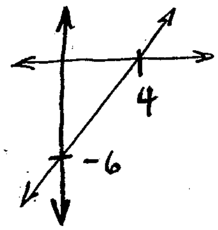
Diagnostic Exam

(1) From the problem we have  Thus the points $(4,0)$ and $(0,-6)$ must satisfy the equation of the line. $(4,0)$ satisfies only (C) and (D), so (A) and (B) are out. $(0,-6)$ satisfies (C) so that is the answer.

(2) $\csc\theta = -\frac{8}{5} = \frac{1}{\sin\theta} \Rightarrow \sin\theta = -\frac{5}{8} \Rightarrow \theta = \sin^{-1}\left(-\frac{5}{8}\right)$

$\theta = \sin^{-1}\left(-\frac{5}{8}\right) \approx -0.68 \Rightarrow 2\theta \approx -1.36 \Rightarrow \cos 2\theta \approx 0.22 \approx \frac{7}{32}$

The answer is (A). (This can be done in degrees or radians.)

(3) Let $\theta = 0 \Rightarrow r^2 = 1 - \tan^2 0 = 1 \Rightarrow r = \pm 1$. Thus the graph of $r^2 = 1 - \tan^2\theta$ contains the points with xy-coordinates $(1,0)$, $(-1,0)$. $(1,0)$ satisfies only (D).

(4) Any arithmetic that can be done with numbers can be done with matrices except that AB and BA are not necessarily equal for matrices. Thus $(B+C)A = BA + CA$, which may not equal $AB + AC$. The answer is (C).

(5) Note first that $A = 2C$, so A and C point in the same direction. $B \times C$ is a vector that is perpendicular to both B and C, so it is perpendicular to A as well. Thus $A \cdot (B \times C) = 0$ since the dot product of perpendicular vectors is 0. The answer is (A).

(6) $\_\_, \frac{3}{10}, \_\_, \_\_, \_\_, \frac{243}{160}$

obtained by multiplying the previous term by the same thing. Looking at the numerator, try multiplying by 5: $3 \cdot 5 = 15$, $15 \cdot 5 = 75$, $75 \cdot 5 =$ too big (> 243)

(6) Continued: Try 3: $3\cdot3=9$, $9\cdot3=27$, $27\cdot3=81$, $81\cdot3=243$

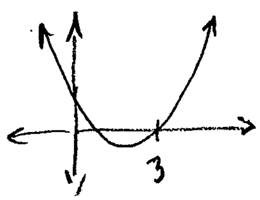
3rd term numerator, 4th, 5th, 6th

Try multiplying denominators by 2: $10\cdot2=20$, $20\cdot2=40$, $40\cdot2=80$, $80\cdot2=160$. Bingo! So each term in the sequence is $\frac{3}{2}$ times the previous term, so $\_\_\cdot\frac{3}{2}=\frac{3}{10}$. The blank is $\frac{1}{5}$, the answer is (B).

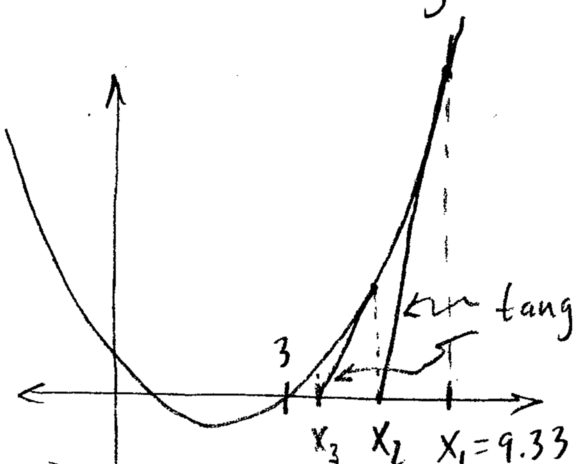
(9) $$\frac{1-e^{3(0.001)}}{4(0.001)}\approx-0.75=-\frac{3}{4} \quad (B)$$

(13) The equation is a second order diff. eqn. with no initial conditions, so the solution must contain two arbitrary constants — (A) is out. Solve $m^2-8m+16=0$ $\Rightarrow (m-4)(m-4)=0 \Rightarrow y=e^{4x}$ is only solution from this, so second solution is obtained by $y=xe^{4x}$. The answer is (B).

