Intermediate Algebra

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Contents

10 More With Polynomial and Rational Expressions

10.1 Dividing Polynomials

10. (a) Divide polynomial expressions.

Suppose that you were going to divide $252 \div 3$, without using a calculator. You would use "long division," beginning with a setup like (a) below. You would probably check to see if 3 goes into 2 and, since it doesn't, you would see how many times it goes into 25 without going over that value. That would be 8 times, so you would put an 8 above the 5, which signifies the tens place, as shown in (b) below. *There is another way to think about this step.* Rather than asking how many times 3 goes into 25 we could ask the equivalent question "What do we have to multiply 3 by to get as close to 25 as possible, without going over?" This gives the same result, 8, but will soon be a better way of thinking of this computation.

$$\begin{array}{r} 3\overline{)\,252} \end{array} \qquad \begin{array}{r} 8 \\ 3\overline{)\,252} \end{array} \qquad \begin{array}{r} 8 \\ 3\overline{)\,252} \\ 24 \end{array} \qquad \begin{array}{r} 8 \\ 3\overline{)\,252} \\ \underline{24} \\ 1 \end{array} \qquad \begin{array}{r} 8 \\ 3\overline{)\,252} \\ \underline{24}\downarrow \\ 12 \end{array} \qquad \begin{array}{r} 84 \\ 3\overline{)\,252} \\ \underline{24} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

(a) (b) (c) (d) (e) (f)

At this point we multiply 8 times 3 and put the result, 24, below the 25, as shown in (c) above. We then subtract $25 - 24$ to get the result shown in (d). *Here we can think of* $25 + (-24)$ *instead of* $25 - 24$, *which is also helpful later.* The 2 in the ones place is then brought down, (e), and we then see that $4 \cdot 3 = 12$. This is placed at the bottom of the "stack" and subtracted to give zero at (f) above. Since there is no remainder, we are done.

Now suppose that we want to divide $(x^3 + 8x^2 + 14x - 3) \div (x + 3)$. The method here is much like the process we used above with numbers. We set up as shown to the left below and ask what we need to multiply times x to get x^3. That would be x^2, so we put x^2 above the bar in the x^2 column. Then we multiply x^2 times $x + 3$ and put the result below $x^3 + 8x^2 + 14x - 3$ with like terms aligned. This is shown to the right below.

$$x+3\,\overline{)\,x^3 + 8x^2 + 14x - 3} \qquad\qquad \begin{array}{l} x^2 \\ x+3\,\overline{)\,x^3 + 8x^2 + 14x - 3} \\ x^3 + 3x^2 \end{array}$$

At this point we need to subtract $(x^3 + 8x^2) - (x^3 + 3x^2)$, which can also be thought of as $(x^3 + 8x^2) + (-x^3 - 3x^2)$. Here we have used the idea that subtraction is addition of the opposite. The result is $5x^2$, and we then carry the $14x$ down to get $5x^2 + 14x$. This is shown below.

