College Algebra

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Contents

1 Equations in One and Two Unknowns

Outcome/Performance Criteria:

1. Solve equations in one unknown, and apply equations to solve problems. Graph solution sets of equations in two unknowns.

   (a) Solve a linear, quadratic, polynomial and absolute value equation.

   (b) Solve an equation containing rational or radical expressions.

   (c) Solve an applied problem by writing and solving an equation modeling the given situation.

   (d) Use a formula to solve a problem.

   (e) Solve a formula for a given unknown.

   (f) Find solution pairs for an equation in two unknowns.

   (g) Graph the solution set of an equation in two unknowns without using a graphing calculator.

   (h) Identify x- and y-intercepts of an equation in two unknowns from its graph; find x- and y-intercepts of an equation in two unknowns algebraically.

   (i) Graph a line; determine the equation of a line.

   (j) Solve a system of two linear equations in two unknowns by the addition method; solve a system of two linear equations in two unknowns by the substitution method.

   (k) Solve a problem using a system of two linear equations in two unknowns.

   (l) Graph $y = ax^2 + bx + c$ by finding and plotting the vertex, y-intercept, and several other points.

   (m) Identify or create the graphs of $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$ and $y = \frac{1}{x}$.

   (n) Graph variations of $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$ and $y = \frac{1}{x}$.

1.1 Solving Equations in One Unknown, Part I

Performance Criterion:

1. (a) Solve a linear, quadratic, polynomial or absolute value equation.

The key idea in solving an equation is the following:

We create a sequence of new equations that are equivalent to the original equation, until we get one that is simple enough that we can see what the solution(s) is (are).

There is no single procedure that can be used to solve every equation, but there are some general guidelines to keep in mind. The guidelines will be illustrated in this section and the next one, and summarized at the end of that section.

Linear Equations

Linear equations are ones in which the unknown is *NOT* to a power, under a root, or in the bottom of a fraction. Here are some examples of linear equations:

$$5-2t=1 \qquad 20=8-5(2x-3)+4x \qquad \tfrac{1}{2}x+\tfrac{1}{4}=\tfrac{1}{3}x+\tfrac{5}{4}$$

Here are some examples of equations that *ARE NOT* linear:

$$3x^3+36x=21x^2 \qquad \frac{x}{x-4}+\frac{2x}{x^2-7x+12}=\frac{15}{x-3}$$

$$t+5=\sqrt{t+7} \qquad (x-2)(x+1)=5$$

Let's look a bit more at the two equations

$$(x-2)(x+1)=5 \qquad \text{and} \qquad 20=8-5(2x-3)+4x$$

The equation on the left might appear to be linear, since the two places we see an x it is to the first power and not under a root or in the bottom of a fraction. However, if we distribute to eliminate parentheses, the two equations become

$$x^2-x-2=5 \qquad \text{and} \qquad 20=8-10x+15+4x$$

and we can now see that the first of these two is not linear.

To solve linear equations we usually follow these steps:

1) Eliminate parentheses by distributing; eliminate fractions by multiplying both sides by the least common denominator of all fractions.

2) Combine like terms on each side of the equation.

3) Add or subtract terms to or from both sides of the equation to get a term with the unknown on one side, and just a number on the other.

4) Divide by the coefficient of the unknown (the number multiplying it) on both sides to finish.

Let's demonstrate the above steps with some examples.

◇ **Example 1.1(a):** Solve $5 - 2t = -1$. Another Example

Solution: Here is how we would probably solve this equation:

$5 - 2t = -1$	the original equation
$-2t = -6$	subtract 5 from both sides
$t = 3$	divide both sides by -2

To avoid having negatives at the second step we could instead add $2t$ and 1 to both sides:

$5 - 2t = -1$	the original equation
$6 = 2t$	add $2t$ and 1 to both sides
$3 = t$	divide both sides by 2

◇ **Example 1.1(b):** Solve $20 = 8 - 5(2x - 3) + 4x$. Another Example

Solution:

$20 = 8 - 5(2x - 3) + 4x$	the original equation
$20 = 8 - 10x + 15 + 4x$	distribute to eliminate parentheses
$20 = -6x + 23$	combine like terms
$6x = 3$	add $6x$ and subtract 20 to/from both sides
$x = \frac{1}{2}$	divide both sides by 6 and reduce

◇ **Example 1.1(c):** Solve $\frac{1}{2}x + \frac{1}{4} = \frac{1}{3}x + \frac{5}{4}$. Another Example

Solution:

$\frac{1}{2}x + \frac{1}{4} = \frac{1}{3}x + \frac{5}{4}$	the original equation
$\frac{12}{1}\left(\frac{1}{2}x + \frac{1}{4}\right) = \frac{12}{1}\left(\frac{1}{3}x + \frac{5}{4}\right)$	multiply both sides by 12 in the form $\frac{12}{1}$
$\frac{12}{1}\left(\frac{1}{2}x\right) + \frac{12}{1}\left(\frac{1}{4}\right) = \frac{12}{1}\left(\frac{1}{3}x\right) + \frac{12}{1}\left(\frac{5}{4}\right)$	distribute the $\frac{12}{1}$ to each term
$6x + 3 = 4x + 15$	multiply - fractions are gone!
$2x = 12$	get x terms on one side, numbers on other
$x = 6$	divide both sides by 2

Quadratic and Polynomial Equations

A **polynomial expression** consists of some whole number powers of an unknown, each multiplied by a number (called a **coefficient**), and all added together. Some examples would be

$$x^4 - 7x^3 + 5x^2 + \frac{2}{3}x - 1 \qquad \text{and} \qquad 3.72x^2 + 14.7x$$

Polynomial equations are equations in which each side of the equation is a polynomial expression (which can be just a number). Here are some examples of polynomial equations:

$$2x^2 + x = 3 \qquad 3x^3 + 36x = 21x^2 \qquad 9x^4 = 12x^3 \qquad x^2 - 10x + 13 = 0$$

The highest power of x is called the **degree** of the polynomial equation. The highest power of the unknown *CAN* be first power, so linear equations are in fact polynomial equations. We generally don't want to think of them as such, since our method for solving them is a bit different than what we do to solve polynomial equations of degree two and higher. Polynomial equations of degree two are the most commonly encountered polynomial equations, and they have their own name: **quadratic** equations. The general procedure for solving polynomial equations is this:

1) Add or subtract all terms to get them on one side of the equation and zero on the other side.

2) Factor the non-zero side of the resulting equation.

3) Determine the values of the unknown that will make each factor zero; those values are the solutions to the equation.

Here are some examples of this process in action:

◇ **Example 1.1(d):** Solve $2x^2 + x = 3$. Another Example

Solution:

$2x^2 + x$	$=$	3	the original equation
$2x^2 + x - 3$	$=$	0	get all terms on one side and zero on the other
$(2x + 3)(x - 1)$	$=$	0	factor
$2x + 3 = 0$	or	$x - 1 = 0$	set each factor equal to zero to get linear equations
$x = -\frac{3}{2}$	or	$x = 1$	solve linear equations

It should have been clear after factoring that one of the solutions is 1, and there is really no need to show the equation $x - 1 = 0$.

Note that we should have been able to see at the third step above that if x was one we would have $(something)(0)$ on the left, which of course is zero regardless of what the "something" is. This could be called "solving by inspection" at that point. As another example, if we had $(x + 5)(x - 2) = 0$, we could see that the left side would be zero if x is either of -5 or 2, so those are the solutions to the equation. This method will be used in the following example:

◇ **Example 1.1(e):** Solve $3x^3 + 36x = 21x^2$. Another Example

Solution:

$3x^3 + 36x$	$=$	$21x^2$	the original equation
$3x^3 - 21x^2 + 36x$	$=$	0	get all terms on one side and zero on the other
$3x(x^2 - 7x + 12)$	$=$	0	factor out the common factor of $3x$
$3x(x - 3)(x - 4)$	$=$	0	factor $x^2 - 7x + 12$
x	$=$	$0,\ 3,\ 4$	obtain solutions by inspection

We should make one observation at this point. In each of the examples we have seen so far, the number of solutions to each equation is the degree of the equation. This is not always the case, but it is "close." The facts are these:

> The number of solutions to a polynomial equation is the degree of the equation or less. A linear equation has one solution or no solution.

Consider now the equation $x^2 = 25$. From what we just read we would expect this equation to perhaps have two solutions. We might think that we can just square root both sides of the equation to get the solution $x = 5$, but then we have one less solution than we might expect. Moreover, we can see that $(-5)^2 = 25$, so -5 is also a solution, giving us the two solutions that we expect. The moral of this story is that we *can* take the square root of both sides of an equation if we then consider both the positive and negative of one side, usually the side with only a number. Here are two examples:

◇ **Example 1.1(f):** Solve $9x^2 = 16$.

Solution:

$9x^2 = 16$	the original equation
$x^2 = \frac{16}{9}$	divide both sides by 9 to get x^2 alone on one side
$x = \pm\sqrt{\frac{16}{9}}$	take the square root of both sides, including both the positive and negative square root on the right side
$x = \pm\frac{4}{3}$	evaluate the square root

◇ **Example 1.1(g):** Solve $3x^2 = 60$.

Solution:

$3x^2 = 60$	the original equation
$x^2 = 20$	divide both sides by 3 to get x^2 alone on one side
$x = \pm\sqrt{20}$	take the square root of both sides, including both the positive and negative square root on the right side
$x = \pm 2\sqrt{5}$ or $x = \pm 4.47$	simplify the square root and give the decimal form

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Of course we should be able to solve at least the equation from Example 1.1(f) by getting zero on one side and factoring, our standard approach to solving polynomial equations:

◇ **Example 1.1(h):** Solve $9x^2 = 16$.

Solution:

$9x^2$	$=$	16	the original equation
$9x^2 - 16$	$=$	0	get all terms on one side and zero on the other
$(3x+4)(3x-4)$	$=$	0	factor the difference of squares
$3x+4=0$	or	$3x-4=0$	set each factor equal to zero to get linear equations
$x = -\frac{4}{3}$	or	$x = \frac{4}{3}$	solve linear equations

Sometimes quadratic equations can't be solved by factoring. In those cases we use the quadratic formula.

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

◇ **Example 1.1(i):** Use the quadratic formula to solve $x^2 - 10x + 13 = 0$. Give your answers as two separate numbers, in simplified square root form and in decimal form, rounded to the nearest hundredth. Another Example

Solution: We first determine the values of a, b and c: $a = 1$, $b = -10$, $c = 13$. We then substitute them into the quadratic formula and evaluate carefully as follows:

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(13)}}{2(1)} = \frac{10 \pm \sqrt{100-52}}{2} = \frac{10 \pm \sqrt{48}}{2}$$

At this point we simplify $\sqrt{48}$ to $4\sqrt{3}$, Then either factor a common factor of two out of the top and bottom and reduce, or split the fraction in two and reduce each part:

$$x = \frac{10 \pm 4\sqrt{3}}{2} = \frac{2(5 \pm 2\sqrt{3})}{2} = 5 \pm 2\sqrt{3}$$

or

$$x = \frac{10 \pm 4\sqrt{3}}{2} = \frac{10}{2} \pm \frac{4\sqrt{3}}{2} = 5 \pm 2\sqrt{3}$$

Decimal solutions are best computed from the form $\dfrac{10 \pm \sqrt{48}}{2}$, giving $x = 1.54,\ 8.46$.

Let's now look at an application of quadratic equations and the quadratic formula. For most of the remainder of this course we will be seeing the following situation: We will have, or create, an equation containing two related unknown quantities. We will be given a value for one of the quantities, and we will need to evaluate an expression or solve an equation to find the corresponding value of the other unknown quantity.

Projectile Motion

Suppose that an object is projected straight upward from a height of h_0 feet, with an initial velocity of v_0 feet per second. Its height (in feet) at any time t seconds after it is projected (until it hits the ground) is

$$h = -16t^2 + v_0t + h_0$$

There is some terminology associated with the above formula that many of you will hear again. The quantities h_0 and v_0 are called **parameters**, which means that they can change from situation to situation, but for a given situation they remain constant. (The -16 is also a parameter, but it would only change if we went to a different planet!) The quantities h and t are **variables**; they vary as the situation we are studying "unfolds." They are related to each other in the way described mathematically by the given equation. Let's look at a couple examples using this formula.

◇ **Example 1.1(j):** A ball is thrown upward from a height of 5 feet, with an initial velocity of 60 feet per second. How high is the ball after 1.5 seconds?

Solution: Here the parameters h_0 and v_0 have values 5 and 60, respectively, so the equation becomes $h = -16t^2 + 60t + 5$. Now we substitute 1.5 in for t and compute the corresponding value for h:

$$h = -16(1.5)^2 + 60(1.5) + 5 = 59$$

The height of the ball after 1.5 seconds is 59 feet.

◇ **Example 1.1(k):** For the same ball, at what time or times is the ball 50 feet above the ground? Round the answers to the nearest hundredth of a second.

Solution: In this case we can see that we are given h and asked to find t, so we must solve the equation $50 = -16t^2 + 60t + 5$. Subtracting 50 from both sides we get $-16t^2 + 60t - 45 = 0$. We can then solve with the quadratic formula, to get $t = 1.04$ seconds and $t = 2.71$ seconds. There are two times because the rock is at a height of 50 feet on the way up, and again on the way down!

Absolute Value Equations

Most of you are probably familiar with the concept of absolute value, but let's do a brief review. First, when we see something like $-a$, this means "the opposite of a." If a is positive, then $-a$ is negative; however, if a is negative, $-a$ is then positive. The upshot of all this is that something like $-x$ does not necessarily represent a negative number.

The **absolute value** of a number x is just x itself if x is zero or positive. If x is negative, then the absolute value of x is $-x$, a positive number. We use the notation $|x|$ to represent the absolute value of x. All the words just given can be summarized concisely using mathematical notation:

Absolute Value

Let $|x|$ represent the absolute value of a number x. Then $|x| = x$ if $x \geq 0$, and $|x| = -x$ if $x < 0$.

◇ **Example 1.1(l):** Find $|4.2|$ and $|-37|$.

Solution: $|4.2| = 4.2$ and $|-37| = -(-37) = 37$

◇ **Example 1.1(m):** Find all values of x for which $|x| = 5$.

Solution: $x = \pm 5$, since both $|5| = 5$ and $|-5| = 5$.

The only absolute value equations that we will solve are *linear* absolute value equations, which basically means they are linear equations with some absolute values in them. Suppose that we have an equation of the form $|\,stuff\,| = 7$. Based on the last example, that means that *stuff* has to be either 7 or -7. That is the key to how we solve absolute value equations.

◇ **Example 1.1(n):** Solve $|3x - 5| = 7$. Another Example

Solution:

$\lvert 3x-5\rvert$	$=$	7	the original equation
$3x - 5 = -7$	or	$3x - 5 = 7$	write as two separate equations
$3x = -2$	or	$3x = 12$	add 5 to both sides of each equation
$x = -\frac{2}{3}$	or	$x = 4$	divide both sides of each equation by 3

Section 1.1 Exercises

To Solutions

1. Solve each of the following linear equations.

   (a) $4x + 7 = 5$

   (b) $2x - 21 = -4x + 39$

   (c) $\dfrac{9}{5}x - 1 = 2x$

   (d) $7(x - 2) = x + 2(x + 3)$

   (e) $8x = 10 - 3x$

   (f) $\dfrac{x}{4} + \dfrac{1}{2} = 1 - \dfrac{x}{8}$

   (g) $\dfrac{2}{3}x + \dfrac{1}{2} = \dfrac{3}{2}x - \dfrac{1}{6}$

   (h) $3(x - 5) - 2(x + 7) = 3x + 1$

2. Solve each polynomial equation.

(a) $x^2 - 13x + 12 = 0$ (b) $x^3 + 3x^2 + 2x = 0$ (c) $5x^2 + x = 0$

(d) $\frac{2}{3}x^2 + \frac{7}{3}x = 5$ (e) $3a^2 + 24a = -45$ (f) $8x^2 = 16x$

(g) $3x^2 - 48 = 0$ (h) $\frac{1}{15}x^2 = \frac{1}{6}x + \frac{1}{10}$ (i) $x^3 = 9x^2 + 22x$

3. For each of the following, use the quadratic formula to solve the given equation. Give your answers in both simplified square root form and decimal form rounded to the nearest tenth.