(15) The graph of $f(x)$ is



The approximations of the root starting with $x_1=9.33$ will be (graphically)



Thus x_3 must be greater than 3, so (D) is the answer.

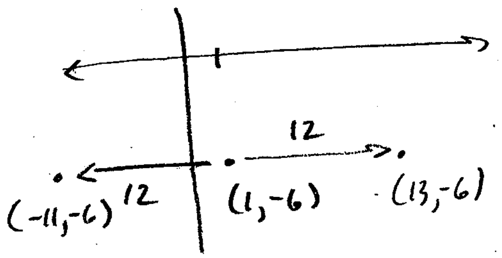
Geometry/Trig Exams

Sample Problems

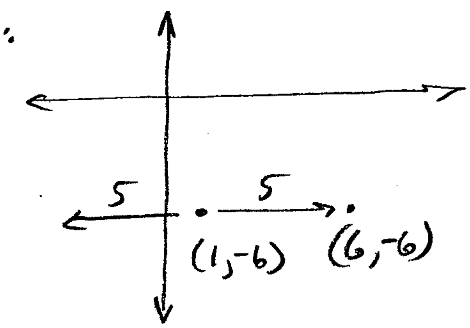
1. Slope of $y = \frac{1}{4}x + 6$ is $\frac{1}{4}$; parallel line must have same slope. (A), (B) and (C) don't, so (D) is the answer. Note that when you solve $x = 4y - 3$ for y you get $y = \frac{1}{4}x + \frac{3}{4}$.

3. Test the given values, using your calculator.

4. Suppose the answer was (A). Plot the center at $(1,-6)$ and go left and right by 12 to get two points:



Test these in the equation. $(13,-6)$ doesn't satisfy, so move on. I'd avoid (B) for now, and I'd try (D) to keep the numbers small:



$(6,-6)$ satisfies the equation, so select (D).

5. Test values. For (A), $x_1 = 1$ is not a solution, so move on. For (B), $x_1 = 4$ is a solution, so check $x_2 = -3$. It is a solution as well, so B is the answer.

Why is the author using the quadratic formula when factoring is easy?

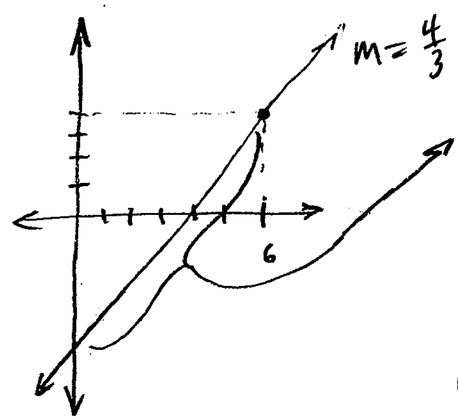
Actually, I'd solve by factoring for this one:

$x^2 - x - 12 = 0 \Rightarrow (x-4)(x+3) = 0 \Rightarrow x = 4, x = -3 \Rightarrow$ (B)

Geometry/Trig Exams

FE-Style Exam

(1) Draw a picture:



Distance is clearly more than 6 units, but not as big as 25. Answer is (A).

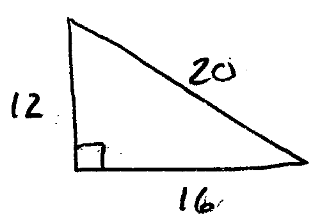
(2) The line $y=4x+10$ has slope 4, so a perpendicular line must have slope $-\frac{1}{4}$. The answer is (B).

(3) The solution given in the book is the most efficient, probably. Another way to do this is to pick any value for θ and plug it into the given expression and each answer, see if any are equal. Choosing $\theta=20$ (it doesn't matter whether you use degrees or radians for this one, so we'll go radians),

$\csc(20)\cos^3(20)\tan 20 \approx 0.17$ $\qquad$ $\sin 20 \approx 0.91$

$\cos 20 \approx 0.41$ $\qquad$ $1-\sin^2 20 \approx 0.17$ $\qquad$ The answer is (C).

(4) Note that $BD<BC$, so (A) and (C) are out. Check (B):



Ratio is 3:4:5, so (B) is the answer.

OR

$12^2+16^2 \stackrel{?}{=} 20^2$

$144+256=400$ yes

(6) Test the points (2,0), (0,3), (-2,0) in each equation. (2,0) satisfies only (B) and (D). From here, either

i) recognize that added terms mean ellipse, subtracted mean hyperbola, OR

ii) test (0,3) in (B) and (D).

