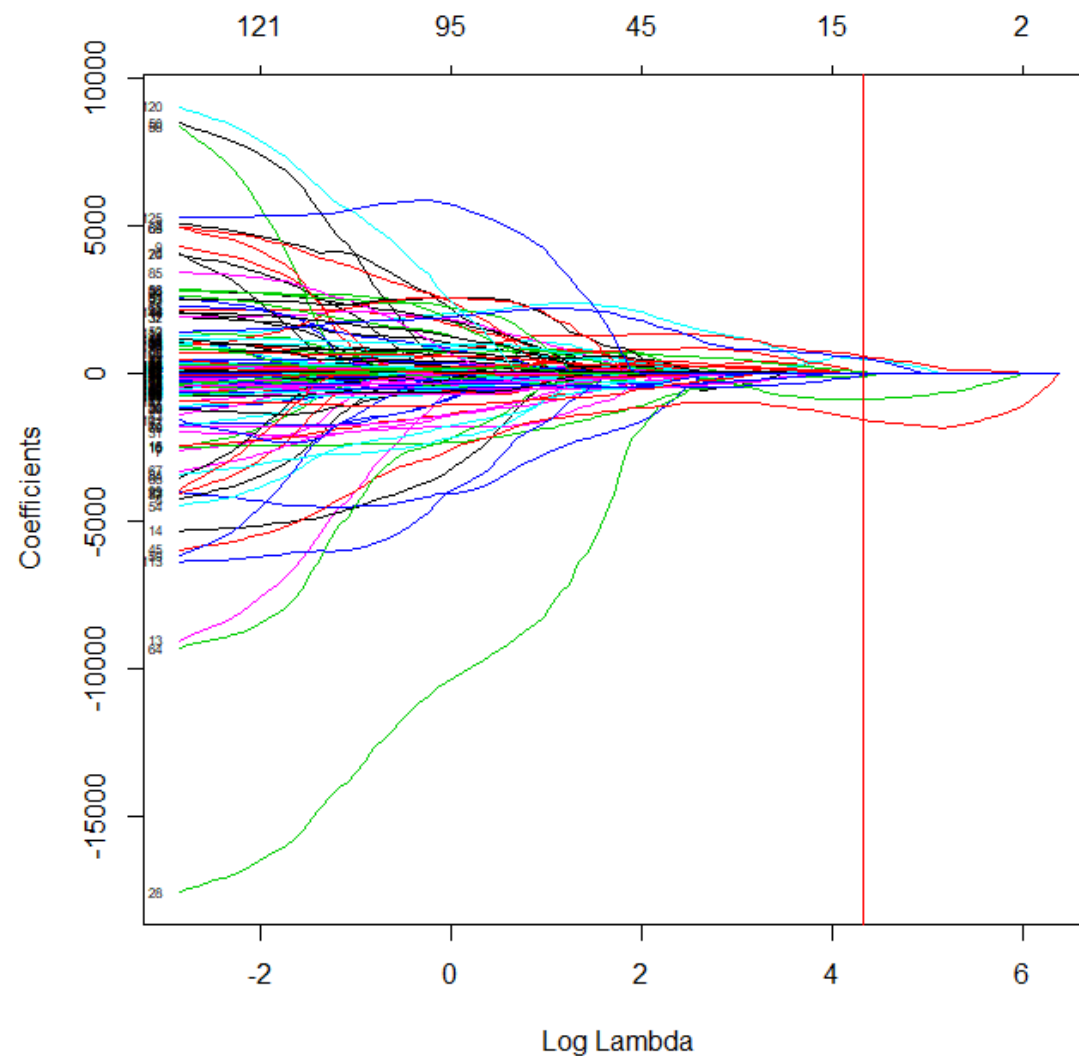
Best cross-validated MAE:  
381.5 at  $\lambda = 75.26325$

*The model is off, on average, by  
382 violent crimes per 100,000 people*



# Coefficients of “Best” LASSO Model

(Intercept)	2297.9344232	X47	.	X93	.
X1	.	X48	.	X94	.
X2	.	X49	.	X95	.
X3	.	X50	.	X96	.
X4	-864.6335292	X51	592.8828027	X97	.
X5	.	X52	.	X98	.
X6	.	X53	.	X99	.
X7	.	X54	.	X100	.
X8	.	X55	.	X101	.
X9	.	X56	.	X102	.
X10	.	X57	.	X103	.
X11	.	X58	.	X104	.
X12	.	X59	.	X105	.
X13	.	X60	.	X106	.
X14	.	X61	.	X107	.
X15	.	X62	.	X108	.
X16	-87.1812780	X63	.	X109	.
X17	.	X64	.	X110	.
X18	.	X65	.	X111	.
X19	.	X66	.	X112	.
X20	.	X67	.	X113	.
X21	.	X68	.	X114	.
X22	.	X69	.	X115	.
X23	.	X70	35.9197947	X116	.
X24	.	X71	.	X117	.
X25	.	X72	.	X118	.
X26	.	X73	.	X119	513.2171350
X27	.	X74	.	X120	.
X28	.	X75	.	X121	.
X29	.	X76	.	X122	.
X30	.	X77	.	X123	.
X31	.	X78	.	X124	.
X32	.	X79	.	X125	.
X33	0.3007172	X80	.		
X34	.	X81	.		
X35	-67.9043955	X82	.		
X36	.	X83	.		
X37	.	X84	.		
X38	.	X85	.		
X39	.	X86	.		
X40	.	X87	.		
X41	.	X88	.		
X42	494.3948635	X89	.		
X43	.	X90	.		
X44	.	X91	.		
X45	-1569.8071172	X92	.		
X46	.				





The “best” model out of kNN, Linear, Ridge and LASSO is...

LASSO, both in terms of performance (lowest MAE) and easy of interpretability:

Predicted number of Violent Crimes = 2298

-865 \* standardized percentage of population that is Caucasian

-87 \* standardized percentage of households with investment / rent income in 1989

0.3 \* standardized percentage of people 16 and over, in the labor force, and unemployed

-68 \* standardized percentage of people 16 and over who are employed in manufacturing

+492 \* standardized percentage of females who are divorced

-1570 \* standardized percentage of families (with kids) that are headed by two parents

+593 \* standardized number of kids born to never married

+36 \* standardized percent of persons in dense housing

+513 \* standardized percent of people using public transit for commuting

# What could we do to improve LASSO's performance?

Try

- Using communities with missing data
- Interactions or transformations of predictors
- Making minimal assumptions about the form of the relationship between  $Y$  and  $X$ ...tomorrow!

# What could we really do to improve the model's performance?

