# Math 243

Day 10

Inv. 1.7 – Power, cont’d

# Announcements

- HW 3 Due Monday, Oct. 15.
- Fill out survey on Blackboard by 5pm on Thursday, Oct. 11
- Take-home portion of midterm exam due Monday, Oct. 15
- In-class portion of midterm exam on Monday, Oct. 15
    - Know definitions of statistical terms
    - Read two handouts
    - Make a single sided, handwritten 8.5 by 11 inch page of notes
    - Bring questions to class on Friday, Oct. 12
    - Need more examples? Solved problems from chapter 1 in Blackboard and previous exam.

# Inv. 1.7 part c

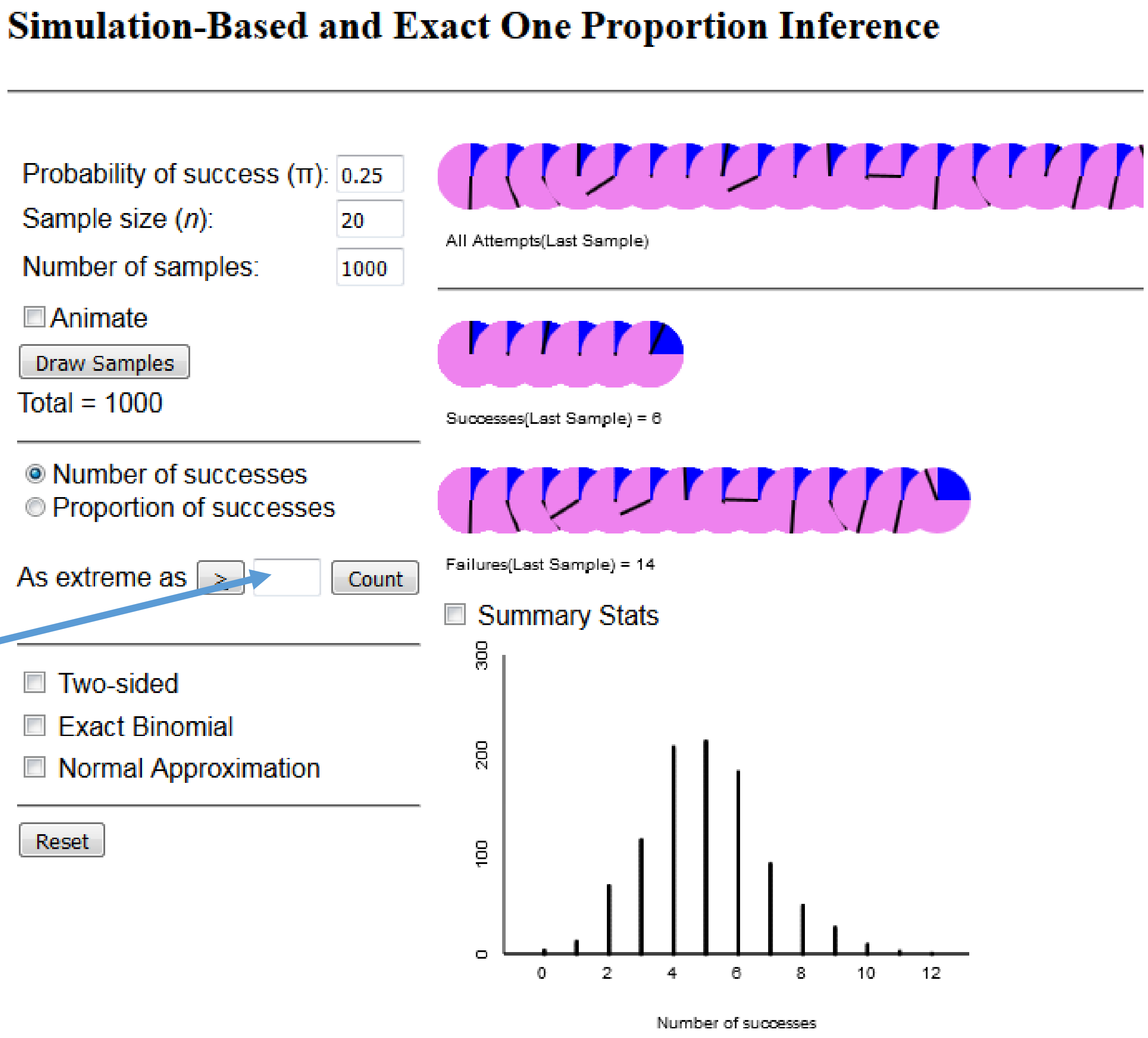
Suppose the manager decides to give the player 20 at-bats in which to prove his improvement.