Either way, (D) is the answer.

(8) Test points again. (0,0) satisfies only (B) and (C). (0,4) satisfies both of those, but (-4,0) satisfies only (C), so that is the answer.

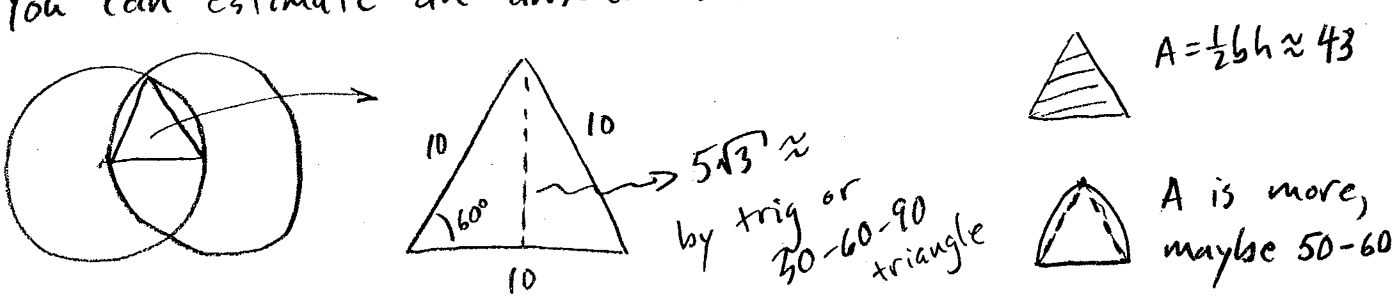
(9) Test (-3,4) first. It only satisfies (D), so that is it!

(10) Solve as in the book, but note that you can get close enough to select the correct answer by taking the angle to be either 57° (rounded correctly) or 58°. You can also just tell from the picture that BC > AC, so the answer must be (C) or (D).

(12) Find some points by letting $t=0$ and $t=\frac{\pi}{2}$: (0,6), (8,0). (0,6) satisfies only (A).

(14) CD is longer than AB, but not by too much. (B) is it!

(15) You can estimate an answer like this:

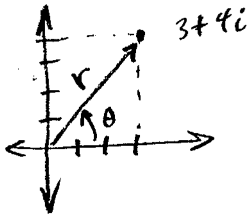


Total area is about 100-120. Choose (B) or (C).

(16) The coefficients of x^2 and y^2 have opposite signs — hyperbola (D)

Algebra

Sample Problems



(2) A couple methods besides the author's can be used. Here's one →

θ is clearly greater than 45° and less than 90°. So (C) and (D) are the possibilities. $r > 4$, so D is the answer.

Here's another:

(A): $(3)(\cos 36.87° + i \sin 36.87°) \approx 3\cos 37° + 3i\sin 37° \approx$ so (A) is not the answer. Repeat for (B), (C), etc. until you hit the answer.

(3) Learn to use your calculator!

(5) Test solutions. For (A) $2(-2)+3(-3)-5 \stackrel{?}{=} -10 \Rightarrow -13 \neq -10$ So try (B), (C), ... until one "works".

FE-Style Problems

(1) Test values, using your calculator. Note that $A^{-\frac{6}{8}} = A^{-.75}$

(A): $0^{-0.75} = 0$, so (A) is out. (B): $100^{-0.75} \approx$

(2) Use your reference manual, or let $x=1, y=2, z=3$:

$\log\left(\frac{x}{y+z}\right) = \log\left(\frac{1}{5}\right) \approx -0.70$

(A): $\log(x) - \log(y) - \log(z) = \log(1) - \log(2) - \log(3) \approx -0.78$

(B): $\log(x) - \log(y+z) = \log(1) - \log(5) \approx -0.70$ The answer is (B).

(5) Enter the coefficient matrix in your calculator:

$$A = \begin{bmatrix} 10 & 3 & 10 \\ 8 & -2 & 9 \\ 8 & 1 & -10 \end{bmatrix} \xrightarrow{\text{calculator}} A^{-1} = \begin{bmatrix} 0.0136 & 0.0496 & \\ 0.1886 & -0.2233 & \text{etc.} \\ 0.2978 & & \end{bmatrix}$$

Signs of these entries tell us that (D) is the answer.

Algebra, continued

(6) Test solutions or use your calculator.

