Math 243

t-distribution – Inv. 2.5

Group work: Inv. 2.5 (a)-(c)

What is a healthy body temperature? Researchers have cited problems with Carl Wunderlich's "axioms on clinical thermometry" and claimed that the traditional value of 98.6°F is out of date (Mackowiak, Wasserman, & Levine, *Journal of the American Medical Association*, 1992). Body temperatures (oral temperatures using a digital thermometer) were recorded for healthy men and women, aged 18-40 years, who were volunteers in Shigella vaccine trials at the University of Maryland Center for Vaccine Development, Baltimore. For these adults, the mean body temperature was found to be 98.249°F with a standard deviation of 0.733°F.

(a) Explain (in words, in context) what is meant by the following symbols as applied to this study: n, $\bar{x}$, s, μ, σ. If you know a value, report it. Otherwise, define the symbol.

n =

$\bar{x}$ =

s =

μ =

σ =

(b) Write a null hypothesis and an alternative hypothesis for testing Wunderlich's axiom using appropriate symbols.

H_o: H_a:

(c) Suppose the axiom is correct and many different random samples of 13 adults are taken from a large normally distributed population with mean 98.6°F. What does the Central Limit Theorem tell you about the theoretical distribution of sample means? (Indicate any necessary information that is missing.)

Inv. 2.5: What's a Normal Body Temperature?

<u>Research Question:</u> Is Wunderlich's axiom (98.6 °F) correct?.

<u>Parameter of Interest:</u> Let μ be the mean body temperature of healthy adults

<u>Test:</u> H_0: μ = 98.6 vs. H_a: μ ≠ 98.6

<u>Data:</u> have $\bar{x}$ = 98.249, s=0.733 for n=13 healthy adults

Can we use the CLT to draw the null distribution of the sample mean?

CLT for sample means

If the population is normally distributed

or

the sample size large enough ($n>30$),

then

the distribution of sample means is approximately normal with

Mean = μ

SD = $\sigma / \sqrt{n}$

where μ is the population mean and σ is the population SD

Apply the CLT to body temperatures, assuming Wunderlich is correct, and H_0: $\mu = 98.6$

If the population is normally distributed

then

the distribution of sample means is approximately normal with

Mean = $\mu = 98.6$

SD = $\sigma / \sqrt{n}$ = ? / $\sqrt{13}$

Problem: we don't know what the standard deviation of the population body temperature is

Apply the CLT to body temperatures, assuming Wunderlich is correct, and H_0: $\mu = 98.6$

If <u>the population is normally distributed</u>

then

the distribution of sample means is approximately normal with

Mean = $\mu = 98.6$

SD ≈ $s / \sqrt{13} = 0.203$

Solution? Try using s=0.733, the standard deviation from the sample of n=13 volunteers (inv. 2.5 (d))

Definition: Standard Error

Standard Error (SE) = standard deviation of a *statistic*

Example: one statistic we've seen before is the sample proportion, $\hat{p}$.