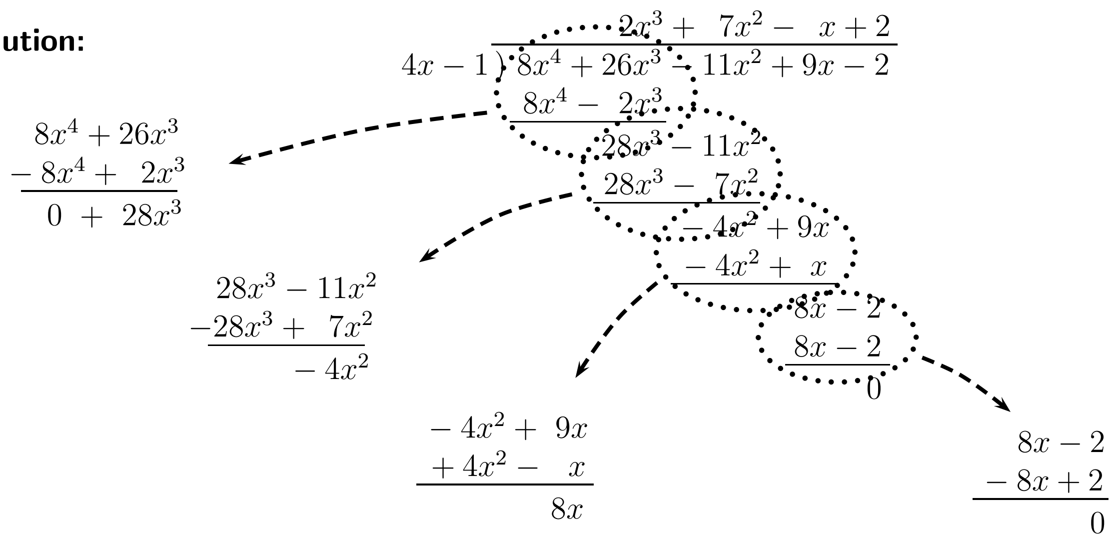
$$
\begin{array}{r}
x^2 \\
x+3 \,\overline{)\, x^3 + 8x^2 + 14x - 3} \\
\underline{x^3 + 3x^2} \\
5x^2 + 14x
\end{array}
\qquad\qquad
\begin{array}{r}
x^2 + 5x - 1 \\
x+3 \,\overline{)\, x^3 + 8x^2 + 14x - 3} \\
\underline{x^3 + 3x^2} \\
5x^2 + 14x \\
\underline{5x^2 + 15x} \\
-x - 3 \\
\underline{-x - 3} \\
0
\end{array}
$$

We now ask what we need to multiply by x to get $5x^2$. That amount, $+5x$, is put in the x place above the bar. *Note that the sign is included.* $5x$ is then multiplied times $x+3$ to get $5x^2 + 15x$, which is put below $5x^2 + 14x$ and subtracted to get $-x-3$ after the -3 is brought down. To get the $-x$ we have to put -1 above the top bar. This is multiplied by $x+3$ to get the $-x-3$ that is subtracted from the $-x-3$ we already have. Of course this can also be thought of as $(-x-3)+(x+3)$ instead. The result is zero, which tells us that $x+3$ goes into $x^3+8x^2+14x-3$ evenly. The final result is shown to the right at the bottom of the previous page.

Many people find it easier, after multiplying and putting the result at the bottom, to change the signs of the second expression and add (instead of subtracting). This works because adding the opposite is equivalent to subtraction. The idea is probably best seen "live," but I've attempted to show it below. Each dotted oval encloses part of the original computation, but then the dashed arrows take you to where the signs of the second expression have been changed, and then the two expressions added.

◇ **Example 10.1(a):** Divide $(8x^4 + 26x^3 - 11x^2 + 9x - 2) \div (4x - 1)$

Solution:



The example at the beginning of this section, of dividing $252 \div 3$, is a bit special in that 3 goes evenly into 252. When this does not happen we need to express our result in some other way. The method we will use is demonstrated at the top of the next page for $257 \div 3$. Things proceed as they did in the other example until we reach the point seen to the left there. We might be inclined to stop at that point, since three will not go into two. But in fact it does, *just not evenly*. Two divided by three is the fraction $\frac{2}{3}$, so we simply get the result that $257 \div 3 = 85\frac{2}{3}$, as shown below. For future reference, note that this could be thought of as $85 + \frac{2}{3}$.

$$
\begin{array}{r}
85 \\
3\,)\overline{257} \\
\underline{24} \\
17 \\
\underline{15} \\
2
\end{array}
\qquad\qquad
\begin{array}{r}
85\tfrac{2}{3} \\
3\,)\overline{257}\phantom{\tfrac{2}{3}} \\
\underline{24}\phantom{7\tfrac{2}{3}} \\
17\phantom{\tfrac{2}{3}} \\
\underline{15}\phantom{\tfrac{2}{3}} \\
2\phantom{\tfrac{2}{3}}
\end{array}
$$