(a) $x^2 - 2x - 4 = 0$ (b) $x^2 = 2x + 17$

(c) $2x^2 = 2x + 1$ (d) $25x^2 + 10x = 6$

4. Solve each of the following absolute value equations.

(a) $|3x + 4| = 2$ (b) $|7x - 1| = 3$ (c) $|x^2 - 13| = 3$

5. Solve the equation $2x^2 + x = 3$ from Example 1.1(d) using the quadratic formula. This shows that even if a quadratic equation can solved by factoring, the quadratic formula will give the solutions as well.

6. A baseball is hit upward from a height of four feet and with an initial velocity of 96 feet per second.

(a) When is it at a height of 84 feet? Note that 16 goes into 80 and 96 evenly.

(b) When is it at a height of 148 feet? Note that 16 goes into 96 and 144 evenly.

(c) When is it at a height of 52 feet? Use your calculator and the quadratic formula, and round your answer(s) to the nearest hundredth of a second. (The hundredth's place is two places past the decimal.)

(d) When is the ball at a height of 200 feet, to the nearest hundredth of a second?

(e) When does the ball hit the ground, to the nearest hundredth of a second?

7. A company that manufactures ink cartridges for printers knows that the number x of cartridges that it can sell each week is related to the price per cartridge p by the equation $x = 1200 - 100p$. The weekly revenue (money they bring in) is the price times the number of cartridges: $R = p(1200 - 100p)$. What price should they set for the cartridges if they want the weekly revenue to be \$3200?

8. (a) Use the strategy of getting all terms on one side of the equation and factoring to solve $9x^2 = 12x$.

   (b) It would seem reasonable to try to solve the same equation by first dividing both sides by $3x$. What happens if you do that? What went wrong?

9. Students often think that when they are solving a polynomial equation with zero on one side and they can factor out a common factor, zero is one of the solutions. Give an example where this *IS* the case, and another where it *IS NOT*.

10. Solve the three equations $4x^2 = 25x$, $4x^2 = 25$, and $4x^3 = 25x^2$.

11. Consider the equation $(x-1)(x^2+5x+6) = 0$.

   (a) If you were to multiply the left side out (don't do it!) you would get a polynomial. What would its degree be? How many solutions do you expect to find for this equation?

   (b) The left side of this equation contains the product of two factors A and B. What are they?

   (c) What value or values cause A to be zero?

   (d) What value or values cause B to be zero? (You'll have to work a little harder here!)

   (e) What are the solutions to the original equation? Does this agree with what you got in (a)?

12. Solve $(x-2)(x^2-8x+7) = 0$.

13. Solve each of the following equations. *Note that each is already factored!*

   (a) $(x+1)(x-3)(x-2) = 0$

   (b) $(x-3)(x-4)(x+1)(x-1) = 0$

   (c) $(x+2)^2(x-5) = 0$

14. Solve $|2x-3| = -7$. (Trick question! Explain.)

15. When solving an equation, if we arrive at $(x+3)(x-1) = 0$ we conclude that $x+3 = 0$ or $x-1 = 0$, so $x = -3, 1$.

   (a) A student solving a *different* equation correctly arrives at $(x+3)(x-1) = 12$. They then write $x+3 = 12$ and $x-1 = 12$ and, from those, reach the conclusion that $x = 9, 13$. Convince them that they are wrong, *without solving the equation*.

   (b) Solve the equation $(x+3)(x-1) = 12$, and verify that your answers are correct.

16. Use the quadratic formula to solve each of the following:

   (a) The equation from Example 1.1(g) - check your answer with the example.

(b) The equation $3x^2 = 5x$ - check your answer by also solving by factoring.

17. The yellow box after Example 1.1(e) tells us that a quadratic equation can have two, one or zero solutions. You have seen a number of quadratic equations with two solutions - here you will see equations with one and zero solutions.

    (a) Solve the equation $x^2 + 9 = 6x$. Check your answer by substituting it into the equation.

    (b) Explain why $x^2 + 9 = 0$ doesn't have a solution.

18. Solve the equation $-\frac{1}{4}x^4 + 2x^2 - 4 = 0$ by the following method:

    (a) Factor $-\frac{1}{4}$ out of the left side. (As usual, though, *don't get rid of it!*) Multiply it back in mentally to make sure you factored correctly.

    (b) Factor the remaining part in pretty much the same way as you would if it were $x^2 + bx + c$, except the powers will be a little different. "FOIL" what you get back together to make sure it is correct.

    (c) Now you should be able to do more factoring to finish solving.

19. Solve each equation. For those that require the quadratic formula, give your answers in both exact and decimal form, rounded to the nearest tenth.

    (a) $4x - 3(5x + 2) = 4(x + 3)$

    (b) $x^2 - 19 = 6x$

    (c) $\frac{2}{3}x - 1 = \frac{3}{4}x + \frac{1}{2}$

    (d) $3|x - 1| + 2 = 4$

    (e) $9x^2 - 12x - 1 = 0$

    (f) $x^3 - 7x^2 + 10x = 0$

    (g) $|x + 2| - 5 = 0$

    (h) $11x + 9 = 3x + 11$

    (i) $6x^2 + 13x = 5$

    (j) $x^2 + 8x + 13 = 0$

    (k) $\frac{2}{15}x^2 + \frac{1}{3}x = \frac{1}{5}$

    (l) $21 + 4x = x^2$

1.2 Solving Equations in One Unknown, Part II

Performance Criterion:

1. (b) Solve an equation containing rational or radical expressions.

Equations With Rational Expressions

A **rational expression** is a fraction whose numerator and denominator are both polynomial expressions. Some examples are

$$\frac{24}{x+5} \qquad \text{and} \qquad \frac{x^2-25}{x^2-7x+12}.$$

The general strategy for solving an equation with rational expressions in it is this:

1) Factor the denominators of all rational expressions.
2) Multiply both sides of the equation by *just enough* of the factors of the denominators to eliminate all fractions.
3) The result after (2) will be a linear or polynomial equation. Solve it.
4) Check to see whether any of the solutions that you obtained causes a denominator to be zero in the original equation; if it does, eliminate it.

◇ **Example 1.2(a):** Solve $7 = \dfrac{3x-2}{2x-5}$.

Solution: If we multiply both sides by $2x-5$ the denominator on the right will be eliminated. At that point we will have a linear equation, which we already know how to solve.

$$7 = \frac{3x-2}{2x-5} \qquad \text{the original equation}$$

$$\frac{2x-5}{1}(7) = \frac{2x-5}{1}\left(\frac{3x-2}{2x-5}\right) \qquad \text{multiply both sides by } \frac{2x-5}{1}$$

$$14x-35 = 3x-2 \qquad \text{distribute the } 7 \text{ on the left side, cancel the } 2x-5 \text{ on the right}$$

$$11x = 33 \qquad \text{get } x \text{ terms on the left, number terms on the right}$$

$$x = 3 \qquad \text{finish solving}$$

At this point we check to see that $x=3$ does not cause the denominator $2x-5$ of the original equation to be zero. Since it doesn't, the solution is $x=3$ (providing there are no errors in the computations).

⋄ **Example 1.2(b):** Solve $\dfrac{1}{x+1}+\dfrac{11}{x^2-x-2}=\dfrac{3}{x-2}$.

Solution: Here we first factor the denominator x^2-x-2. At that point we can see that if we multiply both sides by $(x+1)(x-2)$ we will eliminate all fractions.

$$\begin{aligned}
\frac{1}{x+1}+\frac{11}{x^2-x-2} &= \frac{3}{x-2}\\
\frac{1}{x+1}+\frac{11}{(x+1)(x-2)} &= \frac{3}{x-2}\\
\frac{(x+1)(x-2)}{1}\left(\frac{1}{x+1}+\frac{11}{(x+1)(x-2)}\right) &= \frac{(x+1)(x-2)}{1}\left(\frac{3}{x-2}\right)\\
(x-2)+11 &= 3(x+1)\\
x+9 &= 3x+3\\
6 &= 2x\\
3 &= x
\end{aligned}$$

At this point we check to see that $x=3$ does not cause any of the denominators of the original equation to be zero. Since it doesn't, the solution is $x=3$ (providing there are no errors in the computations).

In the next example we will see a situation where two solutions are obtained and one of them is not valid.

⋄ **Example 1.2(c):** Solve $\dfrac{a+4}{a^2+5a}=\dfrac{-2}{a^2-25}$.

Solution: For this example we begin by factoring the denominators of the original equation $\dfrac{a+4}{a^2+5a}=\dfrac{-2}{a^2-25}$ to get $\dfrac{a+4}{a(a+5)}=\dfrac{-2}{(a+5)(a-5)}$. Now we can see that if we multiply both sides by $a(a+5)(a-5)$ we will eliminate the denominators:

$$\begin{aligned}
\frac{a(a+5)(a-5)}{1}\left(\frac{a+4}{a(a+5)}\right) &= \frac{a(a+5)(a-5)}{1}\left(\frac{-2}{(a+5)(a-5)}\right)\\
(a-5)(a+4) &= -2a
\end{aligned}$$

Finally we multiply out the left side and solve the resulting quadratic equation:

$$\begin{aligned}
a^2-a-20 &= -2a\\
a^2+a-20 &= 0\\
(a+5)(a-4) &= 0
\end{aligned}$$

Here we can see that $a=-5$ and $a=4$ are solutions to the last equation. Looking back at the original equation, however, shows that a cannot be -5 because that value causes the denominators of both sides to be zero. Thus the only solution is $a=4$.

Equations With Radical Expressions

A **radical expression** is simply a root with an unknown under it. We will occasionally wish to solve equations containing such expressions. The procedure for solving an equation with just one radical expression in it goes like this:

1) Add or subtract on each side to get the root alone on one side.

2) Square both sides of the equation. Note that *you must square the entire side*, not the individual terms.

3) The result of the previous step will be a linear or quadratic equation. Solve it.

4) *You must check all solutions in the original equation* for this type of equation. The process of squaring both sides of the equation sometimes introduces solutions that are not valid.

◇ **Example 1.2(d):** Solve $\sqrt{3x+7} = 4$.

Solution: Here we begin by simply squaring both sides of the original equation, then just solve the resulting equation:

$\sqrt{3x+7}$	$=$	4	the original equation
$(\sqrt{3x+7})^2$	$=$	4^2	square both sides
$3x+7$	$=$	16	squaring eliminates the square root
$3x$	$=$	9	subtract 7 from both sides
x	$=$	3	divide both sides by 3

Checking, we see that $\sqrt{3(3)+7} = \sqrt{16} = 4$, so $x = 3$ is in fact the solution to the original equation.

◇ **Example 1.2(e):** Solve $t = \sqrt{t+7} - 5$. Another Example - stop at 5:15

Solution: In this case we need to add 5 to both sides of the original equation to get the square root alone on one side, *THEN* square both sides:

t	$=$	$\sqrt{t+7} - 5$	the original equation
$t+5$	$=$	$\sqrt{t+7}$	add 5 to both sides
$(t+5)^2$	$=$	$(\sqrt{t+7})^2$	square both sides
$(t+5)(t+5)$	$=$	$t+7$	squaring the left side means
$t^2+10t+25$	$=$	$t+7$	multiplying by itself
$t^2+9t+18$	$=$	0	get all terms on one side
$(t+3)(t+6)$	$=$	0	factor
$t=-3$		$t=-6$	finish solving

Let's check both of these solutions in the original equation, using $\stackrel{?}{=}$ in place of $=$ until we are certain whether the statements are true:

$$\begin{aligned} -3 &\stackrel{?}{=} \sqrt{(-3)+7}-5 \\ -3 &\stackrel{?}{=} \sqrt{4}-5 \\ -3 &= -3 \end{aligned} \qquad \begin{aligned} -6 &\stackrel{?}{=} \sqrt{(-6)+7}-5 \\ -6 &\stackrel{?}{=} \sqrt{1}-5 \\ -6 &\neq -4 \end{aligned}$$

We can see that $t=-3$ is a solution but $t=-6$ is not.

What happens when we square both sides of an equation is that any solutions to the original equation are still solutions to the new equation, but the new equation will often have additional solutions. A simple example is the equation $x=5$, whose only solution is obviously $x=5$. When we square both sides we get the equation $x^2=25$. Note that $x=5$ is still a solution to this new equation, but $x=-5$ is as well. However $x=-5$ is not a solution to the original equation $x=5$.

General Strategies For Solving Equations

Here is the summary of equation-solving techniques that was promised earlier:

Techniques For Solving Equations

- If there are grouping symbols in the equation, it is *usually* beneficial to distribute in order to eliminate them.
- If the equation contains fractions, multiply both sides by the common denominator to eliminate all fractions.
- If the equation contains roots, get one root alone on one side and apply the corresponding power to both sides of the equation. Repeat if necessary.
- If the equation contains only the unknown, not to any powers, under roots or in denominators of fractions, it is a linear equation. Get all terms with x on one side, other terms on the other side, simplify and solve.
- If the equation contains various powers of x, get all terms on one side of the equation and zero on the other side and factor. Finish solving by inspection or by setting factors equal to zero and solving the resulting equations.
- If the equation is quadratic and you can't factor it, try the quadratic formula.

If the original equation contained x under roots or in the denominators of fractions, one generally needs to check all solutions, since invalid solutions can be created by those methods. If there are solutions, you will be guaranteed to find them, but you might also come up with solutions that are not really solutions.

Section 1.2 Exercises To Solutions

1. Solve each equation. Be sure to check all solutions to see if they are valid.

(a) $\dfrac{x+4}{2x}+\dfrac{x+20}{3x}=3$

(b) $\dfrac{x+1}{x-3}=\dfrac{x+2}{x+5}$

(c) $1-\dfrac{4}{x+7}=\dfrac{5}{x+7}$

(d) $\dfrac{4x}{x+2}=4-\dfrac{2}{x-1}$

(e) $\dfrac{1}{x-1}+\dfrac{1}{x+1}=\dfrac{6}{x^2-1}$

(f) $\dfrac{2x-1}{x^2+2x-8}=\dfrac{1}{x-2}-\dfrac{2}{x+4}$

2. Solve each equation. Be sure to check all solutions to see if they are valid.

(a) $\sqrt{25x-4}=4$

(b) $\sqrt{2x+7}-6=-2$

(c) $\sqrt{3x+13}=x+3$

(d) $\sqrt{2x+11}-4=x$

(e) $\sqrt{3x+2}+7=5$

(f) $\sqrt{x-2}=x-2$

3. Solve each equation.

(a) $\sqrt{x-2}+3=5$

(b) $\dfrac{x+3}{x-5}=-2$

(c) $\dfrac{3(x-2)(x+1)}{(x-4)(x+4)}=0$

(d) $\sqrt{x+5}-1=0$

(e) $\dfrac{x^2-1}{x^2-3x-2}=3$

(f) $\sqrt{x+2}+4=0$

4. Later in the term we will see equations like

$$30=\frac{120}{1+200e^{-0.2t}}$$

that we'll want to solve for t. Using the techniques of this section and Section 1.1, solve the equation for $e^{-0.2t}$.

5. (a) Solve $\dfrac{3x-1}{x^2+2x-15}=0$.

(b) Solve $\dfrac{x^2-3x-4}{x^2+2x+1}=0$

(c) Note what you got after you cleared the fractions in each of the above. From this we can see that in order to solve $\dfrac{A}{B}=0$ we need simply to solve ..."

6. Solve each equation. When the quadratic equation is required, give both exact solutions and approximate (decimal) solutions rounded to the nearest tenth.

(a) $2x^2 = x + 10$

(b) $\dfrac{4}{x-3} - \dfrac{3}{x+3} = 1$

(c) $10x^3 = 29x^2 + 21x$

(d) $\dfrac{3x}{4} - \dfrac{5}{12} = \dfrac{5x}{6}$

(e) $|3x + 2| = 10$

(f) $x^2 + 10x + 13 = 0$

(g) $\dfrac{1}{5}x^2 = \dfrac{1}{6}x + \dfrac{1}{30}$

(h) $\sqrt{5x+9} = x - 1$

1.3 Solving Problems with Equations

Performance Criteria:

1. (c) Solve an applied problem by writing and solving an equation modeling the given situation.
   (d) Use a formula to solve a problem.
   (e) Solve a formula for a given unknown.