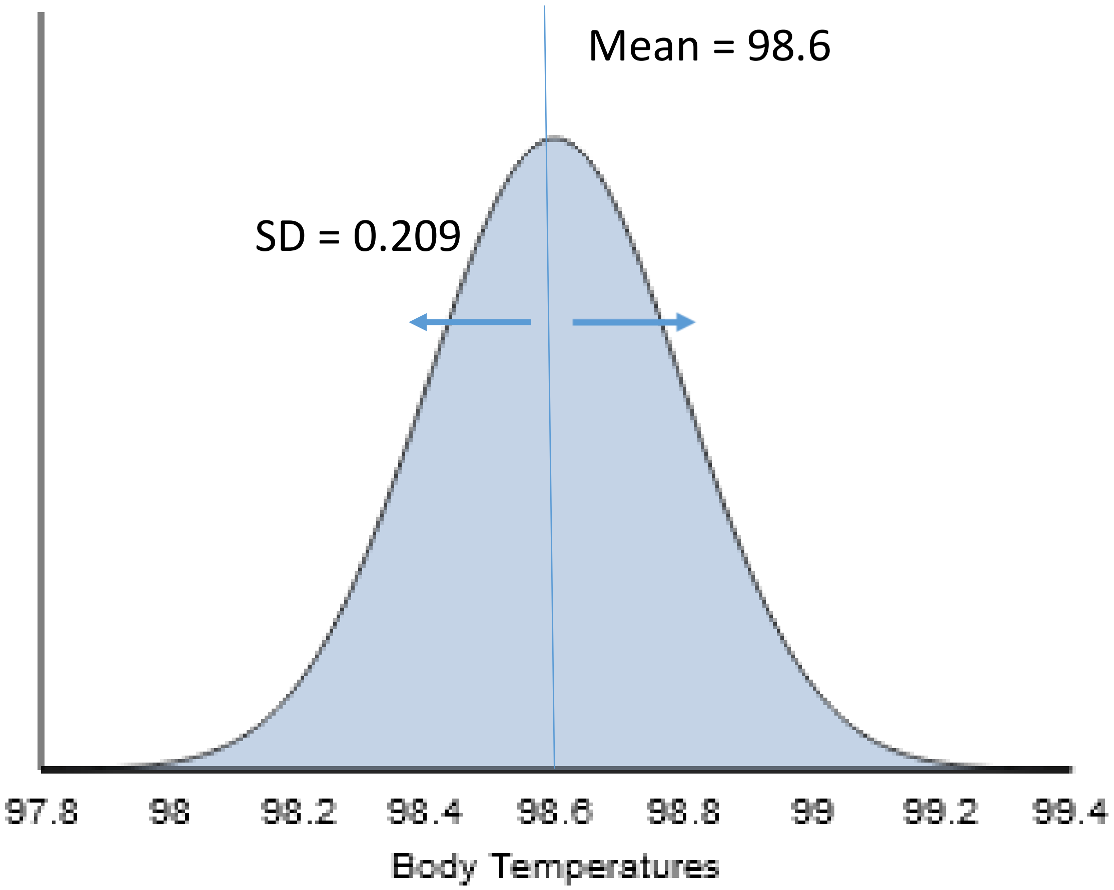
By CLT for sample proportions, its SE is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Example: another statistic is the sample mean, $\bar{x}$.

By CLT for sample means, its SE is $\sigma/\sqrt{n}$

Is our sample consistent with Wunderlich's axiom?

Draw the distribution of sample means assuming H_0: $\mu = 98.6$ is true…



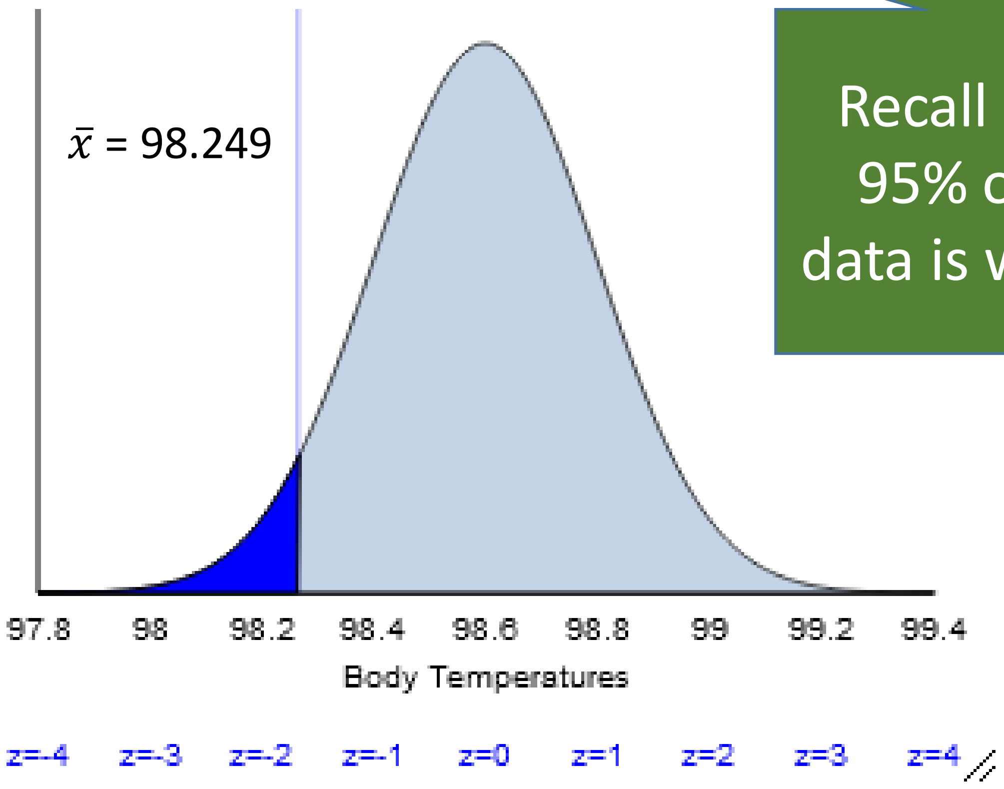
Is our sample consistent with Wunderlich's axiom?