Divide by three and put here

If we want to divide $(3x^3 - x^2 - 13x + 7) \div (x + 2)$ we proceed as usual until we reach the point shown below and to the left. Then, just as in the numerical case, we form the fraction $\frac{5}{x+2}$ as was done in the numerical case, then add it to the expression we have obtained so far, as shown to the right below.

$$
\begin{array}{rl}
 & 3x^2 - 7x + 1 \\
x+2\,) & \overline{3x^3 - x^2 - 13x + 7} \\
 & \underline{3x^3 + 6x^2} \\
 & \quad -7x^2 - 13x \\
 & \quad \underline{-7x^2 - 14x} \\
 & \qquad\qquad x + 7 \\
 & \qquad\qquad \underline{x + 2} \\
 & \qquad\qquad\quad 5
\end{array}
\qquad\qquad
\begin{array}{rl}
 & 3x^2 - 7x + 1 + \frac{5}{x+2} \\
x+2\,) & \overline{3x^3 - x^2 - 13x + 7} \\
 & \underline{3x^3 + 6x^2} \\
 & \quad -7x^2 - 13x \\
 & \quad \underline{-7x^2 - 14x} \\
 & \qquad\qquad x + 7 \\
 & \qquad\qquad \underline{x + 2} \\
 & \qquad\qquad\quad 5
\end{array}
$$

◇ **Example 10.1(b):** Divide $(x^3 + 2x^2 - 38x + 11) \div (x - 5)$

Solution:

$$
\begin{array}{rl}
 & x^2 + 7x - 3 \\
x-5\,) & \overline{x^3 + 2x^2 - 38x + 11} \\
 & \underline{x^3 - 5x^2} \\
 & \quad 7x^2 - 38x \\
 & \quad \underline{7x^2 - 35x} \\
 & \qquad\quad -3x + 11 \\
 & \qquad\quad \underline{-3x + 15} \\
 & \qquad\qquad\quad -4
\end{array}
\qquad\qquad
\begin{array}{rl}
 & x^2 + 7x - 3 - \frac{4}{x-5} \\
x-5\,) & \overline{x^3 + 2x^2 - 38x + 11} \\
 & \underline{x^3 - 5x^2} \\
 & \quad 7x^2 - 38x \\
 & \quad \underline{7x^2 - 35x} \\
 & \qquad\quad -3x + 11 \\
 & \qquad\quad \underline{-3x + 15} \\
 & \qquad\qquad\quad -4
\end{array}
$$

This tells us to add $\dfrac{-4}{x-5}$, but that is the same as $-\dfrac{4}{x-5}$, so we really *subtract* $\dfrac{4}{x-5}$

Section 10.1 Exercises

To Solutions

1. Perform each of the following divisions.

   (a) $(5x^3 - 2x^2 - 12x + 9) \div (x - 1)$
   (b) $(4x^3 + 13x^2 - 2x - 15) \div (x + 3)$
   (c) $(x^3 - 13x + 12) \div (x + 4)$ **Hint:** Write $x^3 - 13x + 12$ as $x^3 + 0x^2 - 13x + 12$.
   (d) $(15x^3 + 16x^2 - x - 2) \div (3x - 1)$
   (e) $(6x^3 + 13x^2 - x + 10) \div (2x + 5)$

2. Perform each of the following divisions.

   (a) $(x^2 + 3x + 7) \div (x + 1)$
   (b) $(x^2 + 9x - 2) \div (x - 5)$
   (c) $(3x^2 + 4x - 7) \div (x - 1)$
   (d) $(2x^2 - 7x + 9) \div (x - 2)$

3. Find each of the following, if possible (complex numbers *NOT* allowed). If not, write DNE for "does not exist." Use the facts that $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$.

