Intermediate Algebra

Gregg Waterman
Oregon Institute of Technology

Contents

4 Equations and Roots, The Quadratic Formula

4.1 Roots

4. (a) Find (without a calculator) real roots of numbers when they exist.
   (b) Simplify a square root.

For any operation we have in mathematics, we always want to know if there is another operation that will reverse it. The operations that reverse powers are called **roots**. Let's start with the root that reverses (not perfectly, as we shall see) squaring.

Square Root

For any number $a \geq 0$, the **square root** of a is the *non-negative* number that can be squared to get a. We denote the square root of a by $\sqrt{a}$.

◇ **Example 4.1(a):** Find $\sqrt{9}$, the square root of nine.

Solution: Because $3^2 = 9$, $\sqrt{9} = 3$. Even though $(-3)^2 = 9$ also, $\sqrt{9}$ cannot be -3 because the square root of a number must not be negative.

◇ **Example 4.1(b):** Find $\sqrt{-16}$.

Solution: -16 is not greater than or equal to zero, so the square root of -16 does not exist. (Well, as a real number anyway. More on this below - for now we'll say that the square root of a negative number does not exist.) In the interest of efficiency (laziness?), we will use the abbreviation DNE for "does not exist."

◇ **Example 4.1(c):** Find $-\sqrt{16}$.

Solution: Here it is 16 that we want the square root of, because the negative sign is *outside* the square root. Because $4^2 = 16$, $\sqrt{16} = 4$. $-\sqrt{16}$ means the negative of the square root of 16, so $-\sqrt{16} = -4$.

Some of you may have previously encountered **imaginary numbers**, which allow us to find a "value" for a root like the one in Example 4.1(b) above. However, *until those numbers are introduced later you should assume that we are working only with* **real numbers**, which are the "ordinary numbers" that you are used to. So when we say "does not exist," we really mean "does not exist as a real number."

It is very important that you understand the difference between the last two examples. In example 4.1(b) we are asked for the square root of a negative number, which goes against the statement in the definition above that a must be greater than or equal to zero. In Example 4.1(c) we are asked for the negative of the square root of 16. Since $16 \geq 0$ we can find its square root, 4. We then take the negative of that to get -4.

Square roots are very useful for many operations. Because of this, it is important that you know (memorize) the perfect squares, starting with zero squared and going at least up until ten squared:

$$0,\ 1,\ 4,\ 9,\ 16,\ 25,\ 36,\ 49,\ 64,\ 81,\ 100$$

for the record, $11^2 = 121$, and $12^2 = 144$, called a *gross*. (you fireworks fans know that one!) Square roots are the kind of root that we will see most often, but there are other roots that reverse other powers also.

nth Root

- For any number $a \geq 0$ and any **even** whole number n, the **nth root** of a is the *non-negative* number whose nth power is a.
- For any number a and any *odd* whole number n, the **nth root** of a is the number (positive or negative, there will only be one) whose nth power is a.

We denote the nth root of a by $\sqrt[n]{a}$.

◇ **Example 4.1(d):** Find $\sqrt[3]{27}$. (We call third roots **cube roots**.)

Solution: Because $3^3 = 27$, $\sqrt[3]{27} = 3$.

◇ **Example 4.1(e):** Find $\sqrt[3]{-27}$.

Solution: $(-3)^3 = -27$, so $\sqrt[3]{-27} = -3$.

◇ **Example 4.1(f):** Find $\sqrt[4]{16}$.

Solution: Because $2^4 = 16$, $\sqrt[4]{16} = 2$.

◇ **Example 4.1(g):** Find $\sqrt[4]{-16}$.

Solution: Any number to the fourth power is positive, so $\sqrt[4]{-16}$ does not exist.