In this section and the next we will use equations to solve problems. (Specifically, what you have probably called "word problems.") Given such a problem, the object will be to write an equation, or use one that we already know, whose solution is the solution to the problem. (In some cases the solution to the equation will only be part of the solution to the problem.) Of course the most difficult part of this is finding the equation. If you are able to write an equation directly, I would welcome you to do so. However, if you have trouble writing the equation, I would suggest you use the following strategy:

- Get to know the situation by guessing a value for the thing you are supposed to find, then doing the appropriate arithmetic to check your guess. (If you are supposed to find several quantities, just guess the value of one of them.) *Do not spend a lot of time and effort trying to decide on your guess - it will probably not be the correct solution, but we don't need it to be!*

- Let x be the correct value that you are looking for. Do everything to x that you did to your guess in order to check it, but set the result equal to what it is really supposed to be. This will be the equation you are looking for!

- Solve the equation to get the value asked for. If asked for more than one value, find the others as well.

Let's look at some examples to see how this process works.

◇ **Example 1.3(a):** A company is billed \$645 for the services of an engineer and a technician for a job. The engineer is billed out at \$90 per hour, and the technician at \$60 per hour. The technician worked on the project for two hours more than the engineer did. How long did each work?

Solution: Let's suppose that the engineer worked on the job for 5 hours. Then the technician worked on the job for $5 + 2 = 7$ hours. (When using this method you should write out every computation so that you can see later what you did.) The amount billed for the engineer is her hourly rate times the number of hours she worked: $90(5) = 450$. Similarly, the amount billed for the the technician is $60(5 + 2) = 420$, and the total amount billed is

$$90(5) + 60(5 + 2) = 870 \tag{1}$$

dollars. Obviously our guess was too high!

Now we replace our guess of 5 hours for the amount the engineer worked with the amount she really did work on the project; since we don't know what it is we'll call it x. We then go back to (1) and replace 5 with x everywhere, and the total amount billed with the correct value, $\$645$. This gives us the equation

$$90x + 60(x + 2) = 645$$

Solving this for x gives us $x = 3.5$ hours, so the engineer worked on the project for 3.5 hours, and the technician worked on it for $3.5 + 2 = 5.5$ hours.

Answers to all word problems MUST include units!

We'll now discuss some specific mathematics for various kinds of problems that we will solve.

Ideas and Formulas From Geometry

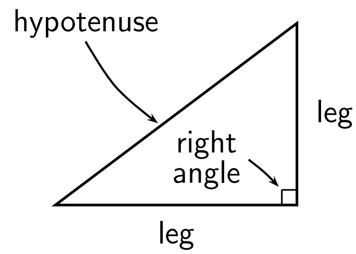
We will be working three geometric shapes: rectangles, triangles and circles. When we discus the amount of surface that a shape covers, we are talking about its **area**. Some standard sorts of practical uses of areas are for measuring the amount of carpet needed to cover a floor, or the amount of paint to paint a certain amount of surface. The distance around a shape is usually called its **perimeter**, except in the case of a circle. The distance around a circle is called its **circumference**.

When computing the circumference or area of a circle it is necessary to use a special number called **pi**. Pi is a little over 3, but in decimal form it never ends or repeats. We use the symbol π for it, and its "exact" value is

$$\pi = 3.141592654...$$

Sometimes we round this to 3.14, but to use it most accurately one should use more places past the decimal. We might not care to type in more, but we don't need to - pi is so important that it has its own key on our calculators! Find it on yours.

There is a special kind of triangle that comes up often in applications, called a **right triangle**. A right triangle is a triangle with one of its angles being a right angle ("square corner"). The side opposite the right angle is called the **hypotenuse** of the triangle, and the other two sides are called the **legs** of the triangle. There is a special relationship between the sides of the right triangle; this relationship is expressed by a famous "rule" called the **Pythagorean Theorem**.



A **formula** is an equation that describes the relationship between several (two or more) values. Here are some important geometry formulas:

Geometry Formulas

- For a rectangle with width w and length l, the perimeter is $P = 2w + 2l$ and the area is $A = lw$.
- For a circle with radius r, the circumference is $C = 2\pi r$ and the area is $A = \pi r^2$.
- **Pythagorean Theorem:** If a right triangle has legs with lengths a and b and hypotenuse of length c, then

$$a^2 + b^2 = c^2$$

Let's look at some examples that use these formulas.

◇ **Example 1.3(b):** One of the legs of a right triangle is 8 inches longer than the other leg and the hypotenuse is 16.8 inches long. How long are the legs, to the nearest tenth of an inch?

Solution: For this problem we actually know the equation, $a^2 + b^2 = c^2$, and we know that $c = 16.8$. If the shorter missing leg has a length of x inches, then the longer leg's length is $x + 8$ inches. We must then solve

$$\begin{aligned} x^2 + (x+8)^2 &= 16.8^2 \\ x^2 + x^2 + 16x + 64 &= 282.24 \\ 2x^2 + 16x - 218.24 &= 0 \end{aligned}$$

To solve this we use the quadratic formula:

$$x = \frac{-16 \pm \sqrt{16^2 - 4(2)(-218.24)}}{2(2)} = \frac{16 \pm \sqrt{2001.92}}{4} = -15.2,\ 7.2$$

Because x cannot be negative, the length of one leg is 7.2 inches and the length of the other is $7.2 + 8 = 15.2$ inches.

◇ **Example 1.3(c):** The longest side of a triangle is five centimeters more than the medium side, and the shortest side is half the medium side. The perimeter is $27\frac{1}{2}$ centimeters. Find the lengths of the sides of the triangle.

Solution: We are asked to find the lengths of all the sides of the triangle; let's pick a value for one of the sides and see what happens. Since both the short side and long side are described in terms of the medium side, let's guess the length of the medium side is 10 centimeters. The longest side is 5 centimeters more than that, or $10+5=15$ centimeters, and the shortest side is half the medium, or $\frac{1}{2}(10)=5$ centimeters. The perimeter is then the sum of the three sides:

$$\text{perimeter} = 10+(10+5)+\frac{1}{2}(10) = 10+15+5 = 30 \text{ centimeters}$$

Our guess was just a little high, but let's write an equation by replacing 10 everywhere in the above equation with x, and using the desired perimeter:

$$x+(x+5)+\frac{1}{2}x = 27\frac{1}{2}$$

If we first multiply both sides of this by 2 to eliminate the fractions we get

$$2x+2(x+5)+x = 55,$$

which is easily solved to get $x=9$ centimeters, the length of the middle side. The length of the longer side is then $9+5=14$ centimeters, and the length of the shorter side is $\frac{1}{2}(9)=4\frac{1}{2}$ centimeters.

Percents

Many quantities of interest are obtained by taking a certain percentage of other quantities. For example, some salespeople earn a certain percentage of their sales. (In this case what they earn is called their **commission**.) Various taxes are computes by a percentage of the item bought (or owned, in the case of property tax).

Percent means "out of one hundred," so seven percent means seven out of 100. If a person bought something for \$200 and they had to pay 7% sales tax, they would pay seven dollars for every \$100, or \$14. When doing math with a percent, it is necessary to change a percent to its decimal form. Since seven percent means seven out of one hundred,

$$7\% = \frac{7}{100} = 0.07$$

This last value is called the **decimal form of the percent**. Note that the decimal in $7=7.0$ has been moved two places to the left.

◇ **Example 1.3(d):** Change 4.25% to decimal form.

Solution: Moving the decimal two places to the left gives us $4.5\% = 0.045$.

Working With Percents

To find p percent of an amount A, change p to decimal form and multiply it times A.

◇ **Example 1.3(e):** The standard tip for waiters and waitresses is 15-20% of the cost of the meal. If you and some friends go out to eat and the total bill is $87.50, how much of a tip should you give your waitress if you wish to give 15%?

Solution: To find 15% of $87.50 we change the percentage to decimal form and multiply it by the amount: $0.15(87.50) = 13.13$. A 15% tip on $87.50 tip would be $13.13.

Note that answers to questions about money *should always be rounded to the nearest cent*, unless told otherwise.

◇ **Example 1.3(f):** A salesperson in a jewelry store gets a *commission* of 15% on all their sales. If their total in commissions for one month is $2602.43, what were her total sales for the month?

Solution: This Example is a little more challenging than the last one. Let's suppose that her sales were $10,000. Then, in the same manner as the last example, her commission would be $0.15(10,000) = 1500$. This is too low, but it allows us to see how to solve the problem. If we let her true sales be s and modify our computation, we arrive at the equation $0.15s = 2602.43$. Solving this, we see that her sales for the month were $17,349.53.

◇ **Example 1.3(g):** The sales tax in a particular city is 4.5%. You buy something there for $78.37, including tax. What was the price of the thing you bought?

Solution: We again begin with a guess. Suppose the price of the item was $70. Then the tax alone would be $0.045(70) = 3.15$ and the total cost would be $70+0.045(70) = 73.15$. Now, letting the actual price be p we get the equation $p + 0.045p = 78.37$. This is solved like this:

$$\begin{aligned} 1p + 0.045p &= 78.37 \\ 1.045p &= 78.37 \\ p = 75.00 \end{aligned}$$

The price of the item was $75.00.

This last example illustrates an important point. The actual value obtained when solving the equation was 74.99521531... When rounding to two place past the zero, the 9 in the hundredth's place changes to ten, making it a zero, which in turn causes the next two places to round up. You might then be tempted to give the answer as just $75, which is the correct amount. However, a person looking at this answer and not knowing how it was rounded might think that the value before rounding was between 74.95 and 75.04 when it was, in fact, far more accurate than that. Including the two zeros indicates that the price was $75 even when rounded to the nearest cent.

Later we will look at compound interest, which is how banks truly calculate interest. For now we'll use the following simpler formula for computing interest.

> **Simple Interest**
>
> Suppose that a **principal** of P dollars is invested or borrowed at an annual interest rate of r percent (in decimal form) for t years. The amount A that is then had or owed at the end of t years is found by
>
> $$A = P + Prt$$

◇ **Example 1.3(h):** You borrow $1000 at 8.5% interest for five years. How much money do you owe at the end of the five years?

Solution: Here we have $P = 1000$, $r = 0.085$ and $t = 5$, giving

$$A = 1000 + 1000(0.085)(5) = 1425$$

The amount owed after five years is $1425.00

◇ **Example 1.3(i):** You are going to invest $400 for 5 years, and you would like to have $500 at the end of the five years. What percentage rate would you need to have, to the nearest hundredth of a percent?

Solution: In this case we know that $P = 400$, $t = 5$ and $A = 500$. We substitute these values and solve for r:

$$\begin{aligned} 500 &= 400 + 400(5)r \\ 100 &= 2000r \\ 0.05 &= r \end{aligned}$$

You would need a rate of 5% to have $400 grow to $500 in five years.

Solving Formulas

Suppose that we wanted to use the simple interest formula $A = P + Prt$ to answer each of the following questions:

1) How long must $300 be invested for at 4.5% in order to have $500?

2) How long must $2000 be invested for at 7.25% in order to have $4000?

3) How long must \$800 be invested for at 5% in order to have \$1000?

To answer the first of these, we would substitute the known values and solve for t:

$$\begin{aligned} 500 &= 300 + 300(0.045)t \\ 200 &= 13.5t \\ 14.8 &= t \end{aligned}$$

It would take 14.8 years for \$300 to grow to \$500 at an interest rate of 4.5%. It should be clear that the other two questions would be answered in exactly the same way. In situations like this, where we might use the same formula over and over, always solving for the same thing, it is usually more efficient to solve the equation *BEFORE* substituting values into it.

◇ **Example 1.3(j):** Solve $A = P + Prt$ for t.

Solution:

$$\begin{aligned} A &= P + Prt \\ A - P &= Prt \\ \frac{A-P}{Pr} &= t \end{aligned}$$

We will call this "solving a formula"; other people might call it other things, but there is not real good name for it. Anyway, if you have trouble doing this you might consider trying the following procedure:

- *If the unknown you are solving for only occurs once in the formula:* Imagine what operations you would do, and in what order, if you knew its value and the values of all other variables and parameters. Then start "undoing" those operations one by one, *in the reverse order that you would do them if you had numbers for everything.*

- *If the unknown that you are trying to solve for occurs to the first power in more than one place:* Get all terms containing it on one side of the equation (and all other terms on the other side) and factor it out. Then divide by whatever it is multiplied by.

Consider again solving for t in the formula $A = P + Prt$. If we knew numbers for all values, on the right side we would multiply t by P and r, then add P. The above tells us that in solving for t we would first "undo" adding P by subtracting it from each side. Then we would undo multiplying by P and r by dividing both sides (the entire side in each case) by Pr. You can see those steps carried out in the example.

Now let's look at the situation addressed by the second bullet above.

◇ **Example 1.3(k):** Solve $ax - 8 = bx + c$ for x.

Solution: Here the unknown that we are solving for, x, occurs (to the first power) in two places. We'll solve for it by following the procedure from the second bullet above:

$ax - 8$	$= bx + c$	the original equation
$ax - bx$	$= c + 8$	get all terms with x on one side, terms without x on the other
$(a-b)x$	$= c + 8$	factor x out of the left side
x	$= \dfrac{c+8}{a-b}$	divide both sides by $a - b$

If the unknown to be solved for occurs in more than one place, and not all to the first power, then there will be no standard method for solving for it.

We will now look at an example of a computation that we will find valuable later.

◇ **Example 1.3(l):** Solve $-2x - 7y = 14$ for y. Give your answer in $y = mx + b$ form.

Solution:

$$\begin{aligned} -2x - 7y &= 14 \\ -7y &= 2x + 14 \\ y &= \frac{2x+14}{-7} \\ y &= \frac{2x}{-7} + \frac{14}{-7} \\ y &= -\tfrac{2}{7}x - 2 \end{aligned}$$

You probably recognize the result as the $y = mx + b$ form of the equation of a line.

Section 1.3 Exercises

To Solutions

Use an equation to solve each problem, whether it is one that has been given or one that you create yourself.

1. The hypotenuse of a right triangle has length 13 inches and one of the legs has length 12 inches. What is the length of the other leg?

2. The circumference of a circle is 16.4 inches. Find the radius of the circle, to the nearest tenth of an inch.

3. Sales tax in a particular city is 5.5%, and you buy an item with a *pre-tax* price of $19.95.
   (a) How much sales tax will you have to pay for the item?
   (b) How much will you have to pay for the item, including tax?
   (c) Suppose that, in the same city, you buy something else, and your total cost *with tax* is $87.48. What was the price of the item?

4. A salesperson in an art gallery gets a monthly salary of $1000 plus 3% of all sales over $50,000. How much do they make in a month that they sell $112,350 worth of art?

5. Suppose that the owner of the gallery from the previous exercise changes the pay structure to the following: A salesperson gets a monthly salary of $1000 plus 2% of all sales between $50,000 and $100,000, and 5% of all sales over $100,000.
   (a) What is the monthly pay for a salesperson in a month that they have $82,000 of sales?
   (b) What is the monthly pay for a salesperson in a month that they have $127,000 of sales?

6. You borrow some money at an interest rate of 8.5%, simple interest. After four years it costs you $1790.25 to pay off the loan. How much did you borrow?

7. The length of a rectangle is one more than three times the width, and the area is 520. Write an equation and use it to find the length and width of the rectangle.

8. Sales tax in a state is 5.5%. If the tax on an item was $10.42, what was the price of the item?

9. One leg of a right triangle is twice as long as the other leg, and the hypotenuse has a length of 15 feet. How long are the legs?

10. After a 10% raise your hourly wage is $7.37 per hour. What was it before the raise?

11. The length of a rectangle is three more than twice the width. The area is 44. Find the length and width.

12. The sales tax in a particular city is 6.5%. You pay $41.37 for an electric can opener, including tax. What was the price of the can opener?

13. The length of the shortest side of a triangle is half the length of the longest side. The middle side is 1.4 inches longer than the shortest side, and the perimeter is 43.4 inches. Find the lengths of all three sides.

14. A retailer adds 40% of her cost for an item to get the price she sells it at in her store. Find her cost for an item that she sells for $59.95.

15. The hypotenuse of a right triangle is two more than one of the legs, and the other leg has length 8. What are the lengths of the sides of the triangle?