   (a) $8^{\frac{4}{3}}$
   (b) $9^{-\frac{1}{2}}$
   (c) $16^{\frac{1}{2}}$
   (d) $(-27)^{\frac{1}{3}}$

4. Perform each division, giving your answers in $a + bi$ form.

   (a) $(5 + i) \div (3 - 2i)$
   (b) $3i \div (5 - i)$
   (c) $(1 + i) \div (6 + 2i)$

5. Solve each of the following quadratic equations, allowing complex solutions.

   (a) $x^2 - 10x + 43 = 0$
   (b) $x^2 = 2x - 8$
   (c) $x^2 - 8x + 28 = 0$

10.2 Adding and Subtracting Rational Expressions

> 10. (b) Add and subtract rational expressions.

We have seen rational expressions, also called algebraic fractions, in several contexts already. Here are some examples of rational expressions:

$$\frac{6}{x-2} \qquad \frac{x^2+5x+6}{x^2+6x+9} \qquad \frac{3x}{x^2-4}$$

We have discussed the domains of such expressions, simplifying them, and solving equations containing them. We have graphed functions of the form $f(x) = \dfrac{6}{x-2}$. In this section we will add and subtract rational expressions.

Since rational expressions are really just fractions, it might be good to start by reminding ourselves how we add and subtract fractions. *To add or subtract fractions, we have to have a common denominator.* The same holds true for rational expressions as well. Let's look at a couple examples involving fractions.

◇ **Example 10.2(a):** Subtract $\dfrac{7}{3} - \dfrac{2}{3}$

Solution: These have a common denominator, so $\dfrac{7}{3} - \dfrac{2}{3} = \dfrac{7-2}{3} = \dfrac{5}{3}$.

◇ **Example 10.2(b):** Add $\dfrac{3}{4} + \dfrac{1}{6}$

Solution: Here we need to get a common denominator. The least common denominator is 12:

$$\frac{3}{4} + \frac{1}{6} = \frac{3}{4} \cdot \frac{3}{3} + \frac{1}{6} \cdot \frac{2}{2} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$$

Note that in the second example we multiply each fraction by some form of the number one so that the denominators become the same. We also could have multiplied the first fraction by $\frac{6}{6}$ and the second by $\frac{4}{4}$, but that does not result in the smallest possible common denominator (which we officially refer to as the **least common denominator**), and we have to reduce when we are done:

◇ **Example 10.2(c):** Add $\dfrac{3}{4} + \dfrac{1}{6}$

Solution:

$$\frac{3}{4} + \frac{1}{6} = \frac{3}{4} \cdot \frac{6}{6} + \frac{1}{6} \cdot \frac{4}{4} = \frac{18}{24} + \frac{4}{24} = \frac{22}{24} = \frac{11}{12}$$

This is no real problem, but *when working with algebraic fractions, not using the least common denominator can make exercises very difficult*.

Adding and subtracting algebraic fractions is pretty straightforward when they have common denominators. The only thing that can trip us up is forgetting to distribute a negative sign when subtracting, like in the second example on the next page.

◇ **Example 10.2(d):** Add $\dfrac{x+2}{x^2-5x+6}+\dfrac{3x}{x^2-5x+6}$

Solution: These two fractions have the same denominator, so we simply add the numerators and put the result over the common denominator:

$$\frac{x+2}{x^2-5x+6}+\frac{3x}{x^2-5x+6}=\frac{(x+2)+3x}{x^2-5x+6}=\frac{x+2+3x}{x^2-5x+6}=\frac{4x+2}{x^2-5x+6}$$

◇ **Example 10.2(e):** Subtract $\dfrac{2x+1}{x-3}-\dfrac{x+1}{x-3}$

Solution: When subtracting algebraic fractions like these, we must be very careful with how the minus sign is handled. In this case we must subtract the entire numerator of the second fraction, so the minus sign must distribute to both parts of that numerator:

$$\frac{2x+1}{x-3}-\frac{x+1}{x-3}=\frac{(2x+1)-(x+1)}{x-3}=\frac{2x+1-x-1}{x-3}=\frac{x}{x-3}$$

Note in both examples how we are careful to add or subtract the entire numerators. In the case of addition we can get away with being a little sloppy about this, but when subtracting we must be careful to distribute the subtraction throughout the numerator of the second fraction.

