College Algebra

Gregg Waterman
Oregon Institute of Technology

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0 Introduction

0.1 About Learning

This textbook is designed to provide you with a basic reference for the topics within. That said, it cannot learn for you, nor can your instructor; ultimately, the responsibility for learning the material lies with you. Before beginning the mathematics, I would like to tell you a little about what research tells us are the best strategies for learning. Here are some of the principles you should adhere to for the greatest success:

- **It's better to recall than to review.** It has been found that re-reading information and examples does little to promote learning. Probably the single most effective activity for learning is attempting to recall information and procedures yourself, rather than reading them or watching someone else do them. The process of trying to recall things you have seen is called *retrieval*.

- **Spaced practice is better than massed practice.** Practicing the same thing over and over (called *massed practice*) is effective for learning very quickly, but it also leads to rapid forgetting as well. It is best to space out, over a period of days and even weeks, your practice of one kind of problem. Doing so will lead to a bit of forgetting that promotes retrieval practice, resulting in more lasting learning. And it has been determined that your brain makes many of its new connections while you sleep!

- **Interleave while spacing.** *Interleaving* refers to mixing up your practice so that you're attempting to recall a variety of information or procedures. Interleaving naturally supports spaced practice.

- **Attempt problems that you have not been shown how to solve.** It is beneficial to attempt things you don't know how to do *if you attempt long enough to struggle a bit*. You will then be more receptive to the correct method of solution when it is presented, or you may discover it yourself!

- **Difficult is better.** You will not strengthen the connections in your brain by going over things that are easy for you. Although your brain is not a muscle, it benefits from being "worked" in a challenging way, just like your body.

- **Connect with what you already know, and try to see the "big picture."** It is rare that you will encounter an idea or a method that is completely unrelated to anything you have already learned. New things are learned better when you see similarities and differences between them and what you already know. Attempting to "see how the pieces fit together" can help strengthen what you learn.

- **Quiz yourself to find out what you *really* do (and don't) know.** Understanding examples done in the book, in class, or on videos can lead to the illusion of knowing a concept or procedure when you really don't. Testing yourself frequently by attempting a variety of exercises without referring to examples is a more accurate indication of the state of your knowledge and understanding. This also provides the added benefit of interleaved retrieval practice.

- **Seek and utilize immediate feedback.** The answers to all of the exercises in the book are in the back. Upon completing any exercise, check your answer right away and correct any misunderstandings you might have. Many of our in-class activities will have answers provided, in one way or another, shortly after doing them.

Through the internet, we now have immediate access to huge amounts of information, some of it good, some of it not! I have attempted to either find or make *quality* videos over many of the concepts and procedures in this book. I have put links to these at my web page,

http://math.oit.edu/~watermang/

(You can also find the web page by doing a search for *waterman oit*.) I would encourage you to view at least a few of them to see whether or not they could be useful for your learning. In keeping with the above suggestions, one good way to use the videos is to view long enough to see the problem to be solved, then pause the video and try solving it yourself. After solving or getting stuck (while trying hard, see the fourth bullet above), forward ahead to see the solution and/or watch how the problem is solved.

It is somewhat inevitable that there will be some errors in this text that I have not caught. As soon as errors are brought to my attention, I will update the online version of the text to reflect those changes. If you are using a hard copy (paper) version of the text, you can look online if you suspect an error. If it appears that there is an uncorrected error, please send me an e-mail at *gregg.waterman@oit.edu* indicating where to find the error.

Gregg Waterman
Oregon Institute of Technology
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0.2 About This Course

This course is an in-depth introduction to functions. Our study will be guided by some goals, and related *essential questions* that are designed to get at the essence of the idea of a function.

Course Goals

The overall goal of this course is to understand the concept of a function. This is a multi-faceted task, to be accomplished by developing several more specific understandings related to functions:

- For any type of function studied in the course students will understand, from both analytical (equation) and graphical points of view,
  - the specific behavior of the function as a relationship between two variable quantities (output from input, input from output, domain and range, intercepts, locations and values of minima/maxima)
  - the general behavior of the function as a relationship between two variable quantities (increasing/decreasing, positive/negative, existence of minima/maxima)
- Students will understand the classification of functions into families (linear, quadratic, polynomial, rational, exponential, logarithmic).
- Students will understand the relationship between analytical and graphical representations of functions, including the effects of parameters in the analytical representation on the appearances of graphs.
- Students will understand the role of functions in modelling "real world phenomena."
- Students will understand the role of notation and organization of written work in both clearly communicating ideas and in understanding concepts.

Essential Questions

- How do we determine specific output and input values of interest from either of an equation or a graph representing a function?
- How do we determine general behaviors of a function from its graph, and what behaviors are we interested in?
- What different types of functions are we interested in? How are the various types alike, and how are they different?
- How are the graphs of functions affected by changes to parameters in their equations?
- How do we use mathematical functions to represent related quantities in science, engineering and finance?
- What are the commonly accepted notations that allow us to communicate mathematics with others? How do we organize work so that others can understand our thought processes?