Note that
$$\sqrt{4 \cdot 9} = \sqrt{36} = 6 = 2 \cdot 3 = \sqrt{4} \cdot \sqrt{9}$$

This is an example of the most important property of square roots:

Product of Square Roots

For any numbers $a \geq 0$ and $b \geq 0$,

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \qquad \text{(and of course} \quad \sqrt{a}\sqrt{b} = \sqrt{ab}\text{)}$$

One thing we use this property for is to simplify square roots; this is something like reducing fractions. When we simplify a square root we don't change its value, we just change its appearance. The idea is to find a perfect square factor of the number whose root we are simplifying. (One reason to learn those perfect squares!) Here is an example of how we do this:

◇ **Example 4.1(h):** Simplify $\sqrt{20}$.

$$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

Solution: Using a calculator, $\sqrt{20} \approx 4.472$ and $2\sqrt{5} \approx 4.472$, verifying our work. ($\approx$ means 'approximately equal to.' We use it in this case because those two values are rounded versions of the actual values.)

Note that $\sqrt{20} = \sqrt{2 \cdot 10}$, but that is not useful because we don't know the square root of either two *or* ten. This process can be done in multiple steps if you don't recognize the *largest* perfect square factor:

◇ **Example 4.1(i):** Simplify $\sqrt{72}$.

Solution: One might do this as

$$\sqrt{72} = \sqrt{9 \cdot 8} = \sqrt{9} \cdot \sqrt{8} = 3\sqrt{4 \cdot 2} = 3\sqrt{4} \cdot \sqrt{2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2}$$

Solution: We could get the same result in fewer steps if we recognize that $72 = 36 \cdot 2$:

$$\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

We will usually show fewer steps when simplifying square roots like this. For $\sqrt{72}$ we might show our calculations like this:

$$\sqrt{72} = \sqrt{9} \cdot \sqrt{8} = 3\sqrt{4} \cdot \sqrt{2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2}$$

Section 4.1 Exercises

To Solutions

1. For each of the following, determine whether the root exists. If it doesn't exist, say so (write DNE for "does not exist") and you are done. If the root does exist, give its value without using your calculator.

   (a) $\sqrt{25}$ (b) $\sqrt{9}$ (c) $\sqrt[3]{8}$ (d) $\sqrt{-16}$ (e) $\sqrt[4]{1}$

   (f) $-\sqrt{49}$ (g) $\sqrt[3]{-27}$ (h) $-\sqrt[3]{27}$ (i) $\sqrt{\frac{16}{25}}$ (j) $\sqrt{-1}$

2. For each of the following, simplify the root if possible.

   (a) $\sqrt{45}$ (b) $\sqrt{8}$ (c) $\sqrt{15}$ (d) $\sqrt{72}$

   (e) $\sqrt{98}$ (f) $\sqrt{12}$ (g) $\sqrt{-50}$ (h) $\sqrt{75}$

3. Determine all values that the unknown is not allowed to have in each of the following.

   (a) $\dfrac{x^2-2x-3}{x-3}$ (b) $\dfrac{x^2-9}{x^2+5x+6}$ (c) $\dfrac{2x^2-2x-24}{x^2+4x+3}$

4. Reduce each of the rational expressions from Exercise 3.

5. Multiply each.

   (a) $\dfrac{3x}{x^2-25}\cdot(x+5)(x-5)$ (b) $\dfrac{x+5}{x+1}\cdot\dfrac{x^2+5x+4}{x-3}$

6. For each equation, tell what values x is not allowed to have, then solve the equation.

   (a) $x+\dfrac{6}{x}=-7$ (b) $\dfrac{8}{x^2-9}+\dfrac{4}{x+3}=\dfrac{2}{x-3}$

4.2 Adding, Subtracting, and Multiplying Expressions With Roots

4. (c) Add, subtract and multiply expressions containing roots.

If we have an expression like $7\sqrt{5} - 4\sqrt{3}$ we can't simplify any further, because the two different roots are unlike terms, like $7x^2$ and $4x$ are. However, $7\sqrt{5} - 4\sqrt{5} = 3\sqrt{5}$. We don't know an exact value for $\sqrt{5}$ but, whatever it is, we start with seven of them and then take away four of them.

◇ **Example 4.2(a):** Simplify $4\sqrt{3} - 2\sqrt{7} + \sqrt{3}$.