To understand how to add or subtract rational expressions that don't have the same denominators, let's take a careful look at the exercise of adding the rational expressions shown below and to the left. The first thing we need to do to add them is to factor the denominators, as shown to the right below.

$$\frac{3x}{x^2-9}+\frac{5}{x^2+3x} \qquad\qquad \frac{3x}{(x+3)(x-3)}+\frac{5}{x(x+3)}$$

This allows us to take a careful look at the denominators. Now we want to make the denominators the same, and we should note right off that *both denominators contain the factor* $x+3$ *already.* What we then need to do is get the other factors, $x-3$ and x in both denominators. Since the first fraction's denominator is missing the x, we multiply that fraction by $\dfrac{x}{x}$ and we multiply the second fraction by $\dfrac{x-3}{x-3}$ to get $x-3$ in its denominator:

$$\frac{3x}{(x+3)(x-3)}\cdot\frac{x}{x}+\frac{5}{x(x+3)}\cdot\frac{x-3}{x-3}$$

We then finish as follows.

$$\frac{3x^2}{x(x+3)(x-3)}+\frac{5x-15}{x(x+3)(x-3)}=\frac{3x^2+5x-15}{x(x+3)(x-3)}$$

We usually distribute and combine like terms in the numerator, but it is customary to leave the denominator in factored form. If the numerator can be factored, we should factor it and see if any factors cancel, like shown in the next example.

◇ **Example 10.2(f):** Subtract $\dfrac{x+5}{4x+12} - \dfrac{x}{x^2-9}$

Solution:

$$\begin{aligned}
\frac{x+5}{4x+12} - \frac{x}{x^2-9} &= \frac{x+5}{4(x+3)} - \frac{x}{(x+3)(x-3)} \\
&= \frac{(x+5)(x-3)}{4(x+3)(x-3)} - \frac{4x}{4(x+3)(x-3)} \\
&= \frac{x^2+2x-15-4x}{4(x+3)(x-3)} \\
&= \frac{x^2-2x-15}{4(x+3)(x-3)} \\
&= \frac{(x-5)(x+3)}{4(x+3)(x-3)} \\
&= \frac{x-5}{4(x-3)}
\end{aligned}$$

◇ **Example 10.2(g):** Add $\dfrac{3x-2}{x^2-x-2} + \dfrac{4x-3}{x^2-4}$

Solution:

$$\begin{aligned}
\frac{3x-2}{x^2-x-2} + \frac{4x-3}{x^2-4} &= \frac{3x-2}{(x+1)(x-2)} + \frac{4x-3}{(x+2)(x-2)} \\
&= \frac{(3x-2)(x+2)}{(x+1)(x-2)(x+2)} + \frac{(4x-3)(x+1)}{(x+2)(x-2)(x+1)} \\
&= \frac{3x^2+4x-4}{(x+1)(x-2)(x+2)} + \frac{4x^2+x-3}{(x+2)(x-2)(x+1)} \\
&= \frac{3x^2+4x-4+4x^2+x-3}{(x+1)(x-2)(x+2)} \\
&= \frac{7x^2+5x-7}{(x+1)(x-2)(x+2)}
\end{aligned}$$

Section 10.2 Exercises

To Solutions

1. Perform the indicated addition or subtraction and simplify your answer.

   (a) $\dfrac{7}{x+5} - \dfrac{2}{x-1}$ (b) $\dfrac{3}{x-2} + \dfrac{4}{x+2}$ (c) $\dfrac{1}{x+4} - \dfrac{5}{x-3}$

2. Graph $f(x) = \dfrac{-4}{x+2}$, indicating clearly at least four points on the graph.

3. Perform each of the following divisions.

   (a) $(x^2 - 5x + 1) \div (x - 2)$

   (b) $(x^3 - 7x^2 + 12x) \div (x - 2)$

4. Solve each of the following quadratic equations, allowing complex solutions.

   (a) $x^2 + 6x + 10 = 0$

   (b) $9x^2 + 30x + 33 = 0$

   (c) $x^2 + 1 = 4x$

10.3 Simplifying Complex Fractions

10. (c) Divide rational expressions; simplify complex rational expressions.