Draw the distribution of sample means assuming H_0: μ = 98.6 is true and check to see if our sample mean is unusual.

Our sample mean of 98.249 is

(98.249-98.6)/0.209
=-1.67 SDs below the mean

This is within 2 SDs of the mean and so is **NOT** unusual if the distribution is normal



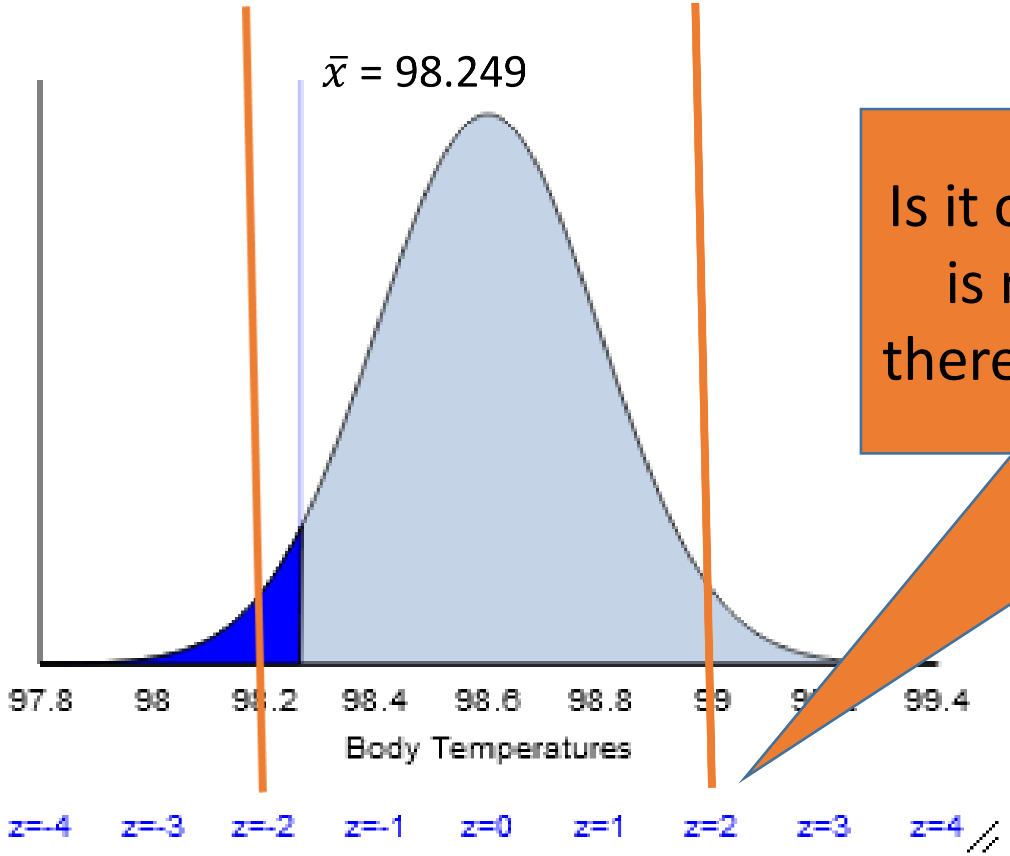
Recall the Empirical Rule says 95% of normally distributed data is within 2 SDs of the mean

Is our sample consistent with Wunderlich's axiom?

Draw the distribution of sample means assuming H_0: μ = 98.6 is true and check to see if our sample mean is unusual.