16. (a) A rectangle has a width of 7 inches and a length of 10 inches. Find the area of the rectangle.
    (b) Suppose that the length and width of the rectangle from part (a) are each doubled. What is the new area?
    (c) When the length and width are doubled, does the area double as well?
    (d) Suppose that the length and width are each multiplied by three, then the area is computed. How many times bigger than the original area do you think the new area will be? Check your answer by computing the new area - were you correct?

17. You invest $1200 at 4.5% simple interest.
    (a) How much money will you have if you take it out after 4 years?
    (b) How many years, to the nearest tenth, would it take to "double your money?"

18. Solve the equation $A = P + Prt$ for r.

19. Solve $PV = nRT$ for R.

20. Solve $ax + 3 = bx - 5$ for b.

21. The formula $F = \frac{9}{5}C + 32$ is used for converting a Celsius temperature C to a Fahrenheit temperature F.

   (a) Convert 25° Celsius to Fahrenheit.

   (b) Even though it doesn't work quite as well, the formula can also be used to convert from Fahrenheit to Celsius. Convert 95° Fahrenheit to Celsius.

   (c) Solve the formula $F = \dfrac{9}{5}C + 32$ for C.

   (d) Use your result from part (c) to convert 77°F (this means 77 degrees Fahrenheit) to Celsius.

22. Solve $P = 2w + 2l$ for l.

23. Solve $C = 2\pi r$ for r.

24. Solve each equation for y. Give your answers in $y = mx + b$ form.

   (a) $3x + 4y = -8$ (b) $5x + 2y = -10$ (c) $3x - 2y = -5$

   (d) $3x - 4y = 8$ (e) $3x + 2y = 5$ (f) $-5x + 3y = 9$

25. Solve $8x + 3 = 5x - 7$ for x.

26. Solve each equation for x.

   (a) $ax + b = cx + d$ (b) $x + 1.065x = 8.99$

   (c) $ax + 3 = bx - 5$ (d) $ax + 7 = b(x + c)$

27. Solve $A = P + Prt$ for P.

28. Read Example 1.3(a) again, where the equation was determined to be

$$90x + 60(x + 2) = 645.$$

   For each of the following, write a brief sentence telling *in words* what the given part of the equation represents.

   (a) x (b) $90x$ (c) $x + 2$ (d) $60(x + 2)$ (e) $90x + 60(x + 2)$

1.4 Equations in Two Unknowns

Performance Criteria:

1. (f) Find solution pairs for an equation in two unknowns.

   (g) Graph the solution set of an equation in two unknowns without using a graphing calculator.

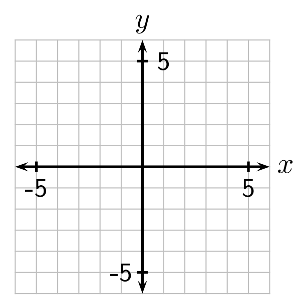
   (h) Identify x- and y-intercepts of an equation in two unknowns from its graph; find x- and y-intercepts of an equation in two unknowns algebraically.

When working with equations in one unknown, our goal has always been to *solve* them. This means we are looking for all values (called **solutions**) of the unknown that make the equation true. Consider the equation $2x+y=5$; how do we "solve" an equation like this? Well, it would seem reasonable to say that a **solution of an equation in two unknowns** is a *pair* of numbers, one for each of the unknowns, that makes the equation true. For the equation $2x+y=5$, the pair $x=2,\ y=1$ is a solution. So are the pairs $x=0,\ y=5$, $x=3,\ y=-1$ and $x=\frac{1}{2},\ y=4$. At this point you probably realize that there are infinitely many such pairs for this one equation! Since there is no hope of listing them all, we will instead draw a picture that describes all of them, to some extent.

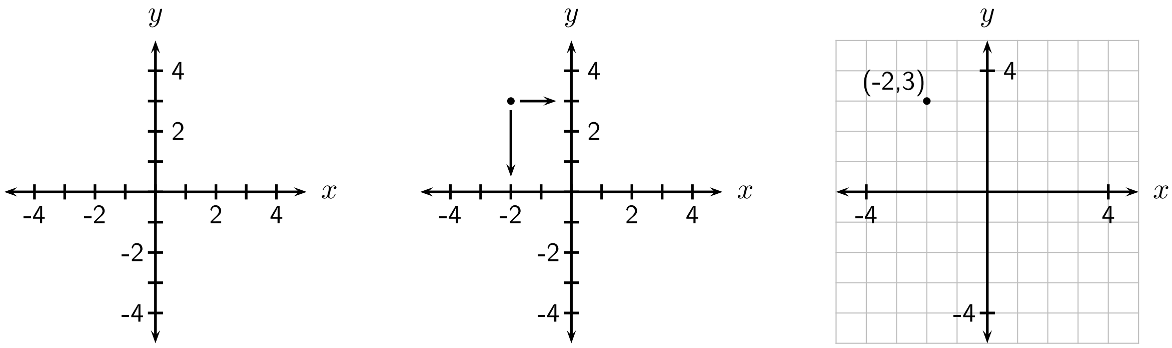
In order to write a bit less, we usually write $(2,1)$ instead of $x=2,\ y=1$. $(2,1)$ is called an **ordered pair**, and it is understood that the first value is for x and the second for y. Finding such pairs is easier if we can get x or y alone on one side of the equation. The equation $y=-2x+5$ is equivalent to $2x+y=5$, but instead of trying to think about what values of x and y make $2x+y=5$ true, we can substitute any value we wish for x into $y=-2x+5$ to find the corresponding value for y. We usually keep track of our results in a table like the one shown to the right, with which you are probably familiar. Note that when displaying solution pairs in such a table they are not necessarily shown in any particular order.

x	y
2	1
0	5
3	-1
$\frac{1}{2}$	4

Since there is no hope of listing all of the solutions, we instead draw a picture, or graph, that gives an indication of what all those pairs are. The basic idea is that every ordered pair of numbers corresponds to a point on a grid like the one shown to the right (with the understanding that the grid continues infinitely far in all directions). This grid is called the coordinate plane, or **Cartesian plane** after the mathematician and philosopher René Descartes. The darkened horizontal line is called the x-**axis** and the darkened vertical line is the y-**axis**.

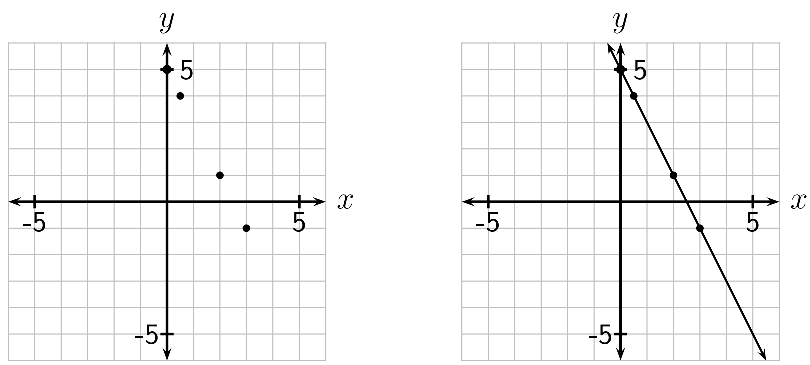


In some sense the coordinate plane is simply two number lines set at right angles to each other, as shown to the left at the top of the next page. Given a point in the plane, we travel to the x-axis in a direction parallel to the y-axis to get a number from the x-number line, then travel from the point to the y-axis in a direction parallel to the x-axis to get a second number. These two numbers are called the **coordinates** of the point. The point shown in the middle picture at the top of the next page has coordinates $x = -2$, $y = 3$ or, as an ordered pair, $(-2, 3)$. The last picture, on the right, shows how easy it is to determine the coordinates of a point when we cover the plane with a grid.



So for an equation containing two unknowns, like $2x + y = 5$, we find enough solution pairs that when we plot a point for each of them in the coordinate plane we can determine where all the others that we haven't found are. We then draw in a line or a curve, every point of which represents a solutions pair. In the previous section we found the solution pairs given in the table on the left below for the equation $y = -2x + 5$, which is equivalent to $2x + y = 5$. The corresponding points have been plotted on the grid in the middle, and we can see that they appear to lie on a line. That line is shown on the grid to the right below.

x	y
2	1
0	5
3	-1
$\frac{1}{2}$	4



Note the use of arrowheads to indicate that the line continues in both directions off the edges of the graph.

In this course we will usually graph solutions to equations having just y on the left side and an expression of x on the right side. Examples of such equations are

$$y = \tfrac{1}{2}x^2 - 2x - 1 \qquad\qquad y = \sqrt{x+3} \qquad\qquad y = |x+1| - 5$$

For now the process will be as follows:

1) Choose (sometimes carefully) values of x and substitute them into the right side of the equation to get corresponding values of y, recording the values in a table as shown above.

2) Plot each pair of values as a point on a coordinate grid. If any appear to not fit a pattern, check the computation of the 'y value.

3) Draw a line or curve through the points you have plotted. Extend it and add arrowheads when appropriate, to indicate that the graph extends beyond the grid.

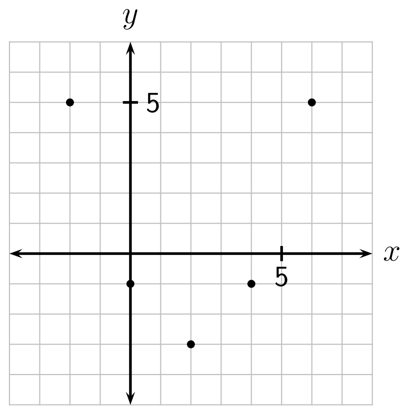
When we carry out this process we are technically graphing the solution set of the equation, but in a slight abuse of language we will say that we are "graphing the equation."

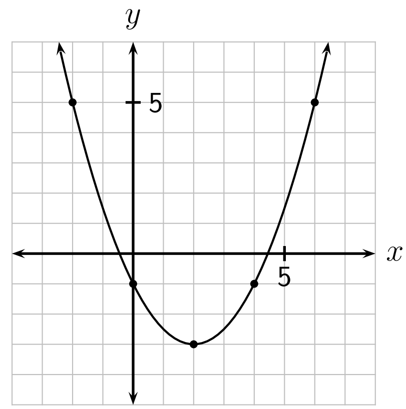
Let's see how this process is carried out for the first equation above.

◇ **Example 1.4(a):** Graph the equation $y = \frac{1}{2}x^2 - 2x - 1.$

Solution: We must choose some values of x for which to find the corresponding y values. A good first choice of x is $x = 0,$ resulting in $y = 1.$ Often we would choose $x = 1$ next, but we can see that results in a fraction for $y,$ so let's use $x = 2$ instead, resulting in $y = -3.$ Continuing like this results in the table of values below and to the left, which are plotted on the left grid below. On the right grid below we connect the dots and extend the curve and add arrowheads on the ends to indicate that the graph keeps going.

x	y
-2	5
0	-1
2	-3
4	-1
6	5

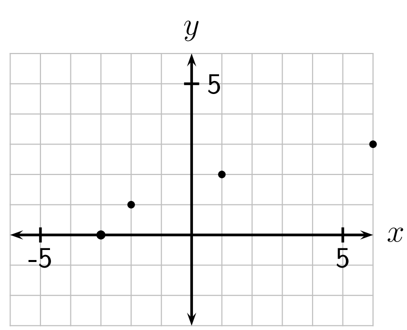


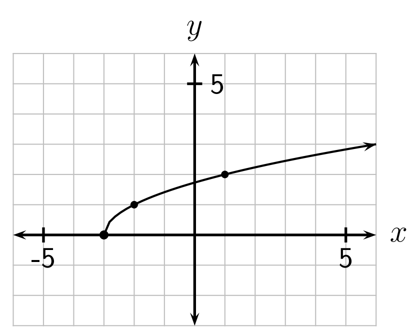


◇ **Example 1.4(b):** Graph the equation $y = \sqrt{x+3}.$

Solution: Here our y values will be "nicer" (that is, will be integers) if we choose values of x such that $x+3$ is a perfect square. Recalling that the perfect squares are $0, 1, 4, 9, ...,$ we can see that when $x = -3,$ $x + 3 = 0$ and we get $y = \sqrt{-3+3} = \sqrt{0} = 0.$ Similarly, $x = -2$ gives $y = \sqrt{-2+3} = \sqrt{1} = 1,$ $x = 1$ gives $y = \sqrt{1+3} = \sqrt{4} = 2,$ $x = 6$ gives $y = \sqrt{6+3} = \sqrt{9} = 3$ and $x = 13$ gives $y = \sqrt{13+3} = \sqrt{16} = 4.$ These values can be organized into the table shown to the left below, and the corresponding points are plotted on the left grid below. In this case the points appear to lie on half of a parabola, as plotted on the right grid. There are no points to the left of $-3, 0)$ because the square root is undefined for x values less than $-3,$ so there is no arrowhead on that end of the curve.

x	y
-3	0
-2	1
1	2
6	3



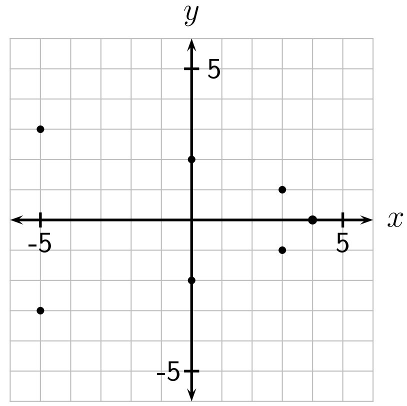


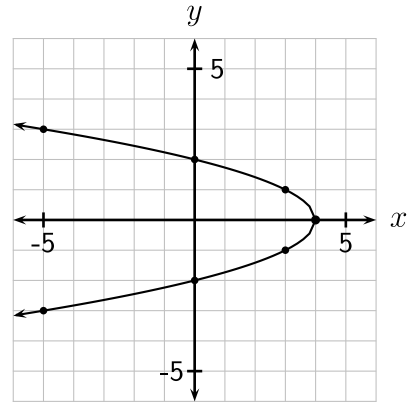
Both examples that we have looked at so far have had y alone on one side. We now look at an example with x alone on one side. You may feel some compulsion to solve for y, but *there is no need to, and if not done carefully it can lead to incorrect results!*

◇ **Example 1.4(c):** Graph the equation $x = 4 - y^2$.

Solution: In this case we should *choose values for* y *instead of* x, and we can see that when $y = 0$ we get $x = 4$. We should also note that we are squaring y and subtracting the result from five to get x, so we will get the same thing for a negative value as we do for the corresponding positive value. For example, when $y = 1$ we get $x = 4 - (1)^2 = 3$, and when $y = -1$, $x = 4 - (-1)^2 = 3$. Letting $y = 1, 2, 3$ and their negatives gives us the values in the table below and to the left. The corresponding points are plotted on the left grid below, and the solution curve is graphed on the right grid.

x	y
-5	-3
0	-2
3	-1
4	0
3	1
0	2
-5	3

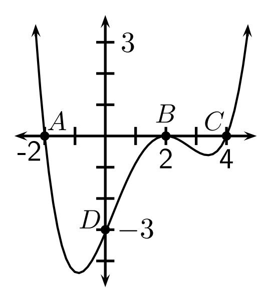




One goal we have is to be able to graph equations quickly,efficiently, and accurately. In later sections you will see that doing so requires first identifying what we expect a graph to look like, then plotting key points, rather than just randomly finding and plotting points.

x- and y-Intercepts

Consider the graph shown to the right, for some unknown equation. Points A, B and C are called x-**intercepts** of the graph or equation, and D is a y-**intercept**. *It is possible to have more than one intercept on each axis.* The points are called intercepts because they are where the graph of the equation intercepts the axes. When asked for the intercepts, we mean to give their coordinates, so in this case we would say that the x-intercepts are $(-2, 0)$, $(2, 0)$ and $(4, 0)$. The y-intercept is $(0, -3)$.



We should observe that the y-coordinate of every x-intercept is zero, and the x-coordinate of the y-intercept (and any other y-intercept, if there were more) is zero. This indicates the following.

Finding Intercepts

- To find the x-intercepts for an equation, let $y = 0$ and solve for x.
- To find the y-intercepts for an equation, let $x = 0$ and solve for y.
- We sometimes give the intercepts as single numbers rather than ordered pairs, because we know that the y-coordinates of x-intercepts must be zero, and vice-versa.