0.3 An Example

Much of mathematics concerns itself with describing, as precisely as possible, relationships between related variable quantities. Although in reality several quantities are often related, we will stick with relationships between two quantities in this course. In such situations, one quantity usually depends on the other. Here are some examples one might run across:

- The cost of building a fence depends on the length of the fence.
- The number of fish in a lake depends on the length of time after some "starting" time.
- The intensity of an ultrasound signal depends on how far it has traveled through soft tissue.
- The demand for some item depends on the cost of the item.

If quantity y depends on quantity x, we call y the **dependent variable** and x the **independent variable**.

We usually want to obtain an equation that describes such a relationship, or to use such an equation that is already known. An equation relating variable quantities is an example of what we call a **mathematical model**. Models (equations) are usually obtained from either statistical data or physical principles.

Lets take a look at an example of a model and what we might do with it. In doing so we will touch on the major themes of this entire course. Suppose that the Acme Company makes and sells Geegaws. Their weekly profit P (in dollars) is given by the equation

$$P = -8000 + 27x - 0.01x^2,$$

where x is the number of Geegaws sold that week. Note that it will be possible for P to be negative (for example, if they make and sell $x = 0$ Geegaws in a week, their profit is $P = -8000$ dollars for the week); of course negative profit is really loss!

◇ **Example 0.1(a):** If 1280 Geegaws are sold in a week, what profit can the Acme Company expect for the week?

Solution: This is the most basic use for our model, and we obtain the profit by **evaluating the equation** for the value $x = 1280$:

$$P = -8000 + 27(1280) - 0.01(1280)^2 = \$10,176$$

◇ **Example 0.1(b):** Determine the number of Geegaws that must be sold in a week in order to obtain a profit of $7,000.

Solution: In this situation the variable P has the value 7000, and we must use that fact to determine the corresponding value of x. Substituting the value of P in, we obtain

$$7000 = -8000 + 27x - 0.01x^2.$$

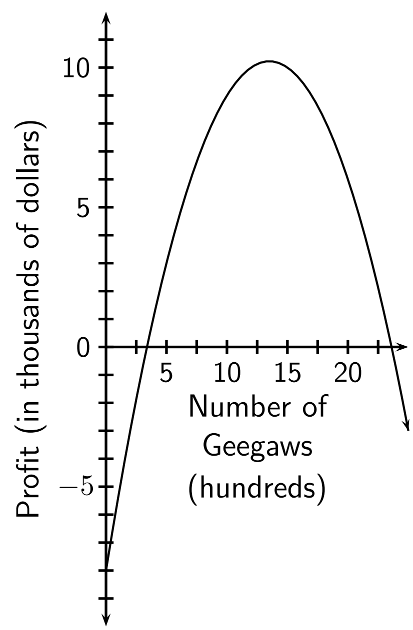
Here we can't simply evaluate like we did in the previous example - we must instead **solve the equation**. You probably already know the technique for doing this, but we will review it in the next section. It turns out that the equation has *two* solutions, $x = 782$ Geegaws or $x = 1918$ Geegaws. Producing and selling either of those numbers of Geegaws will result in a weekly profit of $7,000. It may not be clear why, but we'll save for later the discussion of why that is the case.

The two processes you've just seen carried out will be repeated over and over in this course. Given a model for the relationship between two variables, we often want to find the value of one of the variables, given the value of the other. In some cases we will be able to simply evaluate an equation, and in other cases we'll have to solve it. The techniques of solving equations vary somewhat, depending on the type of equation we have. Determining the correct technique for solving an equation, and carrying that technique out, will be one of our areas of focus.

Evaluating or solving are often useful, but they fail to give us an overall understanding of the nature of a relationship between two variables. When we wish to "see the forest rather than individual trees" we will create and evaluate a **graph of the equation**. You are undoubtedly familiar with this idea, but let's take a look at it for our equation. Below and to the right is a graph of the equation. Note that the independent variable is (always) on the horizontal axis and the dependent variable is (always) on the vertical axis. From the graph we can make a number of observations:

- The profit can actually be negative if few Geegaws are made and sold, and it is also negative when too many are made. When few Geegaws are produced and sold, the amount of money brought in (called the **revenue**) is not high enough to pay the fixed costs of maintaining a facility and paying employees.

- Initially the profit increases as more Geegaws are made and sold, but eventually the profit starts decreasing as more Geegaws are produced. Making and selling more Geegaws results in greater profit until too many are produced. At that point the supply exceeds the demand and the excess Geegaws cannot be sold. Eventually the profit decreases to the point that it is negative, because the cost of producing unsold Geegaws exceeds the revenue generated by the ones that are sold.



- Where the profit quits increasing and starts decreasing, the maximum profit is obtained. We can see that this occurs when the number of Geegaws produced and sold is about 1300 or 1400. We'll see later how to determine the exact number of Geegaws that need to be produced and sold to maximize the profit. (This can be done precisely from the equation itself, rather than relying on the graph to obtain an approximate value.)

One other thing we might wish to do is determine the number of Geegaws that need to be produced in order to generate some minimum weekly profit, like at least $5000. In this case we want the values of x for which the profit is greater than or equal to $5000:

$$-8000 + 27x - 0.01x^2 \geq 5000$$

From the graph we can estimate that Acme needs to make and sell between about 700 and 2200 Geegaws to obtain a profit of at least $5000, but we will see later that we can use the equation to get a more precise answer. For now, let's get on with the basics of solving equations and graphing relationships.