(8) Note that the magnitude of a unit vector is one, so no component can have absolute value greater than one. This rules out (C) and (D). The unit vector must also be proportional to the original. Since $18 < 29$ and $0.892 > 0.416$, (B) is out. Choose (A).

(9) The cross product must be perpendicular to either original vector, so test with dot product:

(A): $(\vec{i} - \vec{j} - \vec{k}) \cdot \vec{A} \neq 0$, so (A) is out.

(B): $(-\vec{i} + \vec{j} + \vec{k}) \cdot \vec{A} \neq 0$

(C): $(2\vec{i} + 7\vec{j} - 5\vec{k}) \cdot \vec{A} = 0$, $(2\vec{i} + 7\vec{j} - 5\vec{k}) \cdot \vec{B} = 0$ $\boxed{(C)}$

OR ... learn to do cross products on your calculator.

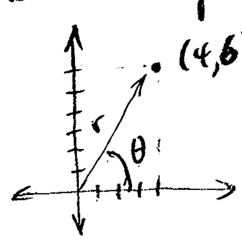
(11) $\log_{10} 4 = 0.703x$
$0.60 = 0.703x$
$x \approx 0.86$

$\log_{10} \frac{1}{4} \approx -0.60$

Now put 0.86 into the choices for x.

$(-0.703)(0.86) \approx -0.60$ so (B) is it.

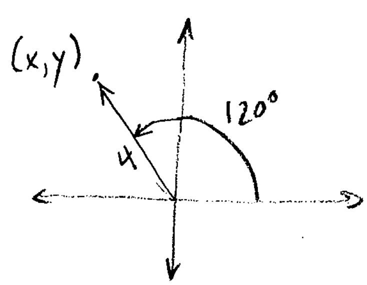
(12) Draw a picture:



From this we see $45° < \theta < 90°$, $r > 0$.

This leads to (D)

(13) Plot $(4, 120°)$:



The only choice that is in quadrant II is (C)!

(14), (15) I'd learn to multiply matrices by hand. You can find the correct answers more quickly than using your calculator.

17 Enter the matrix in your calculator, using the value of a from (A). See if the determinant is -5. If so, done. If not, change a to the value from (B) (learn to edit matrices on your calculator), and so on until the correct answer is found.

18 Use your calculator to find the determinant of B.

19, 20 Use your calculator.

30 Again, the cross product of two vectors is a vector that is perpendicular to both original vectors. See Exercise 9.

Calculus

Sample Problems

1. The question is this: If we put numbers less than or equal to zero into $y = 2x^3+12x^2-30x+10$ for x, what is the largest value we can get out for y? Let's just try $x=0 \Longrightarrow y=10$. The max must be at least 10, so (D) is the correct answer.

4. Consider $f(x) = x^2y^3 + xy^4 + \sin x + \cos^2 x + \sin^3 y$. When taking the partial derivative with respect to x, any terms containing only y will "become" zero. The answer should then have 4 terms, which leads to (D).

5. Let $x = 2.01$: $\dfrac{(2.01)^2-4}{2.01-2} \approx 4.01$ Choose (C).

FE-Style Exam

1. This problem is incorrectly stated. The interval should be $[-2,2]$, rather than $(-2,2)$. I'd start by

$$f(-2) = 5(-2)^3 - 2(-2)^2 + 1 = -47$$
$$f(2) = 5(2)^3 - 2(2)^2 + 1 = 33$$

The min is less than or equal to -47 and the max is greater than or equal to 33, so (A) is the answer.

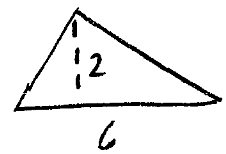
3. You can see the slope of the line is negative, at less than $45°$. Thus $-1<m<0$, which narrows it down to (C) and (D). The y-intercept for (C) is $\frac{9}{4} = 2.25 < 5$ (look at the picture to see why 5), so (C) is out. The answer is (D).

Calculus, continued

5 "Plug in" $x = 3.1$: $\dfrac{(3.1)^2 - 3.1\pi + \sin 3.1}{-\sin 3.1} \approx 2.10$

Make sure your calculator is in radians.
The result is clearly not supposed to be 0 or 1, so (A) and (B) are out. Since $\pi - 1 \approx 2.14$, the answer is (C).