How many hits would the player need to make out of 20 at-bats in order to convince the manager that he has improved to the point of being in the top 5% of 0.250 hitters?

In statistics jargon, what is the ***rejection region*** for the null hypothesis that corresponds to a ***level of significance*** of 0.05?

**Inv. 1.7 part (e):** How many hits would the player need to make out of 20 at-bats in order to convince the manager that he has improved to the point of being in the top 5% of 0.250 hitters?

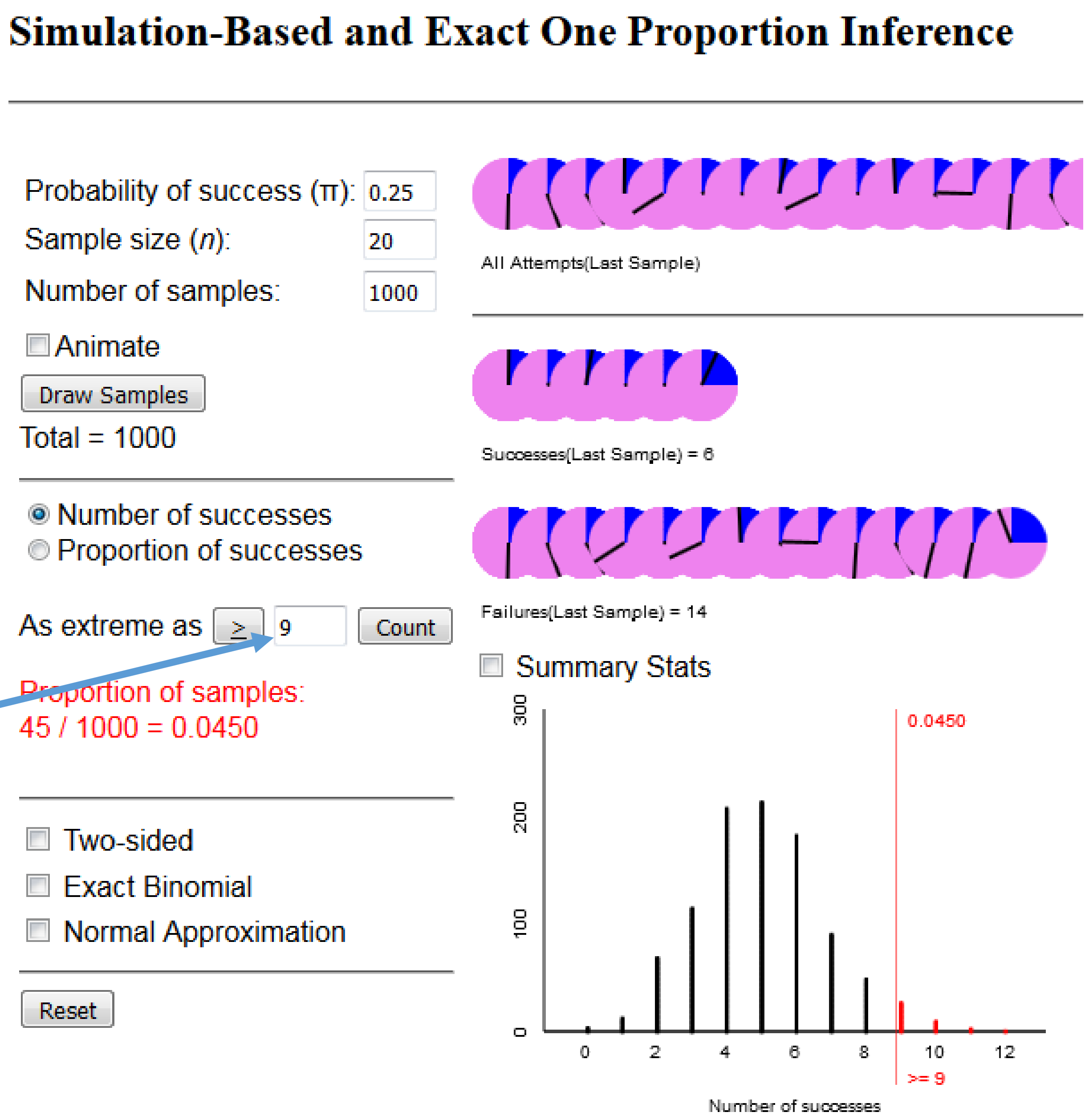
*How many hits should we put in the box so that the probability is no more than 0.05?*



