Math 341 - Winter 2014 **Final** **Name:** ____________________

Randall Paul **Part II** **No Calculators (120 minutes)**

1. Find an orthogonal basis for the span of the vectors below using the Gram-Schmidt process.

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{bmatrix} \right\}$$

Not covered on the 2015 final

2. Write the vector form of the solution to the system:

$$\begin{array}{rrrl} x & +3y & -2z & = 1 \\ 2x & +5y & -z & = 5 \end{array}$$

$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 2 & 5 & -1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 7 & 10 \\ 0 & 1 & -3 & -3 \end{bmatrix}$$

So $x = -7z + 10$ and $y = 3z - 3$, thus

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 10 \\ -3 \\ 0 \end{bmatrix}$$

3. Let $W \subset \mathbb{R}^3$ be the subspace with orthogonal basis

$$W = \text{span}\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix} \right\} \qquad \vec{v} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

(a) Find the projection of $\vec{v}$ onto W.
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(b) Write the standard matrix for the linear transformation: $\text{proj}_W(\vec{u})$.
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4. Find the LU decomposition of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & -2 & -3 \\ 3 & -9 & 0 & -9 \\ -1 & 2 & 4 & 7 \\ -3 & -6 & 26 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & -3 \\ 3 & -9 & 0 & -9 \\ -1 & 2 & 4 & 7 \\ -3 & -6 & 26 & 2 \end{bmatrix} \begin{matrix} \\ R_2 - 3R_1 \\ R_3 + R_1 \\ R_4 + 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & -3 \\ 0 & -3 & 6 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & -12 & 20 & -7 \end{bmatrix} \begin{matrix} \\ \\ \\ R_4 - 4R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & -3 \\ 0 & -3 & 6 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -4 & -7 \end{bmatrix} \begin{matrix} \\ \\ \\ R_4 + 2R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & -3 \\ 0 & -3 & 6 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -3 & 4 & -2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -2 & -2 & -3 \\ 0 & -3 & 6 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Let B be the matrix $\begin{bmatrix} 9 & -10 \\ 5 & -5 \end{bmatrix}$

(a) Find the eigenvalues of B.

$$\det \begin{bmatrix} -\lambda+9 & -10 \\ 5 & -\lambda-5 \end{bmatrix} = (-\lambda+9)(-\lambda-5)+50 = \lambda^2 - 4\lambda + 5 = 0$$

$$\Rightarrow \lambda = \frac{4 \pm \sqrt{16-20}}{2} = 2 \pm i$$

(b) Find matrix P and matrix $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ so that $B = PCP^{-1}$.

$$\lambda = 2 - i : \begin{bmatrix} 7+i & -10 \\ 5 & -7+i \end{bmatrix} \sim \begin{bmatrix} 50 & -70+10i \\ 5 & -7+i \end{bmatrix} \sim \begin{bmatrix} 1 & (-7+i)/5 \\ 0 & 0 \end{bmatrix}$$

$x = (7/5 - i/5)y$, so the eigenvector is:

$$\begin{bmatrix} 7 \\ 5 \end{bmatrix} + i \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} 9 & -10 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 7 & -1 \\ 5 & 0 \end{bmatrix}^{-1}$$