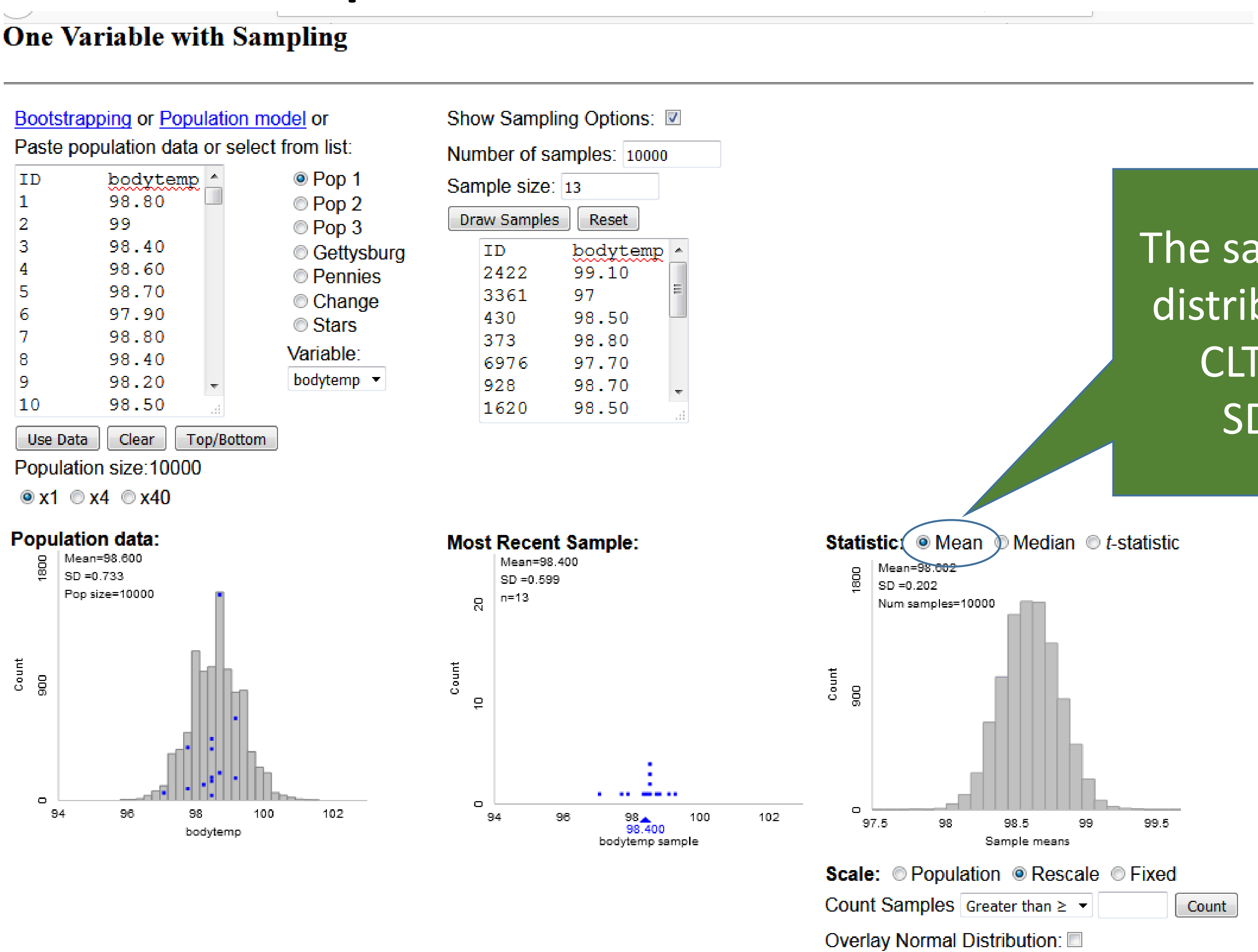
Definition:

The **t-statistic** = $\frac{(\bar{x}-\mu)}{s/\sqrt{n}}$ measures the number of sample SDs from the hypothesized mean



Is it okay to assume the t-statistic is normally distributed? (and therefore use the empirical rule?)