**Inv. 1.7 part (e):** How many hits would the player need to make out of 20 at-bats in order to convince the manager that he has improved to the point of being in the top 5% of 0.250 hitters?

**9 hits**

*If we put 9 in the box so then the probability is no more than 0.05*



# Terminology

**Rejection Region:** the values of the statistic that correspond to rejecting the null hypothesis.

**Example:** the rejection region is **hits ≥ 9**

*If the manager observes **more than 9** hits he will reject the idea that the player is a typical 0.250 hitter and in fact is in the top 5%.*

# More terminology

**Level of significance** (notation α) is the value such that

- If p-value ≤ α we "reject" $\mathbf{H_0}$
- If p-value > α we "fail to reject" $\mathbf{H_0}$

Example: $\alpha = 0.05$

*the manager rejected* $H_0$*:π = 0.250 if the player appeared to be in the top 5%*

# Back to the types of errors

Recall that the player really did improve and is now a 0.333 hitter.

Which type of error is the player worried about the manager making?

| | | Reality | |
|---|---|---|---|
| | | **$H_0 : \pi = 0.250$ is true** | **$H_a : \pi > 0.250$ is true** |
| Decision of the Jury | **$H_a : \pi > 0.250$ is true** | **Type I Error** | **Correct** |
| | **$H_0 : \pi = 0.250$ is true** | **Correct** | **Type II Error** |

# Inv. 1.7: Power

**Probability of a Type II error:**

the probability of incorrectly rejecting $H_0$

**Notation:** β

**Power**: probability of correctly rejecting $H_0$ when $H_a$ is true

**Notation:** 1-β

The player wants to minimize the manager's probability of a type II error and therefore maximize power.

# How can we compute the **Power** of a test?

**Power**: probability of correctly rejecting $H_0$ when a specific $H_a$ is true

1. Find the rejection region for a given level of significance.
2. Simulate the distribution assuming the *alternative* hypothesis is true.
3. Compute the probability of the rejection region assuming $H_a$ is true.

