

1. Find an orthogonal basis for the span of the vectors below using the Gram-Schmidt process.

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{bmatrix} \right\}$$

2. Write the vector form of the solution to the system:

$$\begin{array}{rrcr} x & +3y & -2z & = 1 \\ 2x & +5y & -z & = 5 \end{array}$$

3. Let  $W \subset \mathbb{R}^3$  be the subspace with orthogonal basis

$$W = \text{span} \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix} \right\} \quad \vec{v} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

(a) Find the projection of  $\vec{v}$  onto  $W$ .

(b) Write the standard matrix for the linear transformation:  $\text{proj}_W(\vec{u})$ .

4. Find the  $LU$  decomposition of the matrix  $A$ .

$$A = \begin{bmatrix} 1 & -2 & -2 & -3 \\ 3 & -9 & 0 & -9 \\ -1 & 2 & 4 & 7 \\ -3 & -6 & 26 & 2 \end{bmatrix}$$

5. Let  $B$  be the matrix  $\begin{bmatrix} 9 & -10 \\ 5 & -5 \end{bmatrix}$

(a) Find the eigenvalues of  $B$ .

(b) Find matrix  $P$  and matrix  $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$  so that  $B = PCP^{-1}$ .