From the last bullet, we can give the x-intercepts for the graph on the previous page as -2, 2 and 4, and the y-intercept can be given as just -3. More often you will be asked to find the intercepts for a given equation, like in the next example.

◇ **Example 1.4(d):** Find the intercepts of $x = 4 - y^2$

Solution: To find the x-intercepts we set $y = 0$ and solve for x, giving $x = 4$. To find the y-intercepts we set $x = 0$ and solve, giving $y = 2$ or $y = -2$. Therefore the x-intercept of $x = 4 - y^2$ is 4 and the y-intercepts are 2 and -2.

Section 1.4 Exercises To Solutions

Check your answers to all graphing exercises by graphing the equation on a graphing calculator, or with an online grapher like *www.desmos.com*.

1. In this exercise you will graph the equation $y = x^2 - 2x - 2$; the graph will be a parabola similar to the one in Example 1.4(a).

   (a) Begin by letting $x = 0, 1, 2, ...$ and continuing finding the corresponding y values until you obtain values too large to fit on a reasonable sized graph. Put your values in a table.

   (b) Find y values corresponding to some negative x values until those y values are also too large to fit on a reasonable sized graph,adding these solution pairs to your table.

   (c) Plot your points. If any points look out of place, go back and check your calculation of the y. Once you see the points as being on a parabola, draw the parabola. Indicate that the ends keep going by putting arrowheads on them.

   (d) Check your answer as described above.

2. The graph of the equation $y = -x^2 - 4x$ is also a parabola. follow the same procedure as outlined in Exercise 1 to determine its graph.

3. Consider the equation $y = -\frac{3}{2}x + 3$.

   (a) Note that if x is an even number, $-\frac{3}{2}x$ will be an integer. Calculate y values for $x = -2,\ 0,\ 2,\ 4$ and put your results in a table.

   (b) Plot the points listed in your table. They should lie on a (straight) line - if they don't, check your calculations.

   (c) Draw in the line, again with arrowheads to indicate it keeps going. Check your answer, again using a graphing calculator or online grapher.

4. Consider the equation $y = \sqrt{5-x} - 1$

   (a) Note that $\sqrt{5-x}$ is undefined for $x = 10$, so we would not be able to find a y value corresponding to that value of x. What is the largest value that you can use for x? What is y in that case?

   (b) Choose three more values of x for which $5-x$ has the values of $1, 4$ and 9, and find the corresponding y values. Put those pairs, and the pair from (a), in a table.

   (c) The graph in this case should be *half of* a parabola. Plot your points, sketch the graph and check it.

5. You should know the graphs of the following equations, or be able to create them very quickly. For each, (*i*) make a table of solution pairs for $x = -2,\ -1,\ 0,\ 1,\ 2$, (*ii*) plot the corresponding points, (*iii*) sketch the graph and (*iv*) check your graph.

   (a) $y = x$ (b) $y = x^2$ (c) $y = |x|$ (d) $y = x^3$

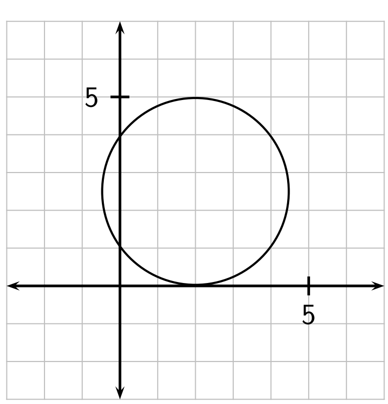
6. You should also know the graphs of $y = \sqrt{x}$ and $y = \sqrt[3]{x}$. Note that we cannot find square roots of negative numbers, but we *CAN* find cube (third) roots of negative numbers.

   (a) Remembering that zero and one are both perfect squares, make a table of x and y values for the first four perfect square values of x. Plot the first three pairs, keep in mind where the fourth pair would be, and graph the equation. Check your graph.

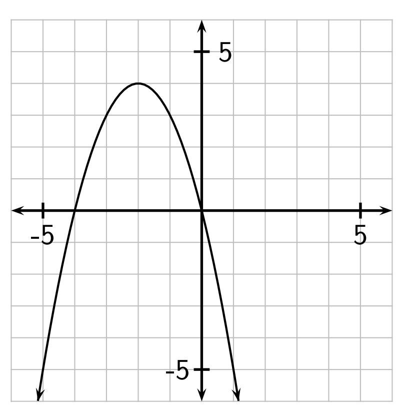
   (b) For the equation $y = \sqrt[3]{x}$, find y values for $x = -2,\ -1,\ 0,\ 1,\ 2$ and plot the resulting points. Sketch the graph and check it.

7. Give the x- and y-intercepts for each of the graphs below, indicating clearly which is which.

   (a)



   (b)



8. Give the x- and y-intercepts for the graphs in each of the following examples. You will have to approximate some of them - use decimals to the nearest tenth.

   (a) Example 1.4(c). (b) Example 1.4(a). (c) Example 1.4(b).

9. Give the intercepts of the line graphed on page 26.

10. Find the intercepts of $-5x + 3y = 15$, being sure to indicate which are x-intercepts and which are y-intercepts.

11. Find the intercepts of $y = x^2 - 2x - 3$.

12. This exercise will reinforce the fact that intercepts are not always integers! (Remember that integers are positive or negative whole numbers, or zero.) Find the intercepts of the equation $2x + 3y = 9$.

13. This exercise illustrates that the an x-intercept can also be a y-intercept.

    (a) Find the x-intercepts of $y = x^2 - 5x$. *Give your answers as ordered pairs.*

    (b) Find the y-intercepts of $y = x^2 - 5x$. *Give your answers as ordered pairs.*

14. Find the intercepts for each of the following equations algebraically.

    (a) $x = y^2 - 2y$ (b) $y = \sqrt{x+4}$ (c) $x = 2y - 3$

    (d) $y = (x+3)^2$ (e) $x^2 + y = 1$ (f) $y = x^3$

    (g) $3x + 5y = 30$ (h) $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ (i) $\sqrt{x+4} = y + 1$

    (j) $y = |x-3| + 1$ (k) $y = |x+2|$

15. Find the x- and y-intercepts of the following equations algebraically. Note that for (c) and (d) you are given two forms of the same equation. One will be easier to use to find x-intercepts and the other will be easier to use for finding y-intercepts. (**Hint for (c) and (d):** Recall the result of Exercise 5 in Section 1.2.)

    (a) $y = \frac{1}{3}(x+3)^2(x-1)(x-2)$

    (b) $y = -\frac{3}{5}(x-5)(x+2)(x+4)$

    (c) $y = \dfrac{2x^2 - 8}{x^2 - 2x - 3} = \dfrac{2(x+2)(x-2)}{(x-3)(x+1)}$

    (d) $y = \dfrac{3x}{x^2 - 2x + 1} = \dfrac{3x}{(x-1)^2}$

1.5 Equations in Two Unknowns: Lines

Performance Criterion:

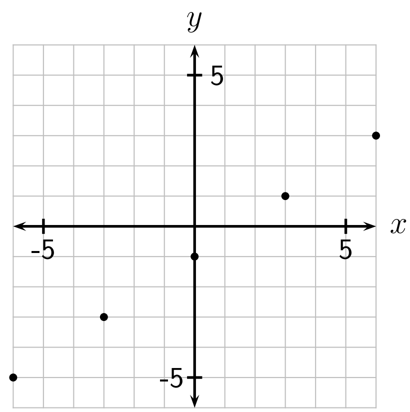
1. (i) Graph a line; determine the equation of a line.

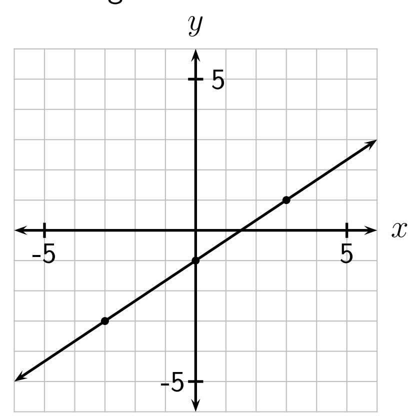
Let's begin with an example.

◇ **Example 1.5(a):** Graph the equation $2x - 3y = 3$.

Solution: If we solve the equation for y we obtain $y = \frac{2}{3}x - 1$. When $x = 0$ we get $y = -1$, and we can avoid fractional values of y if we let x be various multiples of three. The table below and to the left shows some solution pairs obtained in this way, and the corresponding points are plotted on the left graph below. We can then see that the graph of the equation is the line shown below and to the right.

x	y
-6	-5
-3	-3
0	-1
3	1
6	3





It should be clear that any equation of the form $Ax + By = C$ can be put in the form $y = mx + b$, as we did above, and the graph of such an equation is always a line. We will soon see that there is a much faster way to graph a line, that you may recall. To use it we need the idea of the slope of a line.

Slopes of Lines

The slope of a line is essentially its "steepness," expressed as a number. Here are some facts about slopes of lines that you may recall:

- The slope of a horizontal line is zero, and the slope of a vertical line is undefined. (Sometimes, instead of "undefined," we say a vertical line has "no slope," which is not the same the same as a slope of zero!)

- Lines sloping "uphill" from left to right have positive slopes. Lines sloping downhill from left to right have negative slopes.

- The steeper a line is, the larger the *absolute value* of its slope will be.

- To find the numerical value of the slope of a line we compute the "rise over the run." More precisely, this is the amount of vertical distance between two points on the line (*any* two points will do) divided by the horizontal distance between the same two points. If the two points are (x_1, y_1) and (x_2, y_2), the slope is given by the formula

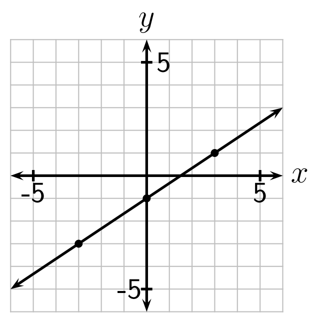
$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

  where m represents the slope of the line. It is standard in mathematics to use the letter m for slopes of lines.

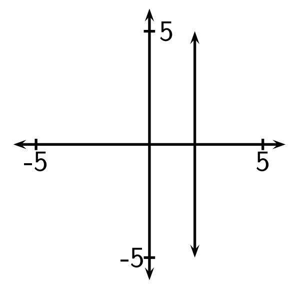
- Parallel lines have the same slope. Perpendicular lines have slopes that are negative reciprocals; for example, lines with slopes $m_1 = -\frac{2}{3}$ and $m_2 = \frac{3}{2}$ are perpendicular. (The subscripts here designate one slope from the other.)

Let's look at some examples involving the slopes of lines.

◇ **Example 1.5(b):** Give the slope of the lines with the graphs shown below.



Line 1



Line 2

Solution: For Line 1 we first note that it slopes uphill from left to right, so its slope is positive. Next we need to determine the rise and run between two points; consider the points $(-3, -3)$ and $(3, 1)$. Starting at $(-3, -3)$ and going straight up, we must go up four units to be level with the second point, so there is a rise of 4 units. We must then "run" six units to the right to get to $(3, 1)$, so the run is 6. We already determined that the slope is positive, so we have $m = \frac{\text{rise}}{\text{run}} = \frac{4}{6} = \frac{2}{3}$.

Suppose we pick any two points on Line 2. The rise will be some number that will vary depending on which two points we choose. However, the run will be zero, regardless of which points we choose. Thus when we try to find the rise over the run we will get a fraction with zero in its denominator. Therefore the slope of Line 2 is undefined.

◇ **Example 1.5(c):** Give the slope of the line through $(-3, -4)$ and $(6, 8)$.

Solution: Here we will let the first point given be (x_1, y_1), and the second point be (x_2, y_2). We then find the rise as $y_2 - y_1$ and the run as $x_2 - x_1$, and divide the rise by the run:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-4)}{6 - (-3)} = \frac{12}{9} = \frac{4}{3}.$$

◇ **Example 1.5(d):** The slope of Line 1 is $-\frac{3}{5}$. Line 2 is perpendicular to Line 1 and Line 3 is parallel to Line 1. What are the slopes of Line 2 and Line 3?

Solution: Because Line 2 is perpendicular to Line 1, its slope must be the negative reciprocal of $-\frac{3}{5}$, or $\frac{5}{3}$. Line 3 is parallel to Line 1, so it must have the same slope, $-\frac{3}{5}$.

◇ **Example 1.5(e):** The slope of Line 1 is 0. If Line 2 is perpendicular to Line 1, what is its slope?

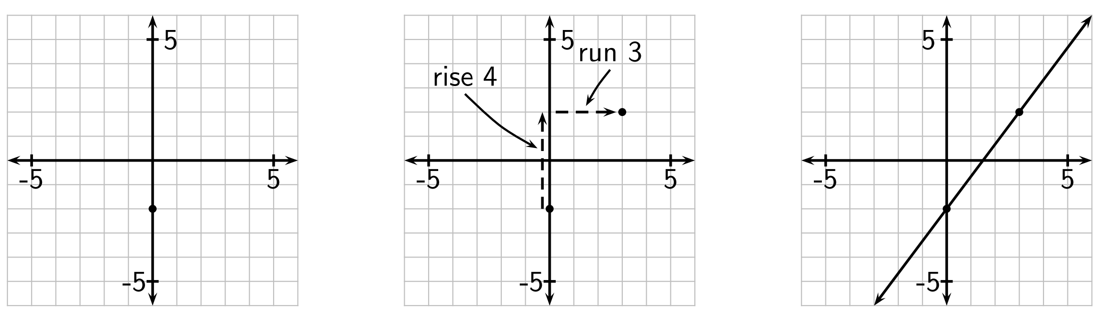
Solution: Because its slope is zero, Line 1 is horizontal. A perpendicular line is therefore vertical, so its slope is undefined.

Equations of Lines

Note that Line 1 from Example 1.5(b) is the graph of $y = \frac{2}{3}x - 1$ obtained in Example 1.5(a). This illustrates that the graph of any equation of the form $y = mx + b$ is a line with slope m. Furthermore, when $x = 0$ the equation $y = mx + b$ yields $y = b$, so b is the y-intercept of the line. This can easily be seen to be true for Example 1.5(a). Let's use these ideas to get another graph.

◇ **Example 1.5(f):** Graph the equation $4x - 3y = 6$.

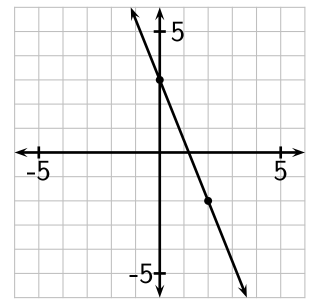
Solution: The equation is of the form $Ax + By = C$, so we know its graph will be a line. Solving for y, we obtain $y = \frac{4}{3}x - 2$. This tells us that the y-intercept is -2. We then plot that point,as shown on the grid to the left below. Next we note that the slope of the line is $\frac{4}{3}$, so we can obtain another point by *starting at* $(0, -2)$ and going up four units and right three units (we would have gone left if the slope was negative) to get a second point. This is shown on the middle grid below. We only need two points to determine the graph of a line, so we can now draw the line, as shown on the grid below and to the right.



We only need two points to determine the graph of a line, but we often get additional points by applying the slope again from the new point obtained, or "backward" from the y-intercept. Had we done that in the above example we would have obtained points located at the tips of the two arrowheads.

◇ **Example 1.5(g):** Give the equation of the line graphed below and to the right.

Solution: Here we begin by noting that the equation of any line can be written in the form $y = mx + b$, and the task essentially boils down to finding m and b. In our case b is the y-intercept 3. Next we use any two points on the line to find the slope of the line, noting that the slope is negative since the line slopes "downhill" from left to right. Using the y-intercept point and the point $(2, -2)$, we see that the vertical change is five units and the horizontal change is two units. Therefore the slope is $m = -\frac{5}{2}$ and the equation of the line is $y = -\frac{5}{2}x + 3$.



Students sometimes give their result from an exercise like the last example as $y = -\frac{5}{2} + 3$. This is the same as $y = \frac{4}{3}$, which *IS* the equation of a line (the horizontal line with y-intercept $\frac{4}{3}$), but it is not the equation of the line graphed above. Be sure to include the x from $y = mx + b$ in your answer to this type of question, or the two that follow.

◇ **Example 1.5(h):** Find the equation of the line through $(-4, 5)$ and $(2, 1)$ **algebraically**.