Inv. 2.5, parts h and i

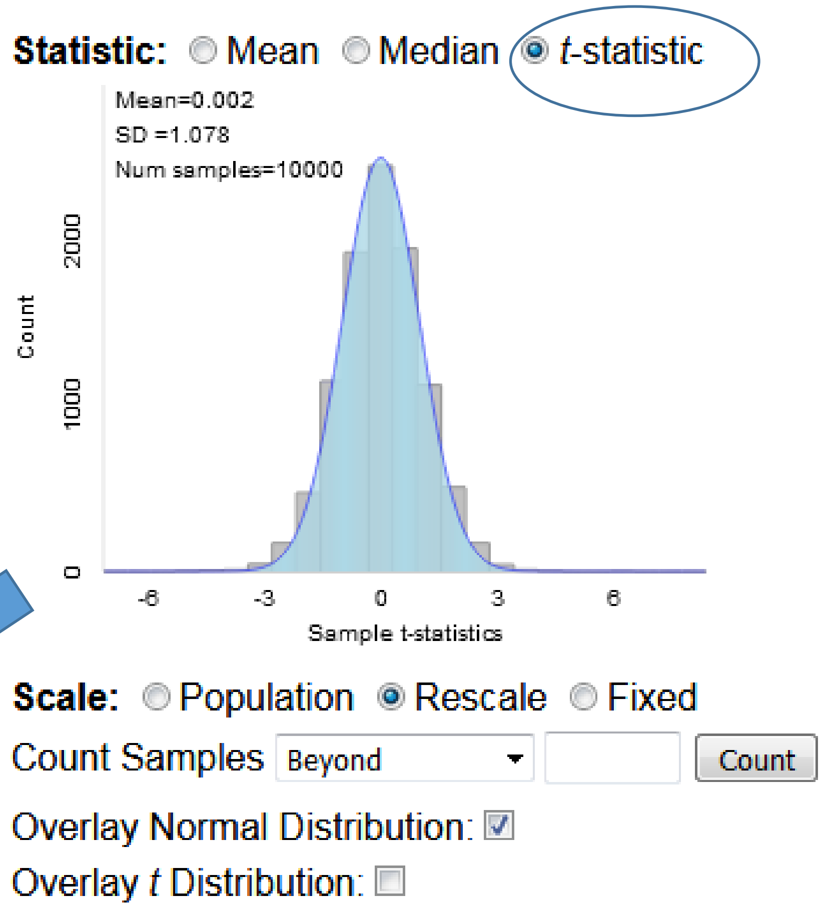


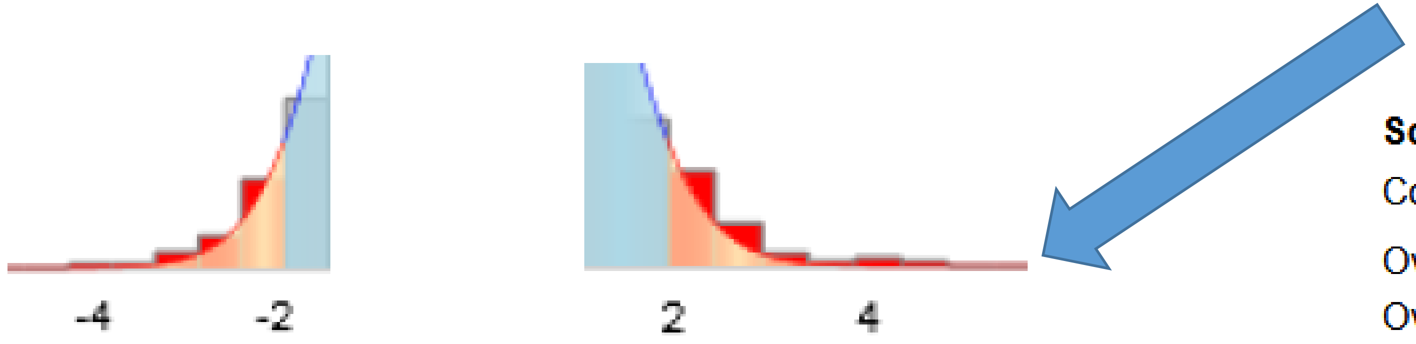
The sample means are normally distributed as predicted by the CLT with mean = 98.6 and SD = 0.733/√13 = 0.203

Inv. 2.5, parts j and k

Is the t-statistic normally distributed?

