Math 361

"methods" table

Test of Significance for 2 proportions

Methods Table

Classify "methods" by number and **type** of variable and your **purpose**.

Types of variables: Binary or quantitative

Purpose: descriptive or inferential

Tests of Significance

Using a p-value or rejection region to accept or reject H_0

So far, we used one of three techniques to calculate a p-value or rejection region:

1. Simulation (e.g. coin tosses)
2. Exact mathematical formula (e.g. Exact Binomial)
3. Approximate formula (e.g. Normal, z, or t)

Confidence Intervals

I am 95% confident that the parameter is between ______ and ______.

- The calculation is often based on "approximate" formulas, such as the normal (z) or t-distribution

- Most have a technical condition that must be checked

Inv. 3.1: part (h)

(h) Let π_{94} represent the proportion of all American teenagers in 1994 with at least some hearing loss, and similarly for π_{06}. Define the parameter of interest to be $\pi_{94} - \pi_{06}$, the difference in the population proportions between these two years. State appropriate null and alternative hypotheses about this parameter to reflect the researchers' conjecture that hearing loss by teens is becoming more prevalent.

H_o:

H_a:

Need to make a test of significance for two proportions…

In general, the steps for testing H_0 are:

1. Assume H_0 is true
2. Compute p-value, the probability of seeing our sample result or one more extreme under H_0
3. Reject H_0 if p-value is small

Need to make a test of significance for two proportions…

Simulation-based Test:

1. Assume H_0 is true
2. Compute p-value **by performing a simulation under H_0 to determine if our sample result is unusual.**
3. Reject H_0 if p-value is small

Simulation under H_0

How could we perform a simulation of our dataset assuming that the null hypothesis is true?

Simulation under H_0

Assume there is no difference in hearing loss over time…

…then our "best guess" for the proportion of the population with some hearing loss is (480+333)/(2928+1771) = 0.173

<u>Simulation:</u>

*Draw two samples, one of size 2928 and one of size 1771 to represent the 1994 and 2006 studies.

Repeat * many times, computing the "difference in sample proportions" for each pair of samples.

The empirical p-value is the proportion of times we got our sample result of -0.024 or one larger in the simulation trials

Simulation under H_0

You will need R or Minitab to carry out this simulation: there is no applet for it

