

# Math 243

Day 8

Binomial Test – Inv. 1.4

# Last time: Inv. 1.3, Bob/Tim

- Assuming random guessing,

$$\pi = P(\text{Tim on left}) = 0.5$$

- In a sample of 20 students, 18 chose Tim to be on the left

$$\hat{p} = 0.9$$

# Terminology Detour – Tim/Bob Survey

## Parameter vs. Statistic

- A numerical summary from a random process or *population*.
- Usually notated with Greek letters, e.g.  $\sigma$ ,  $\mu$ ,  $\pi$ ,...
- Example:  $\pi$  is the probability a **student** randomly picks Tim

**Population** – the entire group of interest

- A numerical summary from a *sample*.
- Usually notated with accented Latin letters, e.g.  $\bar{x}$ ,  $\hat{p}$ ,  $s$
- Example:  $\hat{p}$  is the proportion **of the class** who pick Tim

**Sample** - a subset of the population whose data was recorded

# Inv. 1.3, Bob/Tim

- Assuming random guessing,

Parameter if null  
model is true

$$\pi = P(\text{Tim on left}) = 0.5$$

- In a sample of 20 students, 18 chose Tim to be on the left

$$\hat{p} = 0.9$$

statistic

# Interpretation of p-value

What does our study result,

**the statistic,  $\hat{p} = 0.9$ ,**

tells us about the

**parameter under the null model,  $\pi=0.5$  ?**

- **P-value** = *probability of seeing 18 out of 20 students ( $\hat{p} = 0.9$ ) chose Tim as the guy on the left if they were randomly guessing ( $\pi=0.5$ )*

# Using technology to compute the **p-value** either by **simulation** or **math**

## Simulation-Based and Exact One Proportion Inference

Probability of heads:

Number of tosses:

Number of repetitions:

Animate

Total = 1000

Number of heads

Proportion of heads

As extreme as

Proportion of repetitions:  
 $1 / 1000 = 0.0010$

Two-sided

Exact Binomial

$P(X \geq 18) = 0.0002$

Normal Approximation



All Attempts (Last Repetition)

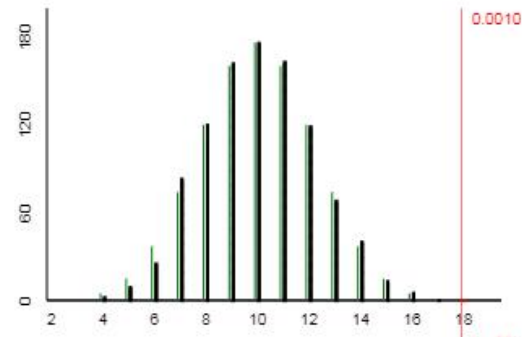


Heads (Last Repetition) = 13



Tails (Last Repetition) = 7

Summary Stats



# Drawing a conclusion from a p-value

What does our study result,

**the statistic,  $\hat{p} = 0.9$ ,**

tells us about the

**parameter under the null model,  $\pi=0.5$  ?**

***There is strong evidence against the null model: students weren't just guessing.***

Let  $X = \#$  of students who put Tim on left out of 20.

Assuming  $X$  is a **binomial random variable**,

$$\text{p-value} = P(X \geq 18) = 0.0002$$

so it would be **very unlikely** to see 18 of 20 students ( $\hat{p} = 0.9$ ) chose Tim on left **if** they were randomly guessing ( $\pi=0.5$ ).

Let's generalize the steps in Inv. 1.3



# A Test of Significance

Research question

**Null Hypothesis:** Nope: there's nothing going on

**Alternative Hypothesis :** Yes, something is going on

Collect data from a sample

Choose a random process that models the data collection well

Compute a p-value, *the probability of seeing results as extreme as the statistic if chance alone is at work*

If p-value is large,  
there's no evidence against the **null hypothesis**.

If p-value is small,  
there's evidence against **the null hypothesis**.

# Binomial Test

Research question involves parameter  $\pi$  from a Binomial Process

$H_0$ :  $\pi =$  some number

$H_a$ :  $\pi \neq$  some number

Collect a binary variable from a sample of size  $n$

Verify that the data collection is modelled well by a binomial process

Compute a binomial p-value, either through simulating a coin toss or the exact formula for a Binomial probability, assuming  
 $\pi =$  some number

If p-value is large,  
there's no evidence against  $H_0$ .  
If p-value is small,  
there's evidence against  $H_0$ .

# Inv. 1.4

Try parts (a), (b), (c), (d), (e) and (f) in class.

Try parts (g)-(q) at home.