Test 2 Part I: Calculators required Name: ____________________

Math 341 - Winter 2015 Randall Paul **(50 minutes)**

Important: Whenever you use your calculator to reduce a matrix, always write down the matrix and then the row reduced form of the matrix, clearly indicating you used your calculator.

1. Consider the matrix A below whose **reduced row echelon form** is also given.

$$A = \begin{bmatrix} 1 & -2 & -1 & 5 & 4 \\ 2 & -1 & 1 & 5 & 6 \\ -2 & 0 & -2 & 1 & -6 \\ 3 & 1 & 4 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) What is the rank of A?

   (b) What is the nullity of A?

   (c) Give a basis $\mathcal{B}$ for the column space of A.

   (d) Give the coordinates with respect to $\mathcal{B}$ of the vector below.

$$\begin{bmatrix} -1 \\ 1 \\ -2 \\ 4 \end{bmatrix}_{\mathcal{B}} =$$

   (e) Give a basis for the null space of A.

2. Use **concrete counterexamples** to prove the following:

   (a) The set, S, of vectors in $\mathbb{R}^2$ satisfying $|x| = |y|$ is **not** a subspace.

   (b) The function $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined to be:

   $$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ x^2 + y^2 \end{bmatrix}$$

   is **not** a linear transformation.

3. Consider the transformations S and T,

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 - x_2 \\ -x_2 \end{bmatrix}, \quad S\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + y_2 \\ 2y_2 \\ y_1 - y_2 \end{bmatrix}$$

(a) Prove S and T are linear transformations.

(b) Find the matrix associated to the linear transformation $S \circ T$.