Any time we see a fraction like $\frac{15}{20}$ we know that we generally wish to have it in its reduced form, sometimes called "lowest terms." We usually just recognize that five will go into both the numerator and denominator, so we divide five into each to get $\frac{3}{4}$. The sequence of equalities below and to the left shows the actual steps of this process. Below and to the right we show how we could "un-reduce" a fraction if we wanted to!

$$\frac{15}{20}=\frac{3\cdot 5}{4\cdot 5}=\frac{3}{4}\cdot\frac{5}{5}=\frac{3}{4}\cdot 1=\frac{3}{4} \qquad\qquad \frac{1}{3}=\frac{1}{3}\cdot 1=\frac{1}{3}\cdot\frac{7}{7}=\frac{7}{21}$$

This "un-reducing" process is exactly what we did in the previous section to get common denominators. In this section we will sometimes do this to put things called **complex fractions** into what is considered a more proper form. A complex fraction is a fraction whose numerator and/or denominator contains fractions, like

$$\frac{1+\dfrac{4}{x}-\dfrac{5}{x^2}}{\dfrac{1}{x}-\dfrac{6}{x^2}}$$

This is considered undesirable, so we correct it by multiplying both the top and bottom of the "larger" fraction by the smallest quantity that eliminates the "smaller" fractions. The following example shows how to do this for the fraction just given.

◇ **Example 10.3(a):** Simplify $\dfrac{1+\dfrac{4}{x}-\dfrac{5}{x^2}}{\dfrac{1}{x}-\dfrac{6}{x^2}}$

Solution:

$$\frac{1+\dfrac{4}{x}-\dfrac{5}{x^2}}{\dfrac{1}{x}-\dfrac{6}{x^2}}=\frac{1+\dfrac{4}{x}-\dfrac{5}{x^2}}{\dfrac{1}{x}-\dfrac{6}{x^2}}\cdot\frac{x^2}{x^2}=\frac{1(x^2)+\dfrac{4}{x}(x^2)-\dfrac{5}{x^2}(x^2)}{\dfrac{1}{x}(x^2)-\dfrac{6}{x^2}(x^2)}=\frac{x^2+4x-5}{x-6}$$

Note that the numerator of the simplified fraction factors to $(x+5)(x-1)$. Because neither of those factors will cancel with the denominator, there is no need to factor the numerator.

If we were to have multiplied the top and bottom both by x we would have eliminated the fractions $\dfrac{4}{x}$ and $\dfrac{1}{x}$, but not $\dfrac{5}{x^2}$ and $\dfrac{6}{x^2}$. However, multiplying the top and bottom by x^2 will eliminate all the fractions. x^3 would as well, but we should always use the smallest power of x that will "get the job done." When finished with eliminating the complex fractions, we should always see if the result will simplify, as shown in the next example.

◇ **Example 10.3(b):** Simplify $\dfrac{1-\dfrac{1}{x}-\dfrac{12}{x^2}}{1-\dfrac{16}{x^2}}$

Solution:

$$\frac{1-\dfrac{1}{x}-\dfrac{12}{x^2}}{1-\dfrac{16}{x^2}} = \frac{1-\dfrac{1}{x}-\dfrac{12}{x^2}}{1-\dfrac{16}{x^2}} \cdot \frac{x^2}{x^2} = \frac{x^2-x-12}{x^2-16} = \frac{(x-4)(x+3)}{(x+4)(x-4)} = \frac{x+3}{x+4}$$

Recall that to divide a fraction by another fraction we simply multiply the first by the reciprocal of the second. But before multiplying we should cancel any like terms in the numerators and denominators. The next example shows this.