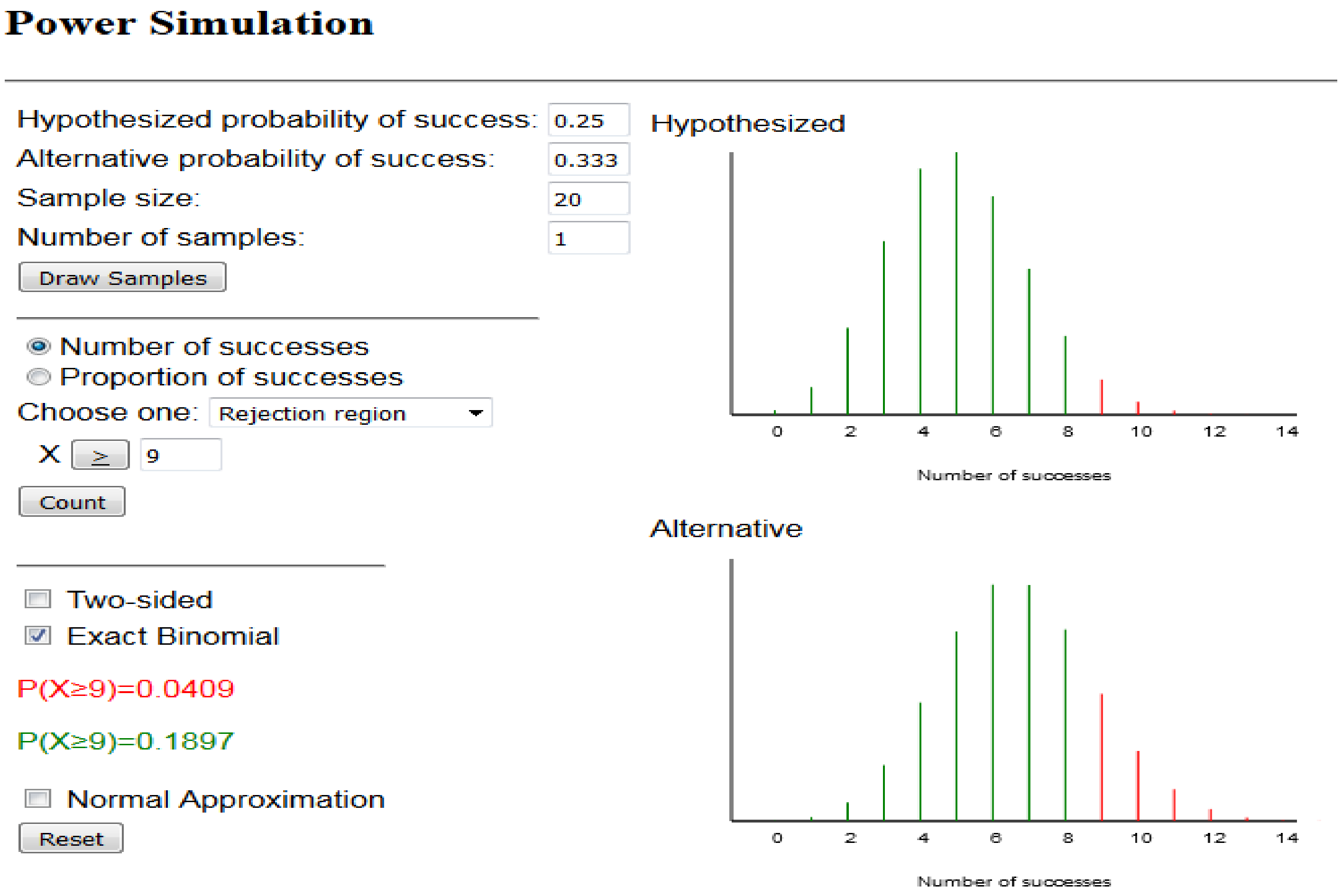
*Sound hard? It's easy when you use an applet!*

# Power Simulation Applet (batting averages)

Power is given in green

*The probability that the manager correctly decides the 0.333 player improved is 0.19…*