Solution: We can combine the terms $4\sqrt{3}$ and $\sqrt{3}$ to get $5\sqrt{3}$, but we cannot combine that with the $-2\sqrt{7}$. The result is then $5\sqrt{3} - 2\sqrt{7}$.

◇ **Example 4.2(b):** Simplify $3\sqrt{2} + 8\sqrt{2} - 4\sqrt{2}$.

Solution: In this case all three terms contain the same root, so they can all be combined to get $7\sqrt{2}$.

Recall that $\sqrt{a}\sqrt{b} = \sqrt{ab}$. Here are some applications of this:

◇ **Example 4.2(c):** Multiply $\sqrt{3}\sqrt{7}$.

$$\sqrt{3}\sqrt{7} = \sqrt{3 \cdot 7} = \sqrt{21}$$

Solution: We need to check $\sqrt{21}$ to see if it can be simplified. Since the only factors of 21 are 3 and 7, neither of which is a perfect square, it *cannot* be simplified.

◇ **Example 4.2(d):** Multiply $3\sqrt{5} \cdot 2\sqrt{7}$.

Solution: Because the only operations are multiplication, all values can be reordered to get the numbers first, followed by the roots. We then multiply the two numbers and the two roots:

$$3\sqrt{5} \cdot 2\sqrt{7} = 3 \cdot 2\sqrt{5}\sqrt{7} = 6\sqrt{35}$$

◇ **Example 4.2(e):** Multiply $\sqrt{6}\sqrt{3}$.

Solution: After multiplying the two roots together, we see that we need to simplify the resulting root:

$$\sqrt{6}\sqrt{3} = \sqrt{6 \cdot 3} = \sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$$

◇ **Example 4.2(f):** Multiply $\sqrt{7}\sqrt{7}$.

$$\sqrt{7}\sqrt{7} = \sqrt{49} = 7$$

Solution: In general, for any positive number a, $\sqrt{a}\sqrt{a} = a$.

◇ **Example 4.2(g):** Multiply $(3+\sqrt{7})(4-\sqrt{2})$.

Solution: Here we just 'FOIL' this out and multiply the two roots in the last term. There are no like terms to combine:

$$(3+\sqrt{7})(4-\sqrt{2}) = 12 - 3\sqrt{2} + 4\sqrt{7} - \sqrt{7}\sqrt{2} = 12 - 3\sqrt{2} + 4\sqrt{7} - \sqrt{14}$$

◇ **Example 4.2(h):** Multiply $(3+\sqrt{7})(5-2\sqrt{7})$.

Solution: This is done in the same way as the previous example, except that there are some like terms to be combined:

$$(3+\sqrt{7})(5-2\sqrt{7}) = 15 - 6\sqrt{7} + 5\sqrt{7} - 2\sqrt{7}\sqrt{7} = 15 - \sqrt{7} - 2(7) = 1 - \sqrt{7}$$

◇ **Example 4.2(i):** Multiply $(3-\sqrt{5})^2$.

Solution: Here we must only remember that $(3-\sqrt{5})^2$ means $3-\sqrt{5}$ times itself, and then proceed in the same way as the previous example:

$$(3-\sqrt{5})^2 = (3-\sqrt{5})(3-\sqrt{5}) = 9 - 3\sqrt{5} - 3\sqrt{5} + \sqrt{5}\sqrt{5} = 9 - 6\sqrt{5} + 5 = 14 - 6\sqrt{5}$$

Section 4.2 Exercises To Solutions

1. Simplify by combining like terms, *if possible*.

   (a) $9 - 3\sqrt{5} - 3\sqrt{5} + 25$ (b) $25 + 10\sqrt{3} - 10\sqrt{3} - 12$

   (c) $15 + 5\sqrt{2} - 6\sqrt{3} - 2\sqrt{6}$ (d) $8 - 4\sqrt{3} + 2\sqrt{7} - \sqrt{21}$

2. Multiply and simplify.

   (a) $\sqrt{6}\cdot\sqrt{3}$ (b) $\sqrt{5}\cdot\sqrt{5}$ (c) $\sqrt{5}\cdot\sqrt{7}$