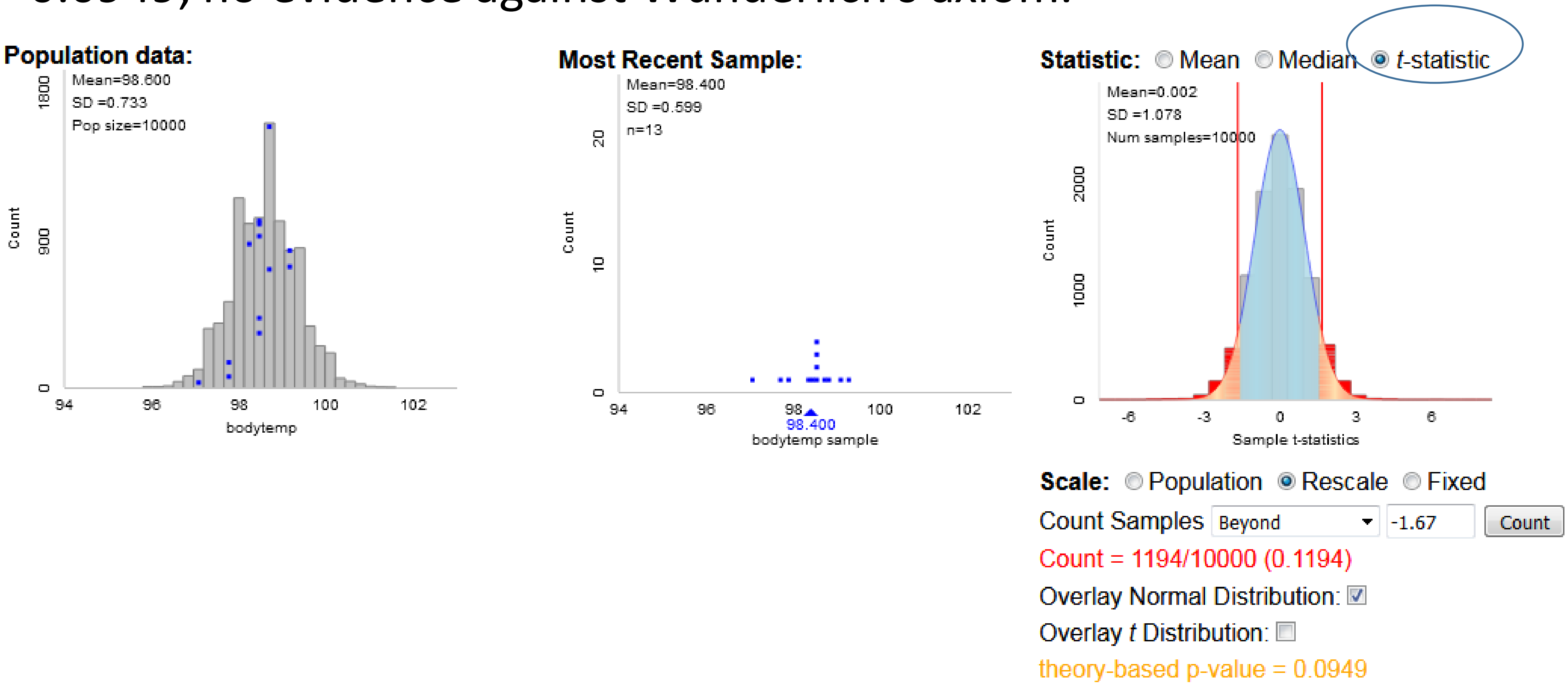
Not exactly, there are more points in the tails of the distribution of the t-statistic than we expect if they were normal.