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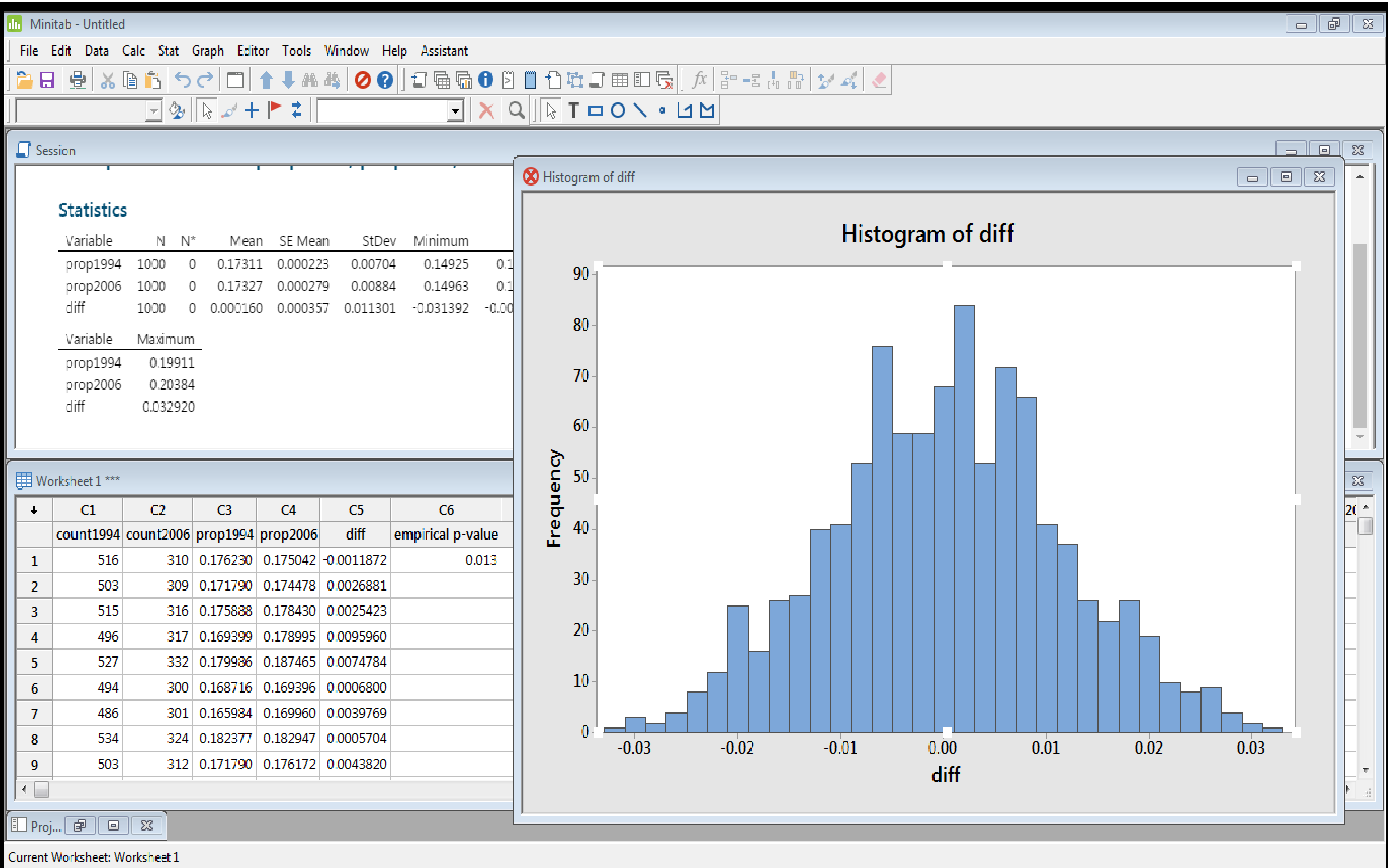
Simulation in Minitab – see steps in 3.1



Two sample z-test

The null distribution from the simulation looked approximately normal…

In fact, it turns out that we can use the normal approximation for it.

Summary of Comparing Two Population Proportions

Parameter: $\pi_1 - \pi_2$ = the difference in the population proportions of success

To test H_0: $\pi_1 - \pi_2 = 0$

1. *Simulation*: Random samples from binomial processes with common π

2. *Two-sample z-test*:

The test statistic $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{X_1 + X_2}{n_1 + n_2} = \dfrac{\textit{total number of successes}}{\textit{total number in study}}$

is well approximated by the standard normal distribution when $n_1 \hat{p} \geq 5$, $n_1(1 - \hat{p}) \geq 5$, $n_2 \hat{p} \geq 5$ and $n_2(1 - \hat{p}) \geq 5$, where $\hat{p}$ is the overall proportion of successes in the two groups put together.

Use the "theory-based Inference applet" to compute this formula and get the p-value