Solution: Again we want to use the "template" $y = mx + b$ for our line. In this case we first find the slope of the line:

$$m = \frac{5 - 1}{-4 - 2} = \frac{4}{-6} = -\frac{2}{3}.$$

At this point we know the equation looks like $y = -\frac{3}{2}x + b$, and we now need to find b. To do this we substitute the values of x and y from *either* ordered pair that we are given into $y = -\frac{2}{3}x + b$ (being careful to get each into the correct place!) and solve for b:

$$\begin{aligned} 1 &= -\tfrac{2}{3}(2) + b \\ \tfrac{3}{3} &= -\tfrac{4}{3} + b \\ \tfrac{7}{3} &= b \end{aligned}$$

We now have the full equation of the line: $y = -\frac{2}{3}x + \frac{7}{3}$.

◇ **Example 1.5(i):** Find the equation of the line containing $(1, -2)$ and perpendicular to the line with equation $5x + 4y = 12$.

Solution: In this case we wish to again find the m and b in the equation $y = mx + b$ of our line, but we only have one point on the line. We'll need to use the other given information, about a second line. First we solve its equation for y to get $y = -\frac{5}{4}x + 3$. Because our line is perpendicular to that line, the slope of our line is $m = \frac{4}{5}$ and its equation is $y = \frac{4}{5}x + b$. As in the previous situation, we then substitute in the given point on our line to find b:

$$\begin{aligned} -2 &= \tfrac{4}{5}(1) + b \\ -\tfrac{10}{5} &= \tfrac{4}{5} + b \\ -\tfrac{14}{5} &= b \end{aligned}$$

The equation of the line is then $y = \frac{4}{5}x - \frac{14}{5}$.

◇ **Example 1.5(j):** Find the equation of the line through $(2, -3)$ and $(2, 1)$.

Solution: This appears to be the same situation as Example 1.5(h), so let's proceed the same way by first finding the slope of our line:

$$m = \frac{-3 - 1}{2 - 2} = \frac{-4}{0},$$

which is undefined. Hmmm... what to do? Well, this means that our line is vertical. Every point on the line has a x-coordinate of 2, regardless of its y-coordinate, so the equation of the line is $x = 2$.

This last example points out an idea that we will use a fair amount later:

Equations of Vertical and Horizontal Lines

The equation of a horizontal line through the point (a, b) is $y = b$, and the equation of a horizontal line through the same point is $x = a$.

Section 1.5 Exercises

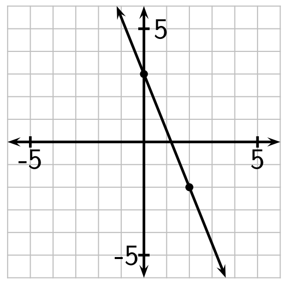
To Solutions

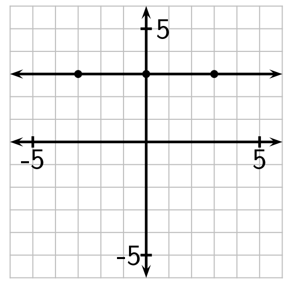
1. Find the slopes of the lines through the following pairs of points.

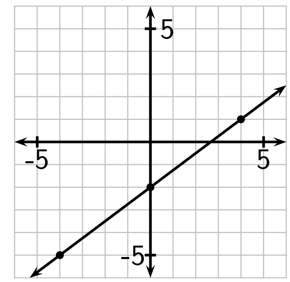
   (a) $(-4, 5)$ and $(2, 1)$ (b) $(-3, 7)$ and $(7, 11)$ (c) $(3, 2)$ and $(-5, 2)$

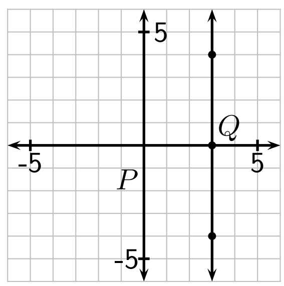
   (d) $(1, 7)$ and $(1, -1)$ (e) $(-4, -5)$ and $(-1, 4)$ (f) $(3, -1)$ and $(-3, 2)$

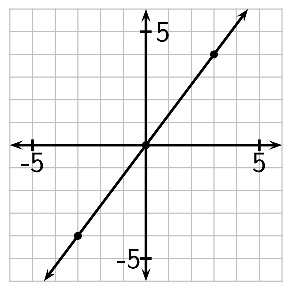
2. Find the slope of each line.

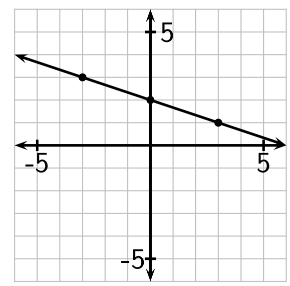
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

3. Give the equation of each line in the previous exercise.

4. Line 1 has slope $-\frac{1}{3}$, and Line 2 is perpendicular to Line 1. What is the slope of Line 2?

5. Line 1 has slope $\frac{1}{2}$. Line 2 is perpendicular to Line 1, and Line 3 is perpendicular to Line 2. What is the slope of Line 3?

6. Sketch the graph of each line.

   (a) $y = -\frac{1}{2}x + 3$ (b) $y = \frac{4}{5}x - 1$ (c) $3x - y = 4$

   (d) $x + y = 2$ (e) $3x + 5y = 10$

7. (a) Sketch the graph of $x = -2$. (b) Sketch the graph of $y = 4$.

8. Find the equation of the line through the two points **algebraically**.

   (a) $(3, 6),\ (12, 18)$ (b) $(-3, 9),\ (6, 3)$ (c) $(3, 2),\ (-5, 2)$

   (d) $(-8, -7),\ (-4, -6)$ (e) $(2, -1),\ (6, -4)$ (f) $(-2, 4),\ (1, 4)$

9. The equation $5x - 3y = 2$ is the equation of a line.

   (a) Give the slope of the line.

   (b) What is the slope of any line parallel to the line with the given equation?

   (c) What is the slope of any line perpendicular to the line with the given equation?

10. Are the two lines $3x - 4y = 7$ and $8x - 6y = 1$ perpendicular, parallel, or neither?

11. Find the equation of the line containing the point $P(2, 1)$ and perpendicular to the line with equation $2x + 3y = 9$. Give your answer in slope-intercept form $(y = mx + b)$.

12. Find the equation of a line with x-intercept -4 and y-intercept 3.

13. Find the equation of a horizontal line through the point $(2, -5)$.

14. Find the equation of the line through the point $(2, -3)$ and parallel to the line with equation $4x + 5y = 15$.

15. Find the equation of the line that contains the point $(3, 4)$ and is perpendicular to the line with equation $3x - 2y = 3$.

16. Find the equation of the line that is parallel to $y = -5x + 7$ and has x-intercept $x = 2$.

17. Find the equation of the line that is perpendicular to the line with equation $3x + 4y = -1$ and contains the point $(3, 2)$.

18. Determine, *without graphing*, whether the points $A(-2, 0)$, $B(2, 3)$ and $C(-6, -3)$ lie on the same line. Show work supporting your answer. (**Hint:** Find the slope of the line through points A and B, then through B and C. How should they compare if the points are all on the same line?)

19. Find the equation of the line through $(-5, 2)$ that is parallel to the line through $(-3, -1)$ and $(2, 6)$.

1.6 Systems of Two Linear Equations

Performance Criteria:

1. (j) Solve a system of two linear equations in two unknowns by the addition method; solve a system of two linear equations in two unknowns by the substitution method.

   (k) Solve a problem using a system of two linear equations in two unknowns.

The Addition Method

Consider the two equations

$$\begin{aligned} 2x - 4y &= 18 \\ 3x + 5y &= 5 \end{aligned}$$

Taken together, we call them a **system of equations**. In particular, this is a system of *linear* equations. From past experience it should be clear that each equation has infinitely many solution pairs. When faced with a system of linear equations, our goal will be to *solve the system*. This means find a value of x and a value of y which, when taken together, make *BOTH* equations true. In this case you can easily verify that the values $x = 5,\ y = -2$ make both equations true. We say that the ordered pair $(5, -2)$ is a solution to the system of equations; it turns out it is the *only* solution.

You might recall that there are two methods for solving systems of linear equations, the **addition method** and the **substitution method**. Here is a simple example of the addition method:

◇ **Example 1.6(a):** Solve the system $\begin{aligned} x - 3y &= 6 \\ -2x + 5y &= -5 \end{aligned}$ using the addition method.

Solution: The idea is to be able to add the two equations and get either the x terms or the y terms to go away. If we multiply the first equation by 2 and add it to the second equation the x terms will go away. We'll then solve for y:

$$\begin{aligned} x - 3y &= 6 \quad \stackrel{\text{times } 2}{\Longrightarrow} \quad & 2x - 6y &= 12 \\ -2x + 5y &= -5 \quad \Longrightarrow \quad & \underline{-2x + 5y} &\underline{= -5} \\ & & -y &= 7 \\ & & y &= -7 \end{aligned}$$

We then substitute this value of y into either of the original equations to get y:

$$\begin{aligned} x - 3(-7) &= 6 \\ x + 21 &= 6 \\ x &= -15 \end{aligned}$$

The solution to the system is then $x = -15,\ y = -7,$ or $(-15, -7)$.

In many cases we must multiply each of the original equations by a different value in order to eliminate one of the unknonwns when adding. This is demonstrated in the next example.

◇ **Example 1.6(b):** Solve the system $\begin{aligned} 2x - 4y &= 18 \\ 3x + 5y &= 5 \end{aligned}$ using the addition method.

Solution: Here if we multiply the first equation by 5 and the second equation by 4 the y terms will go away when we add them together. We'll then solve for x:

$$\begin{aligned} 2x - 4y = 18 &\overset{\text{times } 5}{\Longrightarrow} & 10x - 20y &= 90 \\ 3x + 5y = \ \ 5 &\overset{\text{times } 4}{\Longrightarrow} & \underline{12x + 20y} &\underline{= 20} \\ & & 22x &= 110 \\ & & x &= 5 \end{aligned}$$

We then substitute this value of x into any one of our equations to get y:

$$\begin{aligned} 3(5) + 5y &= \ \ 5 \\ 15 + 5y &= \ \ 5 \\ 5y &= -10 \\ y &= -2 \end{aligned}$$

The solution to the system is then $x = 5,\ y = -2,$ or $(5, -2)$. This can easily be verified by substituting into the two original equations:

$$\begin{aligned} 2(5) - 4(-2) &= 10 + 8 = 18 \\ 3(5) + 5(-2) &= 15 - 10 = 5 \end{aligned}$$

These steps are summarized here:

The Addition Method

To solve a system of two linear equations by the addition method,

1) Multiply each equation by something as needed in order to make the coefficients of either x or y the same but opposite in sign. *Both equations will not be multiplied by the same number!*

2) Add the two equations and solve the resulting equation for whichever unknown remains.

3) Substitute that value into either equation and solve for the other unknown.

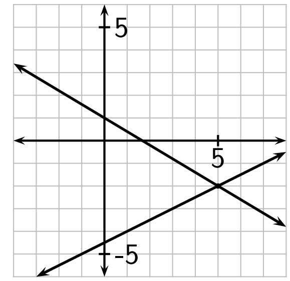
The following example shows the graphical interpretation of what is going on when we solve a system of two linear equations in two unknowns.

◇ **Example 1.6(c):** Solve each of the equations from Example 1.6(b) for y. Then graph the two lines on the same grid and determine where they cross.

Solution: Solving the two equations for y we get

$$y = \frac{1}{2}x - 4\frac{1}{2} \qquad \text{and} \qquad y = -\frac{3}{5}x + 1.$$

Both lines are graphed on the grid to the right. Note that they cross at $(5, -2)$, the solution to the system of equations.



The Substitution Method

We will now describe the substitution method which works best when one of x or y has coefficient 1 or -1 in one of the equations, then give an example of how it works.

> **The Substitution Method**
>
> To solve a system of two linear equations by the substitution method,
>
> 1) Pick the equation in which the coefficient of one of the unknowns is either one or negative one. Solve that equation for that unknown.
> 2) Substitute the expression for that unknown into *the other* equation and solve for the unknown.
> 3) Substitute that value into any equation and solve for the other unknown. *It is generally easiest to substitute it into the solved equation from step one.*

◇ **Example 1.6(d):** Solve the system of equations $\begin{aligned} x - 3y &= 6 \\ -2x + 5y &= -5 \end{aligned}$ using the substitution method.

Solution: Solve the first equation for x: $x = 3y + 6$. Substitute into the second equation and solve for y:

$$\begin{aligned} -2(3y+6) + 5y &= -5 \\ -6y - 12 + 5y &= -5 \\ -y &= 7 \\ y &= -7 \end{aligned}$$

Substitute $y = -7$ into either original equation or $x = 3y + 6$ to find x:

$$x = 3(-7) + 6 = -21 + 6 = -15$$

The solution to the system is $(-15, -7)$.

Applications

Systems of two linear equations can be quite handy for solving problems, as seen in the next example.

◇ **Example 1.6(e):** The length of a rectangle is 3.2 inches less than twice its width, and its perimeter is 51.6 inches. Find the length and width of the rectangle.

Solution: Let l represent the length of the rectangle and let w represent the width. Then we have $l = 2w - 3.2$ and $2l + 2w = 51.6$. This is all set for substitution:

$$\begin{aligned} 2(2w - 3.2) + 2w &= 51.6 \\ 6w - 6.4 &= 51.6 \\ 6w &= 63.5 \\ w &= 10.6 \end{aligned}$$

Substituting this value into $l = 2w - 3.2$ we get $l = 2(10.6) - 3.2 = 18$, so the length is 18 inches, and the width is 10.6 inches.

When "Things Go Wrong"

Consider the the following two systems of equations:

$$\begin{aligned} 2x - 5y &= 3 \\ -4x + 10y &= 1 \end{aligned} \qquad\qquad \begin{aligned} 2x - 5y &= 3 \\ -4x + 10y &= -6 \end{aligned}$$

If we attempt to solve the first system by the addition method, multiplying the first equation by two and then adding, we get $0 = 7$. This is obviously an erroneous statement! It is trying to tell us that the system has no solution. Doing the same for the second system results in $0 = 0$, which is true, but not helpful. In this case the system has infinitely many solutions; any (x, y) pair that is a solution to the first equation is also a solution to the second equation. Both of these situations are of interest in certain applications, but that goes beyond the scope of this course.

Section 1.6 Exercises

To Solutions

1. Solve each of the following systems by both the addition method and the substitution method.

(a) $\begin{aligned} 2x + y &= 13 \\ -5x + 3y &= 6 \end{aligned}$ (b) $\begin{aligned} 2x - 3y &= -6 \\ 3x - y &= 5 \end{aligned}$ (c) $\begin{aligned} x + y &= 3 \\ 2x + 3y &= -4 \end{aligned}$

2. Solve each of the following systems by the addition method.

(a) $\begin{aligned} 7x - 6y &= 13 \\ 6x - 5y &= 11 \end{aligned}$ (b) $\begin{aligned} 5x + 3y &= 7 \\ 3x - 5y &= -23 \end{aligned}$ (c) $\begin{aligned} 5x - 3y &= -11 \\ 7x + 6y &= -12 \end{aligned}$

3. Do the following for each of the systems below.

   (i) Solve each equation for y; do not use decimals!

   (ii) Plot both lines using a graphing calculator or online grapher. Give the solution to the system, based on what you see.

   (iii) Solve the system by either the addition method or the substitution method. If your solution does not agree with what you got in part (b), check your work.

   (a) $\begin{aligned} 2x-3y &= -6 \\ 3x-y &= 5 \end{aligned}$ (b) $\begin{aligned} 2x-3y &= -7 \\ -2x+5y &= 9 \end{aligned}$ (c) $\begin{aligned} 4x+y &= 14 \\ 2x+3y &= 12 \end{aligned}$

4. Consider the system $\begin{aligned} 2x-5y &= 3 \\ -4x+10y &= 1 \end{aligned}$.

   (a) Solve the system by either the addition method or the substitution method.

   (b) Solve each equation for y; again, do not use decimals!

   (c) Plot both lines using a graphing calculator or online grapher.

   (d) Discuss the solution to the system,and how it is illustrated by the graph.