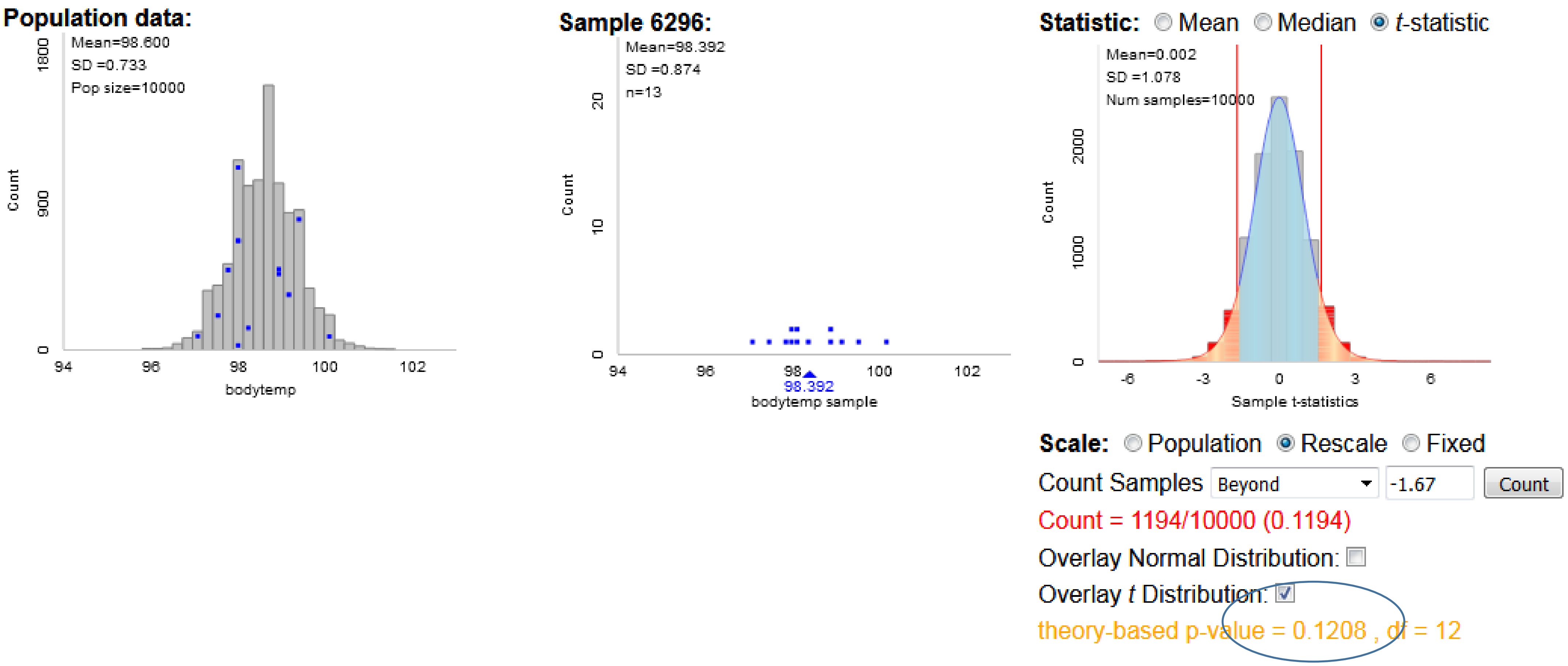
Inv. 2.5, parts j and k

If we did assume the t-statistic was normal, then the p-value would be 0.0949, no evidence against Wunderlich's axiom.



Inv. 2.5, part m

Using the t-distribution, the p-value is 0.1208, much closer to the simulated value of 0.1194.



Use the **t-distribution** to test a population mean when you don’t know the population SD

One-sample *t*-test for μ: So to test a null hypothesis about a population mean when we don’t know the population standard deviation (pretty much always), H_0: $\mu = \mu_0$, we will use the sample standard deviation to calculate the standard error SE($\bar{x}$) and compare the standardized statistic

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

to a *t*-distribution with $n - 1$ degrees of freedom. Theoretically, this approximation requires the population to follow a normal distribution. However, statisticians have found this approximation to also be reasonable for other population distributions whenever the sample size is large. How large the sample size needs to be depends on how skewed the population distribution is. Consequently, we will consider the *t* procedures valid when either the population distribution is symmetric or the sample size is large.

Definition: Test statistic

General Form:

$$\text{Test statistic} = \frac{statistic - null\ value}{standard\ error}$$

This quantity measures the number of standard errors the statistic is from the null value.

Practice Problem 2.5B

A study conducted by Stanford researchers asked children in two elementary schools in San Jose, CA to keep track of how much television they watch per week. The sample consisted of 198 children. The mean time spent watching television per week in the sample was 15.41 hours with a standard deviation of 14.16 hours.

(a) Carry out a one-sample *t*-test to determine whether there is convincing evidence that average amount of television watching per week among San Jose elementary children exceeds fourteen hours per week. (Report the hypotheses, test statistic, p-value, and conclusion at the 0.10 level of significance.)

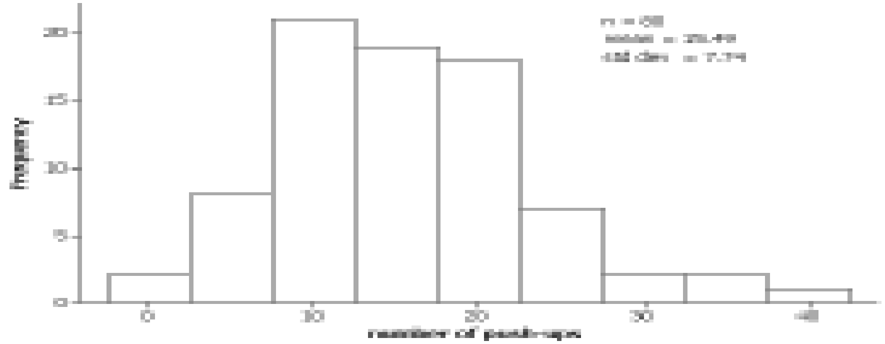
Additional Practice: Example 2.1 on page 175

Example 2.1: Pushing On

Try these questions yourself before you use the solutions following to check your answers.

As in many states, California mandates physical fitness testing at different grade levels. The recommended number of push-ups for 12 year old males is 10-20 and for 13 year old males is 12-25. A sample of 80 7th grade males was obtained at a rural high school in Central California (Wetzel and Hernandez, 2004). Data was gathered using the measurement techniques defined by the state. (The feet are together, hands will be shoulder width apart, the subjects back will be straight, and their eyes will be looking toward the horizon. The arms need to bend to a 90-degree angle, while keeping their back flat. The push-ups are counted at a set tempo without stopping to rest.)