   (d) $\sqrt[3]{4}\cdot\sqrt[3]{20}$ (e) $3\sqrt{10}\cdot 2\sqrt{5}$ (f) $\sqrt{2}\cdot 5\sqrt{6}$

   (g) $4(5+\sqrt{7})$ (h) $(3+\sqrt{5})(2-\sqrt{3})$ (i) $(1-\sqrt{10})(5-\sqrt{2})$

3. For each equation, tell what values x is not allowed to have, then solve the equation.

(a) $\dfrac{4}{x-3} - \dfrac{3}{x+3} = 1$

(b) $\dfrac{x+3}{x-1} + \dfrac{x+5}{x} = \dfrac{3x+1}{x-1}$

4. For each of the following, determine whether the root exists. If it doesn't exist, say so (write DNE for "does not exist") and you are done. If the root does exist, give its value without using your calculator.

(a) $\sqrt[3]{-1}$ (b) $\sqrt[50]{1}$ (c) $\sqrt{0}$ (d) $\sqrt{100}$ (e) $\sqrt{\frac{64}{25}}$

5. For each of the following, simplify the root if possible.

(a) $\sqrt{32}$ (b) $\sqrt{24}$ (c) $\sqrt{18}$ (d) $\sqrt{50}$

4.3 Solving Quadratic Equations With The Quadratic Formula

4. (d) Solve quadratic equations using the quadratic formula.

Consider the quadratic equation $x^2 - 6x + 4 = 0$. Of course we expect this equation to have perhaps two solutions, but a few minutes of effort will convince us that the left side of the equation cannot be factored. Note that if we substitute $3 - \sqrt{5}$ into the equation for x we get (using the result of Example 4.2(i))

$$(3 - \sqrt{5})^2 - 6(3 - \sqrt{5}) + 4 = 14 - 6\sqrt{5} - 18 + 6\sqrt{5} + 4 = 0,$$

so $x = 3 - \sqrt{5}$ is a solution to the equation.

It turns out that $x = 3 + \sqrt{5}$ is another solution. $3 + \sqrt{5}$ and $3 - \sqrt{5}$ are sometimes called **conjugates**. One might ask how we would find that those are the solutions to the equation $x^2 - 6x + 4 = 0$. To find those solutions we use something called the **quadratic formula**. The two solutions to the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad \text{and} \qquad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

If the quantity $b^2 - 4ac$ is negative we will have the square root of a negative number, which does not exist. In that case the equation has no solution. Note that the above two formulas are exactly the same except that one has a minus sign where the other has a plus. We will combine them into one, using the symbol $\pm$ to indicate that both an addition and a subtraction have to take place at that point in order to get both solutions.

Quadratic Formula

If $b^2 - 4ac \geq 0$, the solutions to $ax^2 + bx + c = 0$ are obtained from

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac < 0$ we will say that the equation $ax^2 + bx + c = 0$ has no solution.

◇ **Example 4.3(a):** Solve $x^2 - 4x - 5 = 0$ using the quadratic formula.

Solution: Here $a = 1$, $b = -4$ and $c = -5$, so

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)} = \frac{4 \pm \sqrt{16 + 20}}{2} =$$

$$\frac{4 \pm \sqrt{36}}{2} = \frac{4 \pm 6}{2} = \frac{10}{2}, \frac{-2}{2} = 5, -1$$

Note that the equation from the previous example can be factored to $(x-5)(x+1)=0$, giving us the solutions $x=-1,5$. If an equation can be solved by factoring but we try to use the quadratic formula, we will get the same solutions as we would by factoring. A person should generally try factoring first, and if they can't figure out how to factor fairly quickly, then they should use the quadratic formula. The real advantage of the quadratic formula is that it allows us to get solutions for equations that can't be factored.