5. Repeat the steps of Exercise 6 for the system $\begin{aligned} 2x-5y &= 3 \\ -4x+10y &= 1 \end{aligned}$.

6. Consider the system of equations $\begin{aligned} 2x-3y &= 4 \\ 4x+5y &= 3 \end{aligned}$.

   (a) Solve for x by using the addition method to eliminate y. Your answer should be a fraction.

   (b) Ordinarily you would substitute your answer to (a) into either equation to find the other unknown. However, dealing with the fraction that you got for part (a) could be difficult and annoying. Instead, use the addition method again, but eliminate x to find y.

7. Consider the system of equations $\begin{aligned} \tfrac{1}{2}x-\tfrac{1}{3}y &= 2 \\ \tfrac{1}{4}x+\tfrac{2}{3}y &= 6 \end{aligned}$. The steps below indicate how to solve such a system of equations.

   (a) Multiply both sides of the first equation by the least common denominator to "kill off" all fractions.

   (b) Repeat for the second equation.

   (c) You now have a new system of equations without fractional coefficients. Solve that system by the addition method.

8. An airplane flying with the wind travels 1200 miles in 2 hours. The return trip, against the wind, takes $2\frac{1}{2}$ hours. Assuming that the speed of both the wind and the plane were constant during both trips, what was the speed of the wind, and what was the speed of the plane?

9. You are in line at a ticket window. There are two more people ahead of you than there are behind you. The number of people in the entire line (don't forget to count yourself!) is three times the number of people behind you. How many people are ahead of you in line, and how many are behind you?

10. An economist models the market for wheat by the following equations:

Supply Equation	$y = 8.33p - 14.58$
Demand Equation	$y = -1.39p + 23.35$

Here p is the price per bushel (in dollars) and y is the number of bushels produced and sold (in **millions** of bushels). Find the number of bushels produced and sold at equilibrium. (Equilibrium refers to the situation in which supply and demand are equal.)

1.7 More on Graphing Equations

Performance Criteria:

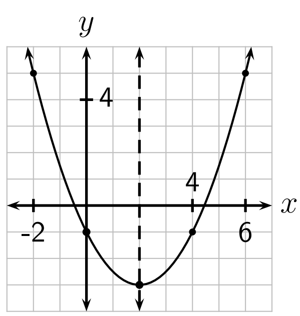
1. (l) Graph $y = ax^2 + bx + c$ by finding and plotting the vertex, y-intercept, and several other points.

   (m) Identify or create the graphs of $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$ and $y = \frac{1}{x}$.

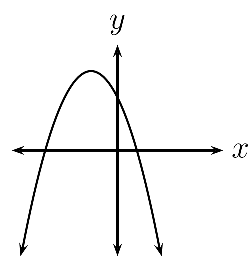
   (n) Graph variations of $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$ and $y = \frac{1}{x}$.

In Section 1.4 we introduced the idea of showing solutions to equations in two unknowns as graphs, and in Section 1.5 we saw how to obtain the graph of $Ax + By = C$ or $y = mx + b$ as efficiently as possible. In this section we will refine our procedure for graphing other equations.

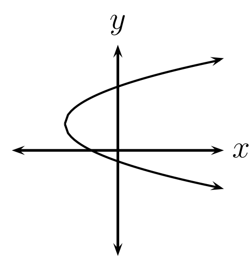
The shapes shown in the graphs below are all **parabolas**. We say that the first two open upward and downward, respectively, and the second two open to the right and left, respectively. Note that we could write $y = mx + b$ as $y = bx + c$; the letters we use for the slope and y-intercept can be any we choose. If we add ax^2 (for a not zero) to the right side of the second of these we get the standard equation $y = ax^2 + bx + c$ of a parabola. When $a > 0$ the graph is an upward opening parabola, and when $a < 0$ the graph is parabola that opens downward. These situations are shown in the first two graphs below. When the equation is $x = ay^2 + by + c$ we get graphs like the two to the right below, the first of them when $a > 0$ and the second one when $a < 0$. We will usually work with equations of the form $y = ax^2 + bx + c$ (where any of a, b or c might be negative and either or both of of b and c can be zero).



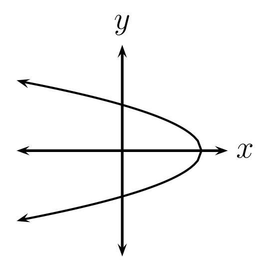
$y = ax^2 + bx + c, \quad a > 0$



$y = ax^2 + bx + c$
$a < 0$



$x = ay^2 + by + c$
$a < 0$



$x = ay^2 + by + c$
$a < 0$

Looking at the first parabola above, the point $(2, -3)$ is called the **vertex** of the parabola. It is the first point we should plot when graphing a parabola and, for parabolas opening up or down, it is important in applications because it is the lowest or highest point on the graph. The x-coordinate of the vertex of a parabola with equation $y = ax^2 + bx + c$ is given by $x = -\dfrac{b}{2a}$, and the y-coordinate of the vertex is found by substituting that x value into the equation.

◇ **Example 1.7(a):** Determine the coordinates of the vertex of the parabola with equation $y = -\frac{1}{2}x^2 + 2x - 1$, and whether the parabola opens up or down.

Solution: The x-coordinate of the vertex is $x = -\dfrac{2}{2(-\frac{1}{2})} = -\dfrac{2}{-1} = 2$. The y-coordinate of the vertex is found by substituting the value 2 that we just found for x into the original equation:

$$y = -\frac{1}{2}(2)^2 + 2(2) - 1 = -2 + 4 - 1 = 1$$

Therefore the vertex of the parabola is $(2, 1)$ and, because $a = -\frac{1}{2} < 0$, it opens down.

Another key idea for graphing parabolas can be seen in the graph of the first parabola on the previous page: The graph is *symmetric about the line with equation* $x = 2$. That line is called the **axis of symmetry** for the parabola. When graphing, the importance of this is that every point on the parabola (besides the vertex) has a "partner" on the other side of the parabola, an equal distance from the axis of symmetry as the original point. In the first graph on the previous page we can see that the point $(4, -1)$ has $(0, -1)$ as its partner, and the points $(-2, 5)$ and $(6, 5)$ pair up in the same way.

The observations just made lead us to the following method for efficiently graphing equations of the form $y = ax^2 + bx + c$:

Graphing $y = ax^2 + bx + c$

1) Recognize from the form of the equation that the graph will be a parabola, and determine whether the parabola opens upward or downward.

2) Use $x = -\dfrac{b}{2a}$ to find the x-coordinate of the vertex, then evaluate the equation for that value of x to find the y-coordinate of the vertex. Plot that point.

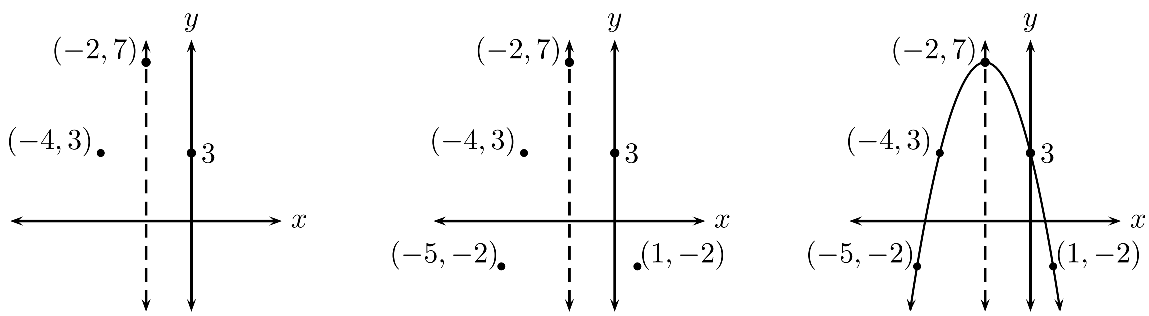
3) Find and plot the y-intercept and the point opposite it. Sketch in the line of symmetry if you need to in order to do this.

4) Pick an x value other than ones used for the three points that you have plotted so far and use it to evaluate the equation. Plot the resulting point and the point opposite it.

4) Draw the graph of the equation. As always, include arrowheads to show where the the graph continues beyond the edges of the graph.

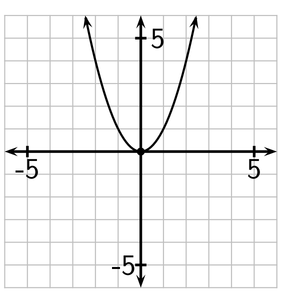
◇ **Example 1.7(b):** Graph the equation $y = -x^2 - 4x + 3$. Indicate clearly and accurately five points on the parabola.

Solution: First we note that the graph will be a parabola opening downward, with y-intercept 3. (This is obtained, *as always*, by letting $x = 0$.) The x-coordinate of the vertex is $x = -\dfrac{-4}{2(-1)} = -2$. Evaluating the equation for this value of x gives us $y = -(-2)^2 - 4(-2) + 5 = -4 + 8 + 3 = 7$, so the vertex is $(-2, 7)$. On the first graph at the top of the next page we plot the vertex, y-intercept, the axis of symmetry, and the point opposite the y-intercept, $(-4, 3)$. To get two more points we can evaluate the equation for $x = 1$ to get $y = -1^2 - 4(1) + 3 = -2$. We then plot the resulting point $(1, -2)$ and the point opposite it, as done on the second graph. The third graph then shows the graph of the function with our five points clearly and accurately plotted.

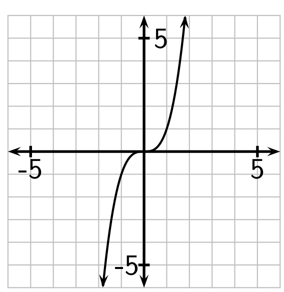


Some Basic Functions

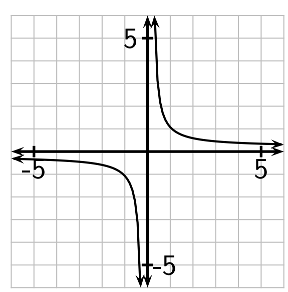
In Section 1.5 we found how to quickly and easily graph any line, and we just found out how to efficiently graph parabolas. We now consider the graphs of some other equations of interest. First let's recall the graphs, that we constructed in the exercises of Section 1.4, of some basic equations:



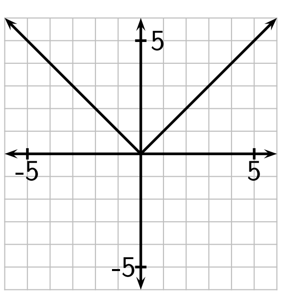
$y = x^2$



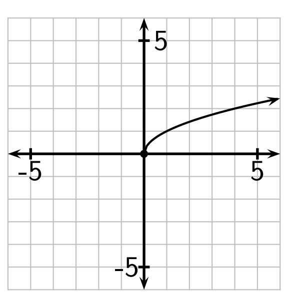
$y = x^3$



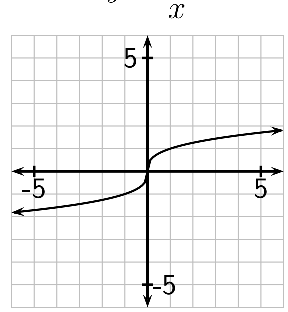
$y = \dfrac{1}{x}$



$y = |x|$



$y = \sqrt{x}$



$y = \sqrt[3]{x}$

The above graphs illustrate a few things to keep in mind, beyond what we already know about the graphs of $y = mx + b$ and $y = ax^2 + bx + c$:

- Graphs of equations containing absolute values are usually vee-shaped.
- Graphs of equations containing square roots are usually half parabolas.
- Graphs of equations whose denominator is zero for one or more values of x will have two or more separate pieces.

We now introduce an important and useful principle, which we will relate to something that we already know. Suppose that we are trying to solve the equation $x^2 = 2x + 3$. We get zero on one side and factor to get $(x - 3)(x + 1) = 0$, for which we can see that $x = 3$ and $x = -1$ are solutions. In general, when we see an expression like $x+a$, $x-a$, or $a-x$ for some number a, the value that makes the expression zero is important for some reason. Consider the equations

$$y = |x - 3|, \qquad y = \sqrt{x + 4}, \qquad \text{and} \qquad y = \frac{6}{2 - x}$$

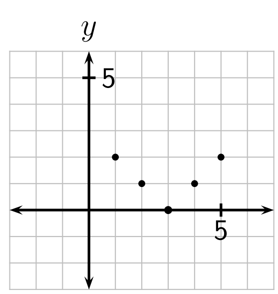
For the first of these, $x = 3$ is an important value for a reason you will soon see. For the second equation $x = -4$ is important because if x is any number less than -4, y will be undefined, but for $x = -4$ and larger, y *IS* defined. y will be undefined for $x = 2$ in the last equation, but y values *CAN* be obtained for *ANY* other value of x. Here is how to use all of this information:

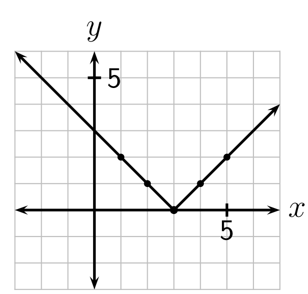
> When graphing an equation containing one of the expressions $x + a$, $x - a$, or $a-x$, first determine the value of x that makes the expression zero, which we will call a **critical value**. If possible, find the value of y for the critical value of x. Then find additional values of y using x values obtained by "working outward" from from the critical value, as illustrated in the following examples.

◇ **Example 1.7(c):** Graph the equation $y = |x - 3|$.

Solution: Because the equation contains an absolute value we expect the graph to be a vee, and $x = 3$ is a critical value because it makes $x - 3$ zero. We find the values of y for $x = 3$ and several values on either side of three, as shown in the table below and to the left. The points are plotted on the left grid, where the vee becomes apparent, and the final graph is shown to the right below.

x	y
1	2
2	1
3	0
4	1
5	2

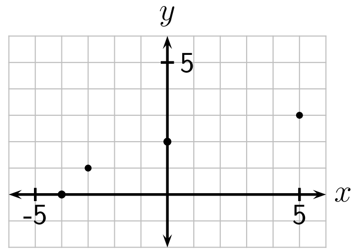


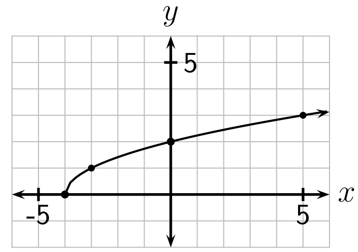


◇ **Example 1.7(d):** Graph the equation $y = \sqrt{x+4}$.

Solution: Here we expect the graph to be a half parabola, and we also see that $(-4)+4 = 0$, so $x = -4$ is a critical value. y is not defined for any number less than negative four, so we want to use -4 and larger for x. We should also choose values of x that make $x+4$ a perfect square like 0, 1, 4, 9, 16,... This is done in the table to the left at the top of the next page, those points are plotted on the middle grid, and the final graph is on the right grid. Note that the graph ends at $(-4, 0)$, as indicated by the solid dot there, but continues in the other direction, as shown by the arrowhead.

x	y
-4	0
-3	1
0	2
5	3

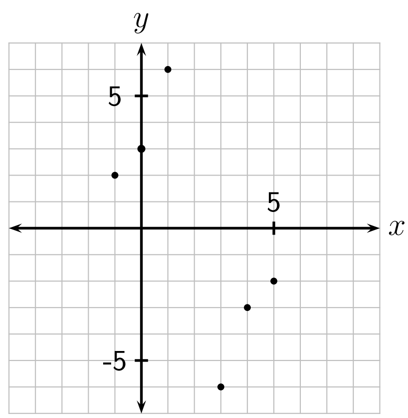


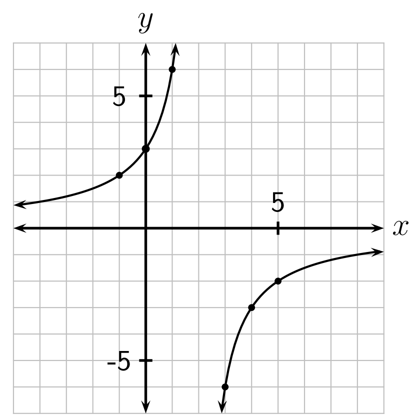


◇ **Example 1.7(e):** Graph the equation $y = \dfrac{6}{2-x}$.