◇ **Example 10.3(c):** Divide $\dfrac{3}{8} \div \dfrac{9}{16}$

Solution: $$\frac{3}{8} \div \frac{9}{16} = \frac{3}{8} \cdot \frac{16}{9} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}} \cdot \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{9}}} = \frac{1}{1} \cdot \frac{2}{3} = \frac{2}{3}$$

To divide two rational expressions we do the same thing, but we need to factor the numerators and denominators of the fractions and cancel any like terms before multiplying.

◇ **Example 10.3(d):** Simplify $\dfrac{\dfrac{x+3}{x-5}}{\dfrac{x-1}{x-5}}$

Solution: $$\frac{\dfrac{x+3}{x-5}}{\dfrac{x-1}{x-5}} = \frac{x+3}{x-5} \div \frac{x-1}{x-5} = \frac{x+3}{\cancel{x-5}} \cdot \frac{\cancel{x-5}}{x-1} = \frac{x+3}{x-1}$$

The next example will be similar to this last one, except that we will be required to factor before cancelling.

◇ **Example 10.3(e):** Simplify $\dfrac{\dfrac{x-1}{x^2+2x-15}}{\dfrac{x^2+x-2}{x+5}}$

Solution:

$$\frac{\dfrac{x-1}{x^2+2x-15}}{\dfrac{x^2+x-2}{x+5}} = \frac{x-1}{x^2+2x-15} \div \frac{x^2+x-2}{x+5} = \frac{x-1}{x^2+2x-15} \cdot \frac{x+5}{x^2+x-2} =$$

$$\frac{\cancel{x-1}}{\cancel{(x+5)}(x-3)} \cdot \frac{\cancel{x+5}}{(x+2)\cancel{(x-1)}} = \frac{1}{(x-3)(x+2)}$$

Section 10.3 Exercises To Solutions

1. Simplify each of the following complex fractions.

   (a) $\dfrac{1-\dfrac{3}{x}}{1+\dfrac{5}{x}}$ (b) $\dfrac{1+\dfrac{3}{x}+\dfrac{2}{x^2}}{1-\dfrac{1}{x^2}}$

2. Simplify each of the following.

   (a) $\dfrac{\dfrac{x-2}{x^2-1}}{\dfrac{x^2+x-6}{x-1}}$ (b) $\dfrac{\dfrac{x^2-1}{x^2+6x+8}}{\dfrac{x^2+2x+1}{x^2-x-20}}$

3. Simplify each of the following.

   (a) $\dfrac{\dfrac{x+2}{x-3}}{\dfrac{x^2-4}{x+3}}$ (b) $\dfrac{1-\dfrac{2}{x}-\dfrac{15}{x^2}}{1+\dfrac{4}{x}+\dfrac{3}{x^2}}$ (c) $\dfrac{\dfrac{x+2}{x-5}}{\dfrac{x^2+5x+6}{x^2-25}}$

   (d) $\dfrac{1-\dfrac{3}{y}}{y-\dfrac{9}{y}}$ (e) $\dfrac{\dfrac{x+2}{x^2-4}}{\dfrac{x+7}{x^2+10x+21}}$ (f) $\dfrac{\dfrac{1}{x}+\dfrac{2}{x^2}}{1-\dfrac{4}{x^2}}$

4. Rationalize each denominator, reduce and/or simplify your answer when possible.

   (a) $\dfrac{10}{\sqrt{14}}$ (b) $\dfrac{6\sqrt{5}}{\sqrt{2}}$

5. Perform each of the indicated computations, giving your answers in $a + bi$ form.

(a) $(3 - 8i)(2 + 5i)$ (b) $(3 - 8i) - (2 + 5i)$ (c) $(3 - 8i) \div (2 + 5i)$

6. Simplify each square root, **allowing complex numbers**.

(a) $-\sqrt{12}$ (b) $\sqrt{-36}$ (c) $\sqrt{-50}$

7. Solve each of the following quadratic equations, allowing complex solutions.

(a) $x^2 - 10x + 29 = 0$ (b) $x^2 + 25 = 8x$ (c) $x^2 + 4x + 5 = 0$

8. Divide $(5x^3 + 8x^2 - 24x + 8) \div (5x - 2)$.

9. Perform the indicated addition or subtraction and simplify your answer.

(a) $\dfrac{8}{x-2} - \dfrac{5}{x-3}$ (b) $\dfrac{2}{x^2 - 5x + 6} + \dfrac{3}{x^2 - 4}$