◇ **Example 4.3(b):** Solve $x^2+10x+23=0$ using the quadratic formula.

$$x=\frac{-10\pm\sqrt{10^2-4(1)(23)}}{2(1)}=\frac{-10\pm\sqrt{100-92}}{2}=\frac{-10\pm\sqrt{8}}{2}$$

Solution: At this point we simplify $\sqrt{8}$ to get $\sqrt{8}=\sqrt{4}\sqrt{2}=2\sqrt{2}$. This is substituted into what we have so far, the 'big' fraction is split into two 'smaller' fractions, and each fraction is reduced:

$$\frac{-10\pm\sqrt{8}}{2}=\frac{-10\pm2\sqrt{2}}{2}=\frac{-10}{2}\pm\frac{2\sqrt{2}}{2}=-5\pm\sqrt{2},$$

so $x=-5+\sqrt{2},\ -5-\sqrt{2}$.

At one point in this last example the big fraction from the formula was broken apart into two separate fractions. This uses a very useful little mathematical manipulation:

"Un-Adding" (and "Un-Subtracting") Fractions

For any numbers a,b and c, $\dfrac{a\pm b}{c}=\dfrac{a}{c}\pm\dfrac{b}{c}$.

Section 4.3 Exercises

To Solutions

1. (a) Solve $2x^2-7x-15=0$ using the quadratic formula. Then solve it by factoring; your answers should, of course, be the same either way.

   (b) Solve $4x^2+13x+3=0$ using the quadratic formula. Then solve it by factoring to check your answers.

2. For each of the following, use the quadratic formula to solve the given equation. Simplify your answers.

   (a) $x^2-2x-4=0$

   (b) $x^2-6x+7=0$

   (c) $x^2+10x+13=0$

   (d) $2x^2=2x+1$

   (e) $9x^2-12x-1=0$

   (f) $25x^2+10x=62$

3. Solve each equation by factoring.

(a) $x^3 + 5x^2 - 9x - 45 = 0$

(b) $2x^2 = 5x + 12$

(c) $9x^2 - 1 = 0$

(d) $x^3 - 7x^2 + 10x = 0$

4. For each equation, tell what values x is not allowed to have, then solve the equation.

(a) $\dfrac{3}{x-1} - \dfrac{2}{x+4} = \dfrac{x^2+8x+6}{x^2+3x-4}$

(b) $\dfrac{3}{x-4} = \dfrac{5x+4}{x^2-16} - \dfrac{4}{x+4}$

5. Simplify by combining like terms, *if possible*.

(a) $1 + \sqrt{5} - \sqrt{5} - 5$

(b) $12 + 2\sqrt{2} - 18\sqrt{5} - 3\sqrt{10}$

4.4 Solving Equations Containing Roots

4. (e) Solve equations containing roots.

Recall that, if $a \geq 0$,

$$(\sqrt{a})^2 = \sqrt{a} \cdot \sqrt{(a)} = \sqrt{a \cdot a} = \sqrt{a^2} = a.$$

We can use this idea to solve and equation when one side of the equation is a square root; we simply square both sides to eliminate the root, then solve the resulting equation:

◇ **Example 4.4(a):** Solve $\sqrt{3x+4} = 5$.

Solution:

$$\begin{aligned} (\sqrt{3x+4})^2 &= 5^2 \\ 3x+4 &= 25 \\ 3x &= 21 \\ x &= 7 \end{aligned}$$

$$\text{Check :} \quad \begin{aligned} \sqrt{3(7)+4} &\stackrel{?}{=} 5 \\ \sqrt{21+4} &\stackrel{?}{=} 5 \\ \sqrt{25} &= 5 \end{aligned}$$

◇ **Example 4.4(b):** Solve $\sqrt{7x-13} = x-1$. Another Example

Solution:

$$\begin{aligned} (\sqrt{7x-13})^2 &= (x-1)^2 \\ 7x-13 &= (x-1)(x-1) \\ 7x-13 &= x^2-2x+1 \\ 0 &= x^2-9x+14 \\ 0 &= (x-7)(x-2) \\ x &= 2, 7 \end{aligned}$$

$$\text{Check} \quad x = 2: \quad \sqrt{7(2)-13} \stackrel{?}{=} 2-1$$

$$\text{Check} \quad x = 7: \quad \begin{aligned} \sqrt{7(7)-13} &\stackrel{?}{=} 7-1 \\ \sqrt{36} &= 6 \end{aligned}$$

When solving equations like these, a funny thing sometimes happens. It is possible that a solution that you find by the procedure we've been using is not actually a valid solution. We'll see this in the next examples.