8 I'd estimate the intersection of the two lines to be at (2,2), which in fact it is. Then



$A = \frac{1}{2}bh = 6$ (C)

9 $\displaystyle\int \frac{4}{8+2x^2}\,dx$ is close to $\displaystyle\int \frac{dx}{k^2+x^2}$ on the integral table, so the answer is probably an arctan, or $\tan^{-1}$! Choose (C).

20 Remember that the gradient of a function at a point is a vector pointing in the direction of greatest increase of the function. Since $\nabla f(1,-2) = 7\vec{i} + 3\vec{j}$, the answer is (B).

21 The greatest slope is the magnitude of the gradient, or $\|\nabla f(1,-2)\| = \sqrt{7^2+3^2} = \sqrt{58} \approx 7.6$ (C)

22 Even if you don't know what divergence is all about, it is easy to compute from the formula in the reference manual. Look at the author's solution.

Curl (see 23) is a little harder, but not too bad if you know how to do 3x3 determinants by hand.

Differential Equations

Sample Problems

① $y''+9y=0$ is second order, no initial conditions, so the solution must have two constants. The characteristic equation is $m^2+9=0 \Rightarrow m^2=-9 \Rightarrow$ complex roots $\Rightarrow$ solution has sines and cosines $\Rightarrow$ (D).

② Test the condition $y(0)=3$, which means $y=3$ when $t=0$. Only $3e^{5t}$ satisfies this so the answer is (B).

③ Even if you've never done Laplace transforms, check out the table in the reference manual. This answer is straight from the table.

FE-Style Exam

① Note that only (A) and (D) solutions satisfy $y(0)=1$. The characteristic equation is $m^2+4m+4=0 \Rightarrow (m+2)(m+2)=0$ $\Rightarrow m=-2$, so the solution is (D) $y=(1+2x)e^{-2x}$

② $m^2+2m+2=0 \longrightarrow$ can't factor $\Rightarrow m=\frac{-2\pm\sqrt{-4}}{2}=-1\pm i$

damping term

damping — causes e^{-x} in solution.

causes sin and cos in solution.

Choose (D)

⑤ The differential equation can be rewritten as

$$2y''+10y'+8y=e^{-2x}$$

We should expect a two-part homogeneous solution to $2y''+10y'+8y=0$ and a particular solution of the form Ae^{-2x}. This leads to (C) and (D). Checking the initial condition $y(0)=1$, we find that the solution is (D)

(15) At the end of the first period, 15% interest is added: \$10,000 + 10,000(.15) = \$11,500. \$3000 is paid: \$11,500 − 3000 = \$8500 balance (C)

(16) As 15, but repeat twice:

$[15{,}500 + .01(15{,}500)] - 350 = 15655 - 350 = 15305$

$[15{,}305 + .01(15{,}305)] - 350 =$

(17) $\overset{f(1)}{4}, \overset{f(2)}{1.3}, 5.3, \overset{f(4)}{6.6}$ (D)

add

add

"The" Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, ... Add any two terms to get the next.

(18) Check the initial condition $y(0)=5$. Only (B) and (C) satisfy it. For (B), $y = e^{-2x} + 4 \Rightarrow y' = -2e^{-2x} \Rightarrow$
$y' + 8y = -2e^{-2x} + 8(e^{-2x}+4) = 6e^{-2x} + 32 \neq 0$. The answer must then be (C). Let's check it to be sure:
$y = 3e^{-8x} + 2 \Rightarrow y' = -24e^{-8x} \Rightarrow y' + 8y = -24e^{-8x} + 8(3e^{-8x}+2) =$
$-24e^{-8x} + 24e^{-8x} + 16 = 0$.

(20) Check (A): $y = (x^2+9)^{\frac{1}{2}} + C \Rightarrow \frac{dy}{dx} = \frac{1}{2}(x^2+9)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+9}}$

$(x^2+9)\frac{x}{\sqrt{x^2+9}} = \frac{x}{\sqrt{x^2+9}} \neq xy = x(\sqrt{x^2+9} + C)$

Check (B): $\frac{dy}{dx} = 2x \quad (x^2+9)\frac{dy}{dx} = 2x(x^2+9) \neq xy = x(x^2+9)$

Check (C): $\frac{dy}{dx} = \frac{Cx}{\sqrt{x^2+9}} \quad (x^2+9)\frac{dy}{dx} = (x^2+9)\frac{Cx}{\sqrt{x^2+9}} = Cx\sqrt{x^2+9} = xy$

See (A)

The answer is (C).