A Solutions to Exercises

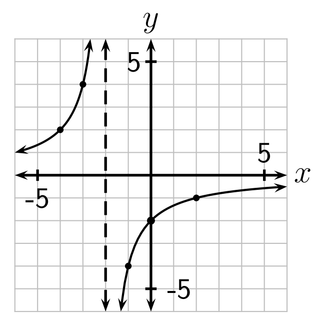
A.10 Chapter 10 Solutions

Section 10.1 Solutions Back to 10.1 Exercises

1. (a) $5x^2 + 3x - 9$ (b) $4x^2 + x - 5$ (c) $x^2 - 4x + 3$
   (c) $5x^2 + 7x + 2$ (d) $3x^2 - x + 2$
2. (a) $x + 2 + \frac{5}{x+1}$ (b) $x + 14 + \frac{68}{x-5}$
   (c) $3x + 7$ (d) $2x - 3 + \frac{3}{x-2}$
3. (a) 16 (b) $\frac{1}{3}$ (c) 4 (d) -3
4. (a) $1 + i$ (b) $\frac{3}{26} + \frac{15}{26}i$ (c) $\frac{1}{5} + \frac{1}{10}i$
5. (a) $x = 5 + 3i\sqrt{2},\ x = 5 - 3i\sqrt{2}$ (b) $x = 1 + i\sqrt{7},\ x = 1 - i\sqrt{7}$
   (c) $x = 4 + 2i\sqrt{3},\ x = 4 - 2i\sqrt{3}$

Section 10.2 Solutions Back to 10.2 Exercises

1. (a) $\dfrac{5x - 17}{(x + 5)(x - 1)}$ (b) $\dfrac{7x - 2}{(x + 2)(x - 2)}$ (c) $\dfrac{-4x - 23}{(x + 4)(x - 3)}$
2. See to the right
3. (a) $x - 3 - \frac{5}{x-2}$ (b) $x^2 - 5x + 2 + \frac{4}{x-2}$
4. (a) $x = -3 + i, \quad -3 - i$
   (b) $x = -\frac{5}{3} + \frac{2\sqrt{2}}{3}i,\ x = -\frac{5}{3} + \frac{2\sqrt{2}}{3}i$
   (c) $x = 2 + \sqrt{3},\ x = 2 - \sqrt{3}$



Section 10.3 Solutions Back to 10.3 Exercises

1. (a) $\dfrac{x - 3}{x + 5}$ (b) $\dfrac{x + 2}{x - 1}$
2. (a) $\dfrac{1}{(x + 1)(x + 3)}$ (b) $\dfrac{(x - 1)(x - 5)}{(x + 1)(x + 2)}$
3. (a) $\dfrac{x + 3}{(x - 3)(x - 2)}$ (b) $\dfrac{x - 5}{x + 1}$ (c) $\dfrac{x + 5}{x + 3}$
   (d) $\dfrac{1}{y + 3}$ (e) $\dfrac{x + 3}{x - 2}$ (f) $\dfrac{1}{x - 2}$
4. (a) $\dfrac{5\sqrt{14}}{7}$ (c) $3\sqrt{10}$

5. (a) $46 - i$ (b) $1 - 13i$ (c) $-\frac{34}{29} - \frac{31}{29}i$

6. (a) $-2\sqrt{3}$ (b) $6i$ (c) $5i\sqrt{2}$

7. (a) $x = 5 + 2i,\ x = 5 - 2i$ (b) $x = 4 + 3i,\ x = 4 - 3i$ (c) $x = -2 + i,\ x = -2 - i$

8. $x^2 + 2x - 4$

9. (a) $\dfrac{3x - 14}{(x-2)(x-3)}$ (b) $\dfrac{5x - 5}{(x-3)(x+2)(x-2)}$