◇ **Example 4.4(c):** Solve $\sqrt{4x-7} = -3$.

Solution:

$$\begin{aligned} (\sqrt{4x-7})^2 &= (-3)^2 \\ 4x-7 &= 9 \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

$$\text{Check :} \quad \begin{aligned} \sqrt{4(4)-7} &\stackrel{?}{=} -3 \\ \sqrt{16-7} &\stackrel{?}{=} -3 \\ \sqrt{9} &\neq -3 \end{aligned}$$

Since the only possible solution does not check, the equation has no solution.

When the root is not alone on one side, we need to do a little before squaring both sides, as shown in the next example.

◇ **Example 4.4(d):** Solve $x = \sqrt{6x+1}+1$. Another Example

Solution: In this case we must first get the square root alone on one side, *THEN* square both sides of the equation:

$$\begin{aligned} x-1 &= \sqrt{6x+1} \\ (x-1)^2 &= (\sqrt{6x+1})^2 \\ x^2-2x+1 &= 6x+1 \\ x^2-8x &= 0 \\ x(x-8) &= 0 \\ x &= 0,\ 8 \end{aligned}$$

Check $x = 0:$

$$0 \stackrel{?}{=} \sqrt{6(0)+1}+1$$

$0 \neq \sqrt{1}+1$, so $x=0$ is *NOT* a solution

Check $x = 8:$

$$8 \stackrel{?}{=} \sqrt{6(8)+1}+1$$

$8 = \sqrt{49}+1$, so $x=8$ *IS* a solution

It is also possible to find two solutions, neither of which checks. In that case there is no solution.

Section 4.4 Exercises

To Solutions

1. Solve each of the following equations.

   (a) $\sqrt{4x+1}=3$ (b) $\sqrt{5x+10}=x+2$ (c) $\sqrt[3]{4x+5}=-1$

2. The following equations are slightly different than the ones in Exercise 1. Begin by adding or subtracting something to both sides in order to get the root alone on one side, before squaring both sides.

   (a) $\sqrt{3x+13}-2=3$ (b) $\sqrt{3x+15}-5=x$

3. Solve each equation. Be sure to check all solutions to see if they are valid.

   (a) $\sqrt[3]{4x+4}+6=7$ (b) $\sqrt{25x-4}=4$

   (c) $\sqrt{3x+13}=x+3$ (d) $\sqrt[3]{4x-15}+5=2$

   (e) $\sqrt{3x+2}+7=5$ (f) $\sqrt{x-2}=x-2$

4. Multiply each.

   (a) $\dfrac{x+2}{x-3}\cdot\dfrac{x^2-9}{x^2+9x+14}$ (b) $\dfrac{x^2+7x+10}{2x^2+10x}\cdot 2x(x+5)$

   (c) $(x+4)(x-1)\cdot\left(\dfrac{3}{x-1}-\dfrac{5}{x+4}\right)$ (d) $3x^2\cdot\left(\dfrac{9}{x^2}+\dfrac{1}{3}\right)$

5. For each of the following, use the quadratic formula to solve the given equation. Simplify your answers.

   (a) $x^2 + 8x + 13 = 0$

   (b) $x^2 + 6x + 7 = 0$

A Solutions to Exercises

A.4 Chapter 4 Solutions

Section 4.1 Solutions Back to 4.1 Exercises

1. (a) 5 (b) 3 (c) 2 (d) DNE (e) 1
   (f) -7 (g) -3 (h) -3 (i) $\frac{4}{5}$ (j) DNE
2. (a) $3\sqrt{5}$ (b) $2\sqrt{2}$ (c) $\sqrt{15}$ (d) $6\sqrt{2}$ (e) $7\sqrt{2}$
   (f) $2\sqrt{3}$ (g) DNE (h) $5\sqrt{3}$
3. (a) $x \neq 3$ (b) $x \neq -2, -3$ (c) $x \neq -1, -3$
4. (a) $x+1$ (b) $\dfrac{x-3}{x+2}$ (c) $\dfrac{2x-8}{x+1}$
5. (a) $3x$ (b) $\dfrac{x^2+9x+20}{x-3}$
6. (a) $x \neq 0, \quad x = -1, -6$ (b) $x \neq 3, -3, \quad x = 5$