*…not very likely.*

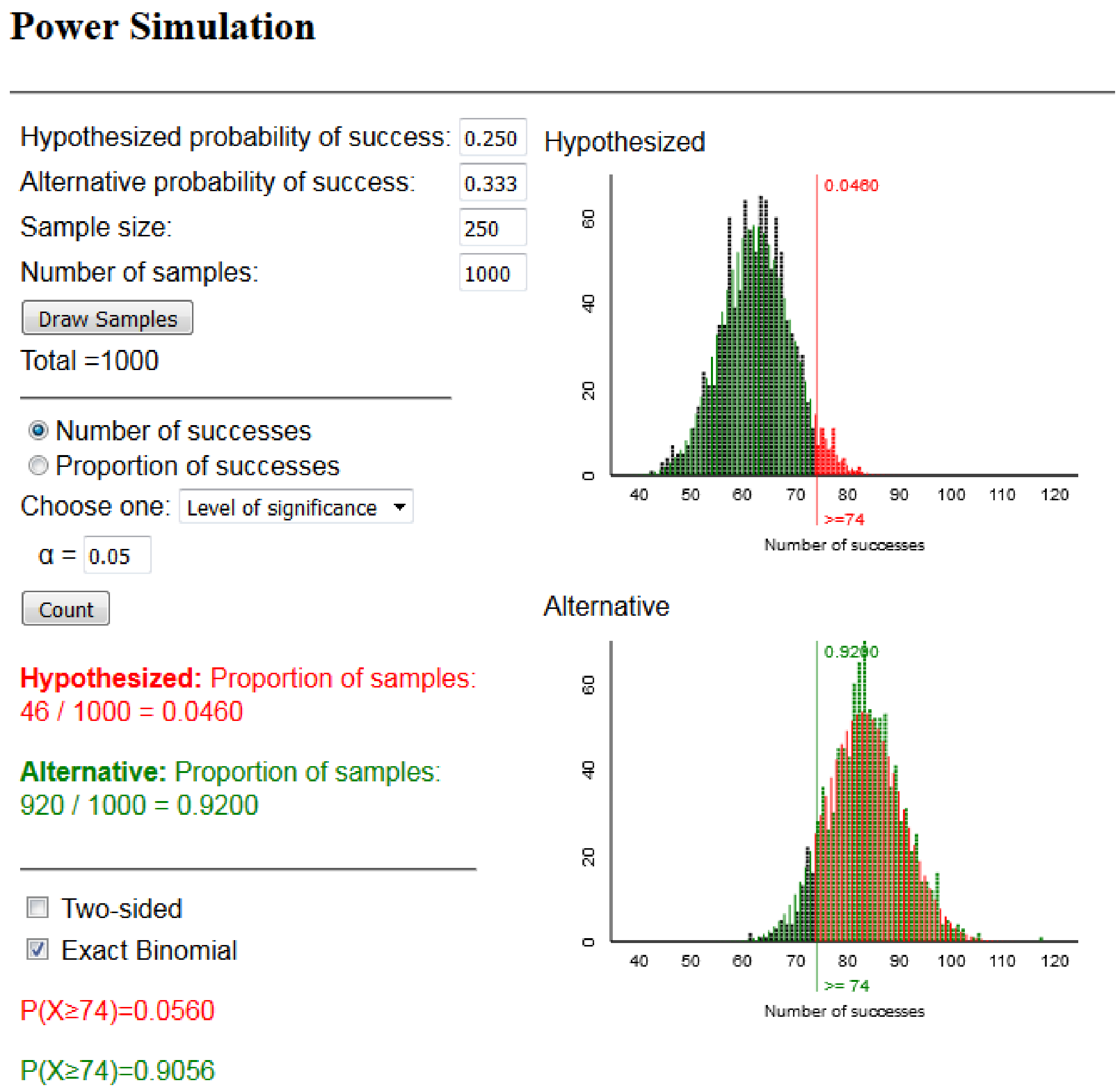


# How can **power** be improved?

Try increasing number of at-bats the manager observes.

*If the manager watches 250 at-bats then the probability he will decide the 0.333 hitter has improved is about 0.91…*

*… much better from the player's perspective*



# What is the probability of a **Type I error**?

The manager is worried about making a **Type I error**, that is, deciding the player improved when really he hasn’t.

The probability Type I error is set by choosing a cutoff value so that if the p-value is below this value then the null hypothesis will be rejected.

This value is called the **level of significance** and is notated $\alpha$.

The manager could decrease the probability of a type I error by using $\alpha = 0.01$ instead of $\alpha = 0.05$.

# Controlling Type I & II error

**Control the probability of Type I error by fixing the level of significance α**:

If you only reject $H_0$ when the p-value is less than α then the probability of making a type I error is at most α

**Control the probability of Type II error (**β**) by adjusting your study design**:

Design your study so that power = 1-β is high (close to 1).

*Try increasing sample size or decreasing α*

# Try practice problem 1.7D on page 61

**Practice Problem 1.7D**

Suppose you want to test a person's ability to discriminate between two types of soda. You fill one cup with Soda A and two cups with Soda B. The subject tastes all 3 cups and is asked to identify the odd soda. You record the number of correct identifications in 10 attempts. Assume a one-sided alternative.

(a) If the subject's actual probability of a correct identification is 0.50, what is the power of this test for a level of significance of $\alpha = 0.50$? [*Hint*: What is the null hypothesis?]

(b) Write a one-sentence interpretation of the power you calculated in (a) in context.

(c) What is the power if you give the subject 20 attempts?

# Next Time: Performing a test of significance **by hand**

We'll develop some tools that will allow us to test

$$H_0: \pi = \text{(some number)}$$

vs.

$$H_a: \pi \neq \text{(some number)}$$

by hand, without a calculator or computer