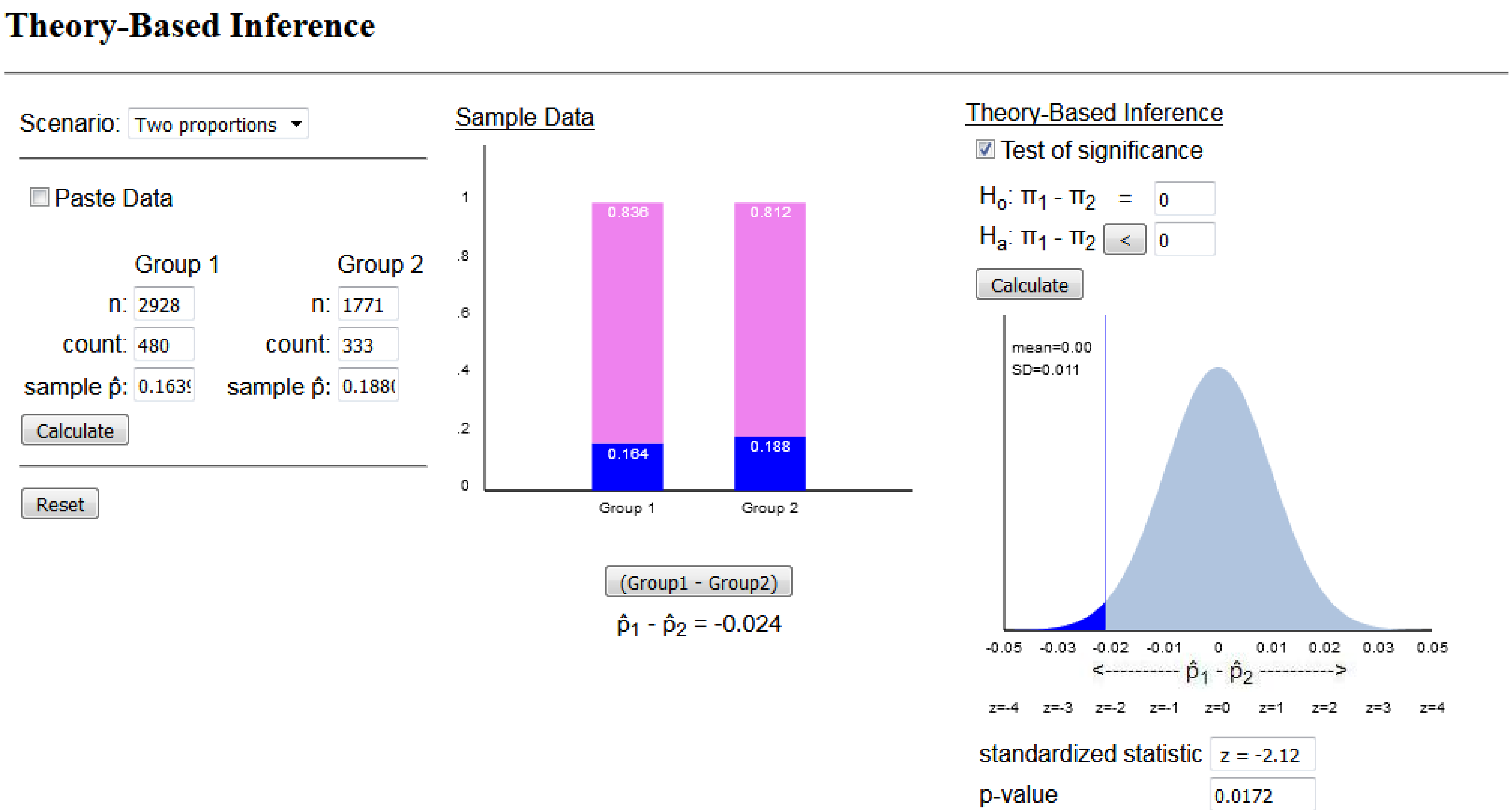
Inv. 3.1, part y

Compute the p-value for the two sample z-test and compare it to your simulation results:

- From the simulation, we got p-value = 0.013

Two sample z-test via applet

Rossman/Chance Applet Collection

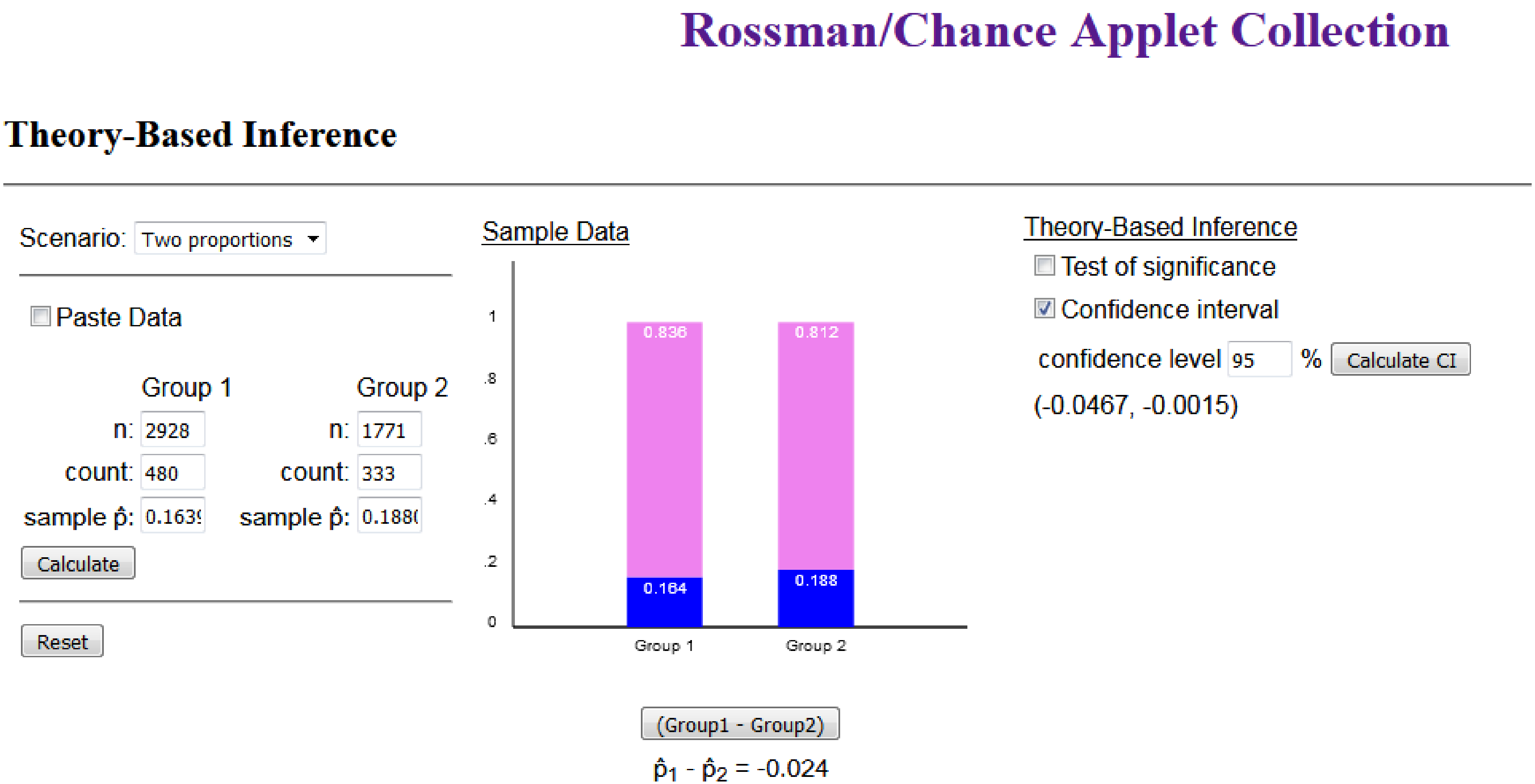


Inv. 3.1, part y

Compute the p-value for the two sample z-test and compare it to your simulation results:

- From the simulation, we got p-value = 0.013
- Using the “theory based inference” applet, p-value = 0.0172

Two sample z-interval via applet



Inv. 3.1, part bb: Interpretation of 95% CI

Both samples were randomly drawn from the population so it is reasonable to assume that the samples are representative of the populations of 1994 and 2006.

Thus…

I am 95% confident that the proportion of people with hearing loss decreased by 0.15% to 4.67% from 1994 to 2006.

Inv. 3.1, part bb: summary of results

Interpretation of p-value ≈ 0.02

There is about a 2% chance of seeing our observed difference in sample proportions of -0.024 or less if the population proportions of hearing loss were equal.

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Since our samples were randomly drawn, we conclude that there **was** a difference in the proportion of the population in 1994 and 2006 with some hearing loss.

Inv. 3.1, part bb

There is about a 2% chance of seeing our observed difference in sample proportions of -0.024 or less if the population proportions of hearing loss were equal.

Since our samples were randomly drawn, we conclude that there was a difference in the proportion of the population in 1994 and 2006 with some hearing loss.

But we have no information on whether the increase in hearing loss was caused by ear buds.

Terminology for 2 variables

Explanatory variable: the variable we think might explain changes in the response variable

Response variable: the outcomes of interest

Terminology for 2 variables

Explanatory variable: the variable we think might explain changes in the response variable

year, 1994 or 2006

Response variable: the outcomes of interest

Hearing loss, some or not