Math 341 - Spring 2014 **Final** **Name:** **<u>Solutions</u>**
Randall Paul **Part I** **(120 minutes)**

1. For the matrix below, give a basis for the corresponding spaces. (You may use you calculator for row reduction. Otherwise show your work.)

$$A = \begin{bmatrix} -28 & -42 & -38 & 34 \\ -96 & -136 & -124 & 110 \\ 164 & 230 & 210 & -186 \\ 40 & 52 & 48 & -42 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5/28 & 1/56 \\ 0 & 1 & 11/14 & -23/28 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) col(A)

$$\text{span}\left\{ \begin{bmatrix} -28 \\ -96 \\ 164 \\ 40 \end{bmatrix}, \begin{bmatrix} -42 \\ -136 \\ 230 \\ 52 \end{bmatrix} \right\}$$

(b) null(A)

$$\text{span}\left\{ \begin{bmatrix} -5 \\ -22 \\ 28 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 46 \\ 0 \\ 56 \end{bmatrix} \right\}$$

(c) The eigenspace for eigenvalue $\lambda = 2$

$$A - 2I = \begin{bmatrix} -30 & -42 & -38 & 34 \\ -96 & -138 & -124 & 110 \\ 164 & 230 & 208 & -186 \\ 40 & 52 & 48 & -44 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1/3 & -2/3 \\ 0 & 1 & 2/3 & -1/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{span}\left\{ \begin{bmatrix} -1 \\ -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix} \right\}$$

2. A car rental company rents cars from two locations in a city: at the airport and downtown. On a typical day 90% of cars rented from the airport are returned at the airport (the remaining 10% downtown). 65% of cars rented from downtown are returned downtown, the rest are returned at the airport.

   (a) Write a difference equations relating the population of cars at the airport on the k-th day (a_k) with the population of cars downtown on the k-th day (d_k) with the corresponding populations on the $k+1$-th day.

   $$a_{k+1} = .9a_k + .35d_k$$

   $$d_{k+1} = .1a_k + .65d_k$$

   (b) What will be the ratio of cars at the airport to cars downtown after a large number of days?

   $$\begin{bmatrix} a_{k+1} \\ d_{k+1} \end{bmatrix} = \begin{bmatrix} .9 & .35 \\ .1 & .65 \end{bmatrix} \begin{bmatrix} a_k \\ d_k \end{bmatrix}$$

   Characteristic poly: $\lambda^2 - 1.55\lambda + .55 = 0$

   So eigenvalues are: 1 and .55. The smaller will fall away as $k \to \infty$, so for $\lambda = 1$,

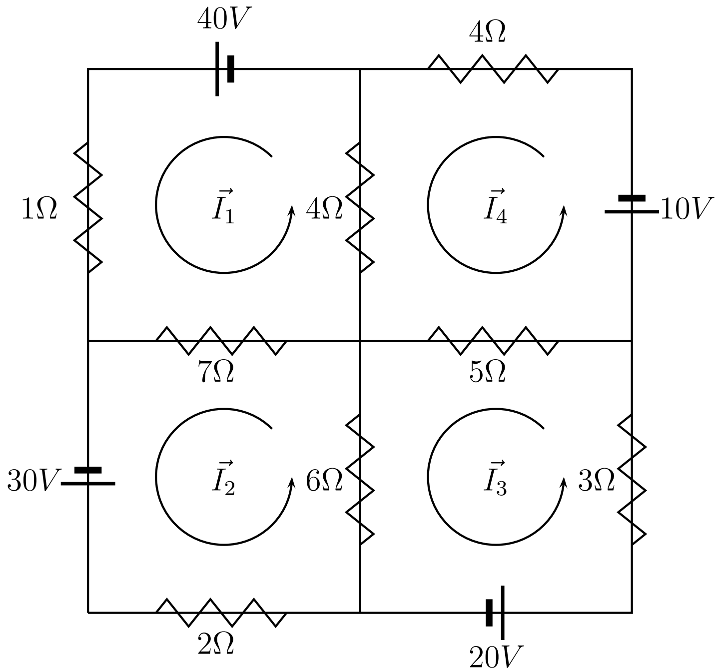
   $$\begin{bmatrix} -.1 & .35 \\ .1 & -.35 \end{bmatrix} \sim \begin{bmatrix} 1 & -3.5 \\ 0 & 0 \end{bmatrix}$$

   So the eigenvector is

   $$\begin{bmatrix} 3.5 \\ 1 \end{bmatrix}$$

   or 7 cars at the airport for every 2 downtown.

3. For the circuit diagramed below write down the equations determined by Kirchhoff's laws, then determine the currents in the circuit.



$$\begin{aligned}
1I_1 + 7(I_1 - I_2) + 4(I_1 - I_4) &= 40 \\
2I_2 + 7(I_2 - I_1) + 6(I_2 - I_3) &= 30 \\
3I_3 + 5(I_3 - I_4) + 6(I_3 - I_2) &= 20 \\
4I_4 + 4(I_4 - I_1) + 5(I_4 - I_3) &= -10
\end{aligned}$$

$$\begin{bmatrix} 12 & -7 & 0 & -4 & 40 \\ -7 & 15 & -6 & 0 & 30 \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 11.43 \\ 0 & 1 & 0 & 0 & 10.55 \\ 0 & 0 & 1 & 0 & 8.04 \\ 0 & 0 & 0 & 1 & 5.84 \end{bmatrix}$$

So $I_1 = 11.43$ amps, $I_2 = 10.55$ amps, $I_3 = 8.04$ amps, and $I_4 = 5.84$ amps.