A histogram and summary statistics of the sample data are below:



(a) Describe the distribution of the results in this sample.

(b) Suppose we take random samples of 80 males from a very large population. According to the Central Limit Theorem, what can you say about the behavior of the sampling distribution of the sample means calculated from these samples?

(c) If this were a random sample from a population, would the sample data provide strong evidence that the population mean differs from 20 push-ups? Conduct a significance test to address this question. Also calculate confidence intervals to estimate the population mean with various levels of confidence.

Similarly, use the t-distribution to compute a 95% CI for a population mean when the population standard deviation is unknown.

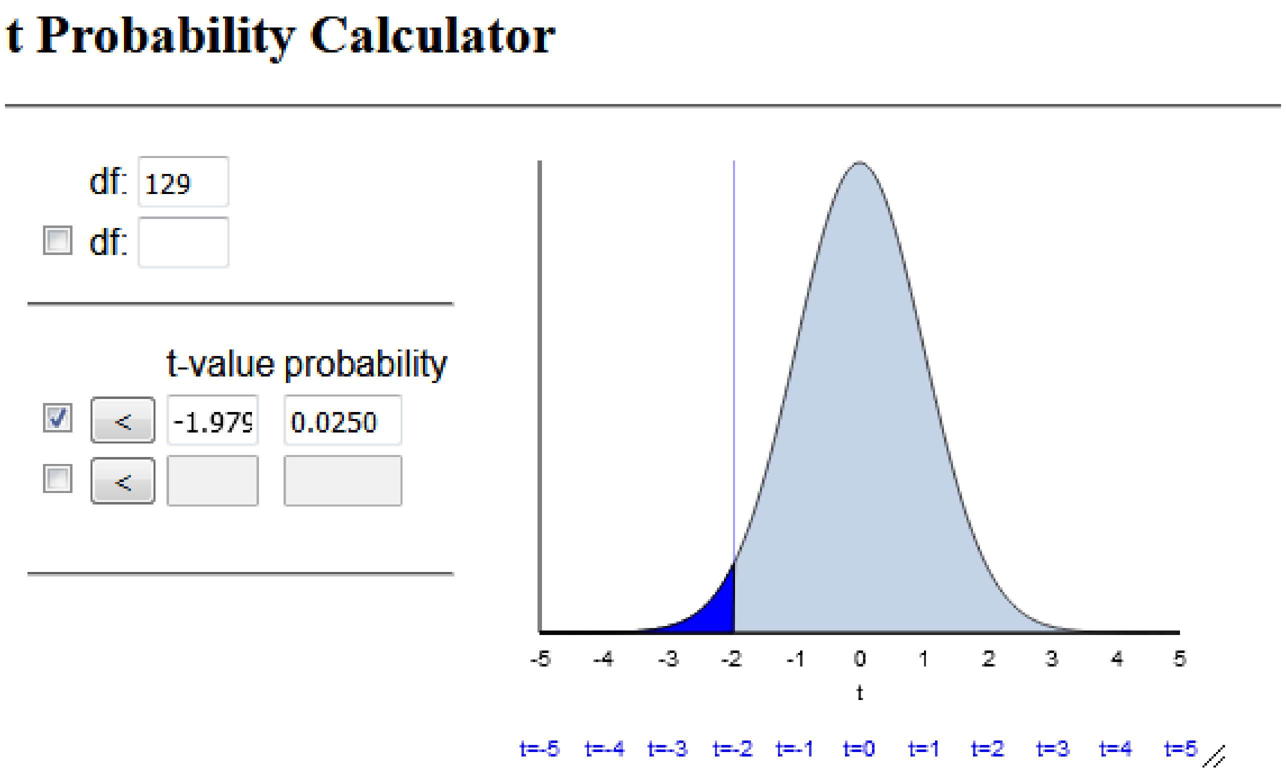
When the sample looks normal or we have a large sample size (n>30), an approximate confidence interval for the population mean is given by

$$\bar{x} \pm t^{*}_{n-1} s / \sqrt{n}$$

Where t^{*}_{n-1} is the **critical value from the t-distribution with n-1 degrees of freedom**

Inv. 2.5, parts (w) and (x)

In a sample of size n=130, the critical value for constructing a 95% CI is 1.979



Inv. 2.5, part x

Construct a 95% CI for the mean body temperature using $\bar{x}$=98.249, s=0.733, n=130 and the t^*_{129}=1.979

$$\bar{x} \pm t^*_{n-1} s / \sqrt{n}$$

Give an interpretation of your interval.