Section 4.2 Solutions Back to 4.2 Exercises

1. (a) $34 - 6\sqrt{5}$ (b) 13 (c) $15 + 5\sqrt{2} - 6\sqrt{3} - 2\sqrt{6}$
   (d) $8 - 4\sqrt{3} + 2\sqrt{7} - \sqrt{21}$
2. (a) $3\sqrt{2}$ (b) 5 (c) $\sqrt{35}$
   (d) $2\sqrt[3]{10}$ (e) $30\sqrt{2}$ (f) $10\sqrt{3}$
   (g) $20 + 4\sqrt{7}$ (h) $6 - 3\sqrt{3} + 2\sqrt{5} - \sqrt{15}$
   (i) $5 - \sqrt{2} - 5\sqrt{10} + 2\sqrt{5}$
3. (a) $x \neq 3, -3, \quad x = -5, 6$ (b) $x \neq 0, 1, \quad x = 5$
4. (a) -1 (b) 1 (c) 0 (d) 10 (e) $\frac{8}{5}$
5. (a) $4\sqrt{2}$ (b) $2\sqrt{6}$ (c) $3\sqrt{2}$ (d) $5\sqrt{2}$

Section 4.3 Solutions Back to 4.3 Exercises

1. (a) $x = -\frac{3}{2}, \ 5$ (b) $x = -\frac{1}{4}, \ -3$
2. (a) $x = 1 + \sqrt{5}, \ 1 - \sqrt{5}$ (b) $x = 3 + \sqrt{2}, \ 3 - \sqrt{2}$
   (c) $x = -5 + 2\sqrt{3}, \ -5 - 2\sqrt{3}$ (d) $x = \frac{1}{2} + \frac{\sqrt{3}}{2}, \ \frac{1}{2} - \frac{\sqrt{3}}{2}$
   (e) $x = \frac{2}{3} + \frac{\sqrt{5}}{3}, \ \frac{2}{3} - \frac{\sqrt{5}}{3}$ (f) $x = -\frac{1}{5} + \frac{3\sqrt{7}}{5}, \ -\frac{1}{5} - \frac{3\sqrt{7}}{5}$

3. (a) $x = -5, -3, 3$ (b) $x = 4, -\frac{3}{2}$ (c) $x = \frac{1}{3}, -\frac{1}{3}$ (d) $x = 0, 2, 5$

4. (a) $x \neq 1, -4,$ $x = -8$ (b) $x \neq 4, -4,$ no solution

5. (a) -4 (b) $12 + 2\sqrt{2} - 18\sqrt{5} - 3\sqrt{10}$

Section 4.4 Solutions

Back to 4.4 Exercises

1. (a) $x = 2$ (b) $x = -2, 3$ (c) $x = -\frac{3}{2}$

2. (a) $x = 4$ (b) $x = -2, -5$

3. (a) $x = -\frac{3}{4}$ (b) $x = \frac{4}{5}$ (c) $x = 1$ $x = -4$ is not a solution
   (d) $x = -3$ (e) no solution ($x = \frac{2}{3}$ doesn't check) (f) $x = 2, 3$

4. (a) $\dfrac{x+3}{x+7}$ (b) $x^2 + 7x + 10$ (c) $-2x + 17$ (d) $27 + x^2$

5. (a) $x = -4 + \sqrt{3}, \ -4 - \sqrt{3}$ (b) $x = -3 + \sqrt{2}, \ -3 - \sqrt{2}$