Solution: Here we expect the graph to be a half parabola, and we also see that $(-4)+4 = 0$, so $x = -4$ is a critical value. y is not defined for any number less than negative four,so we want to use -4 and larger for x. We should also choose values of x that make $x+4$ a perfect square like 0, 1, 4, 9, 16,... This is done in the table to the left at the top of the next page, those points are plotted on the left grid below, and the final graph is on the right grid below. Note that the graph ends at $(-4, 0)$, as indicated by the solid dot there, but continues in the other direction, as shown by the arrowhead.

x	y
-1	2
0	3
1	6
3	-6
4	-3
5	-2
8	-1





Section 1.7 Exercises   To Solutions

1. For each of the following quadratic equations, first determine whether its graph will be a parabola opening upward or opening downward. Then find the coordinates of the vertex.

   (a) $y=-3x^2-6x+2$ (b) $y=x^2-6x+5$ (c) $y=\frac{1}{2}x^2+2x+5$

2. Graph the solution set of each of the equations from Exercise 1.

3. Graph $y=x^2+2x+3$, indicating five points on the graph clearly and accurately.

4. Graph the equation $y=x^2+4x-5$. Indicate clearly and accurately five points on the parabola. How does the result compare with that from Example 1.7(b), in terms of the direction it opens, its intercepts, and its vertex??

5. Discuss the x-intercept situation for equations of the form $y=ax^2+bx+c$. Do they always have at least one x-intercept? Can they have more than one? Sketch some parabolas that are arranged in different ways (but always opening up or down) relative to the coordinate axes to help you with this.

6. (a) A parabola has x-intercepts 3 and 10. What is the x-coordinate of the vertex?
   (b) The vertex of a *different* parabola is $(3,2)$, and the point $(5,4)$ is on the parabola. Give another point on the parabola.
   (c) The vertex of yet another parabola is $(-2,9)$. The y-intercept of the parabola is 5 and one x-intercept is -5. Give the other x-intercept and one other point on the parabola.

7. Consider the equation $y=x^2-6x-7$. Here you will graph the equation in a slightly different manner than used in Example 1.7(b).

   (a) You know the graph will be a parabola - which way does it open?
   (b) Find the y-intercept and plot it.
   (c) Find the x-intercepts, and plot them.
   (d) Sketch the line of symmetry on your graph - you should be able to tell where it is, based on the x-intercepts. What is the x-coordinate of the vertex?
   (e) Find the y-coordinate of the vertex of the parabola by substituting the x-coordinate into the equation.
   (f) Plot the vertex and the point opposite the y-intercept and graph the equation.
   (g) Where (at what x value) does the minimum or maximum value of y occur? Is y a minimum, or a maximum, there? What is the minimum or maximum value of y?

8. Here you will find that the method used in the previous exercise for finding the x-coordinate of the vertex of a parabola will not always work, but that you can always find the x-coordinate of the vertex using the formula $x = -\frac{b}{2a}$.

   (a) Find the x-intercepts of $y = x^2 - 6x + 13$. What goes wrong here?

   (b) Which way does the parabola open?

   (c) Find the vertex or the parabola.

   (d) Tell how your answers to (b) and (c) explain what happened in (a).

   (e) Give the coordinates of four other points on the parabola.

9. Consider the equation $y = 10 + 3x - x^2$.

   (a) What are the values of $a,\ b$ and c for this parabola?

   (b) Use the procedure of Example 1.7(b) to sketch the graph of the parabola.

10. Some equations are given below. For each, determine the most critical value of x for graphing the equation, then either find the corresponding y value or state that the equation is undefined for the critical x value. (In one case there is no obvious critical x value.)

    (a) $y = \dfrac{4}{x+3}$ (b) $y = x^2 - 5x + 6$ (c) $y = \dfrac{6}{2-x}$

    (d) $y = \sqrt{x+5}$ (e) $y = -\dfrac{2}{3}x + 4$ (f) $y = \sqrt{4-x}$

    (g) $y = \dfrac{6}{x-1}$ (h) $y = \dfrac{8}{x^2}$ (i) $y = \sqrt{x+3} - 1$

11. Graph each of the equations from Exercise 10, indicating clearly at least four points *with integer coordinates* on each graph.

1.8 Chapter 1 Exercises

To Solutions

1. Solve each equation. When the quadratic equation is required, give both exact solutions and approximate (decimal) solutions rounded to the nearest tenth.

   (a) $2(x+4)-5(x+10)=6$

   (b) $\dfrac{8}{x^2-9}+\dfrac{4}{x+3}=\dfrac{2}{x-3}$

   (c) $2|x-5|-3=5$

   (d) $x^2-6x+7=0$

   (e) $15x^2=20x$

   (f) $\sqrt{-2x+1}=-3$

   (g) $5x+1=2x+8$

   (h) $2y-8=-y^2$

   (i) $\dfrac{3}{2}+\dfrac{5}{x-3}=\dfrac{x+9}{2x-6}$

   (j) $36-x^2=0$

2. A student is solving the equation $5-2(x+4)=x-2$ and obtains the solution $x=5$.

   (a) Convince them that their solution is incorrect, *without solving the equation*.

   (b) Can you guess what their error was? Show how they obtained their solution, and tell where they went wrong.

   (c) For the same equation, another student got the incorrect solution $x=-7$. Show how they obtained their solution, and tell where they went wrong.

   (d) Solve the equation and verify that your solution is correct.

3. (a) Create an equation of the form $|x+a|=b$ with solutions $x=2,8$.

   (b) Create an equation of the same form with solutions $x=3,11$.

   (c) Suppose that we want an equation of the same form to have solutions x_1 and x_2. Find formulas for obtaining the values of a and b from x_1 and x_2.

   (d) Check your answer to (c) by finding an equation of the given form with solutions $x=-5,1$. Check to make sure that those values really are solutions!

4. The length of a rectangle is 3.2 inches less than twice its width, and its perimeter is 57.2 inches. Find the length and width of the rectangle.

5. The width of a rectangle is three more than half the length. The perimeter is 39. How long are the sides of the rectangle?

6. The cost of a compact disc, with 6% sales tax, was $10.55. What was the price of the compact disc?

7. Solve each equation for y. Give your answers in $y=mx+b$ form.

   (a) $5x+2y=20$

   (b) $2x-3y=6$

   (c) $3x+5y+10=0$

8. Solve each equation for x.

   (a) $ax+bx=c$

   (b) $ax+3=cx-7$

9. A projectile is shot upward from ground level with an initial velocity of 48 feet per second.

   (a) When is the projectile at a height of 20 feet?

   (b) When does the projectile hit the ground?

   (c) What is the height of the projectile at 1.92 seconds? **Round your answer to the nearest tenth of a foot.**

   (d) When is the projectile at a height of 32 feet?

   (e) When is the projectile at a height of 36 feet? Your answer here is a bit different than your answers to (a) and (d). What is happening physically with the projectile?

   (f) When is the projectile at a height of 40 feet?

10. A manufacturer of small calculators knows that the number x of calculators that it can sell each week is related to the price per calculator p by the equation $x = 1300 - 100p$. The weekly revenue (money they bring in) is the price times the number of calculators: $R = px = p(1300 - 100p)$. What price should they set for the cartridges if they want the weekly revenue to be $4225?

11. Find the intercepts for each of the following equations algebraically.

    (a) $y = x^2 - 2x$ (b) $x + 2y = 4$ (c) $x - y^2 = 1$

    (d) $x^2 + y^2 = 25$ (e) $4x - 5y = 20$ (f) $x = \sqrt{y+4}$

    (g) $y = 2|x| - 3$

12. Find the equation of the line through the two points **algebraically**.

    (a) $(-4, -2),\ (-2, 4)$ (b) $(1, 7),\ (1, -1)$ (c) $(-1, -5),\ (1, 3)$

    (d) $(1, 7), (3, 11)$ (e) $(-6, -2),\ (5, -3)$ (f) $(-2, -5),\ (2, 5)$

13. Consider the system of three equations in three unknowns x, y and z:

$$\begin{aligned} x + 3y - 2z &= -4 \\ 3x + 7y + z &= 4 \\ -2x + y + 7z &= 7 \end{aligned}$$

The steps below indicate how to solve such a system of equations - it is essentially the same as solving a system of two equations in two unknowns.

   (a) Use the addition method with the first two equations to eliminate x.

   (b) Use the addition method with the first and third equations to eliminate x.

   (c) Your answers to (a) and (b) are a new system of two equations with two unknowns. Use the addition method to eliminate y and solve for z.

   (d) Substitute the value you found for z into one of the equations containing y and z to find y.

(e) You should now know y and z. Substitute them into ANY of the three equations to find x.

(f) You now have what we call an *ordered triple* (x, y, z). Check your solution by substituting those three numbers into each of the original equations to make sure that they make all three equations true.

14. Solve each of the following systems of equations.

(a) $$\begin{aligned} x+2y-z &= -1 \\ 2x-y+3z &= 13 \\ 3x-2y &= 6 \end{aligned}$$

(b) $$\begin{aligned} 2x-y+z &= 6 \\ 4x+3y-z &= 1 \\ -4x-8y+2z &= 1 \end{aligned}$$

(c) $$\begin{aligned} x-2y+3z &= 4 \\ 2x+y-4z &= 3 \\ -3x+4y-z &= -2 \end{aligned}$$

(d) $$\begin{aligned} x+3y-z &= -3 \\ 3x-y+2z &= 1 \\ 2x-y+z &= -1 \end{aligned}$$

15. The graph of $y = 10 + 3x - x^2$ is a parabola. Determine whether it opens up or down, and give the coordinates of its vertex.

16. For each of the quadratic functions below,

- give the general form of the graph (parabola opening upward, parabola opening downward),
- give the coordinates of the vertex,
- give the y-intercept of the graph and the coordinates of one other point on the graph,
- sketch the graph of the function,
- tell what the maximum or minimum value of the function is (stating whether it is a maximum or a minimum) and where it occurs.

(a) $f(x) = -x^2 + 2x + 15$

(b) $g(x) = x^2 - 9x + 14$

(c) $h(x) = -\frac{1}{4}x^2 + \frac{5}{2}x - \frac{9}{4}$

(d) $s(t) = \frac{1}{2}t^2 + 3t + \frac{9}{2}$

17. Consider the equation $y = (x-3)^2 - 4$.

(a) Get the equation in the form $f(x) = ax^2 + bx + c$.

(b) Find the coordinates of the vertex of the parabola. Do you see any way that they might relate to the original form in which the function was given?

(c) Find the x and y-intercepts of the equation and sketch its graph.

(d) On the same coordinate grid, sketch the graph of $y = x^2$ *accurately*. (Find some ordered pairs that satisfy the equation and plot them to help with this.) Compare the two graphs. Check your graphs with your calculator.

18. Consider the equation $y = -\frac{1}{2}(x-2)(x+3)$. Do all of the following without multiplying this out!

   (a) If you were to multiply it out, what would the value of a be? Which way does the parabola open?

   (b) Find the y-intercept of the equation.

   (c) Find the x-intercepts of the equation. In this case it should be easier to find the x-intercepts than the y-intercept!

   (d) Find the coordinates of the vertex.

   (e) Graph the equation, labelling the vertex with its coordinates.

19. Graph $h(x) = \sqrt{9 - x^2}$. Check your answer using a graphing utility.

A Solutions to Exercises

A.1 Chapter 1 Solutions

Section 1.1 Solutions Back to 1.1 Exercises

1. (a) $x = -\frac{1}{2}$ (b) $x = 10$ (c) $x = -5$ (d) $x = 5$ (e) $x = \frac{10}{11}$
   (f) $x = \frac{4}{3}$ (g) $x = \frac{4}{5}$ (h) $x = -15$
2. (a) $x = 12, 1$ (b) $x = 0, -1, -2$ (c) $x = 0, -\frac{1}{5}$ (d) $x = -5, \frac{3}{2}$
   (e) $a = -3, -5$ (f) $x = 0, 2$ (g) $x = -4, 4$ (h) $x = -\frac{1}{2}, 3$
   (i) $x = -2, 0, 11$
3. (a) $x = 1 + \sqrt{5},\ 1 - \sqrt{5}, \quad x = -1.2, 3.2$
   (b) $x = 1 + 3\sqrt{2},\ 1 - 3\sqrt{2}, \quad x = -3.2, 5.2$
   (c) $x = \frac{1}{2} + \frac{\sqrt{3}}{2},\ \frac{1}{2} - \frac{\sqrt{3}}{2}, \quad x = -0.4, 1.4$
   (d) $x = -\frac{1}{5} + \frac{\sqrt{7}}{5},\ -\frac{1}{5} - \frac{\sqrt{7}}{5}, \quad x = -0.7, 0.3$
4. (a) $x = -2, -\frac{2}{3}$ (b) $x = -\frac{2}{7}, \frac{4}{7}$ (c) $x = -4, 4, -\sqrt{10}, \sqrt{10}$
6. (a) The ball will be at a height of 84 feet at $t = 1$ seconds and $t = 5$ seconds.
   (b) The ball will be at a height of 148 feet at $t = 3$ seconds.
   (c) The ball will be at a height of 52 feet at $t = 0.55$ seconds and $t = 5.45$ seconds.
   (d) The ball never reaches a height of 200 feet.
   (e) The ball hits the ground at $t = 6.04$ seconds.
7. The price should be set at \$4.00 or \$8.00.
8. (a) $x = 0, \frac{4}{3}$ 11. (e) $x = -3, -2, 1$ 12. $x = 1, 2, 7$
13. (a) $x = -1,\ 2,\ 3$ (b) $x = -1,\ 1,\ 3,\ 4$ (c) $x = -2,\ 5$
18. (c) $x = -2, 2$
19. (a) $x = -\frac{6}{5}$ (b) $x = 3 + 2\sqrt{7},\ 3 - 2\sqrt{7}, \quad x = -2.3, 8.3$ (c) $x = -18$
    (d) $x = \frac{5}{3}, \frac{1}{3}$ (e) $x = \frac{2}{3} + \frac{\sqrt{5}}{3},\ \frac{2}{3} - \frac{\sqrt{5}}{3}, \quad x = -0.1, 1.4$ (f) $x = 0, 2, 5$
    (g) $x = 3, -7$ (h) $x = \frac{1}{4}$ (i) $x = \frac{1}{3}, -\frac{5}{2}$
    (j) $x = -4 + \sqrt{3},\ -4 - \sqrt{3}, \quad x = -2.3, -5.7$ (k) $x = -3, \frac{1}{2}$ (l) $x = -3, 7$

Section 1.2 Solutions Back to 1.2 Exercises

1. (a) $x = 4$ (b) $x = -\frac{11}{7}$ (c) $x = 2$
   (d) $x = 2$ (e) $x = 3$ (f) $x = 3$

2. (a) $x = \frac{4}{5}$ (b) $x = \frac{9}{2}$ (c) $x = 1$ ($x = -4$ is not a solution)
   (d) $x = -1$ ($x = -5$ is not a solution) (e) no solution ($x = \frac{2}{3}$ is not a solution)
   (f) $x = 2, 3$
3. (a) $x = 6$ (b) $x = \frac{7}{3}$ (c) $x = -1,\ 2$
   (d) $x = -4$ (e) $x = -\frac{1}{2},\ 5$ (f) no solution
6. (a) $x = -2, \frac{5}{2}$ (b) $x = -5, 6$ (c) $x = \frac{7}{2}, -\frac{3}{5}$ (d) $x = -5$
   (e) $x = \frac{8}{3}$ (f) $x = -5 + 2\sqrt{3},\ \ -5 - 2\sqrt{3},\ \ \ x = -8.5, -1.5$
   (g) $x = 1, -\frac{1}{6}$ (h) $x = 8$ ($x = -1$ is not a solution)

Section 1.3 Solutions Back to 1.3 Exercises

1. The other leg has length 5 inches.
2. The radius is 2.6 inches.
3. (a) The sales tax is \$1.10.
   (b) You will pay \$ 21.05 with tax included.
4. In that month they make \$2870.50.
5. (a) \$1640 (b) \$3350
6. You borrowed \$1650.
7. The length is 40 and the width is 13.
8. The price of the item was \$189.45.
9. The legs of the triangle are 6.7 feet and 13.4 feet long.
10. Your hourly wage was \$6.70 per hour. (If you got \$6.63 your method is incorrect!)
11. The length is 11 units and the width is 4 units.
12. The price of the can opener is \$38.85.
13. The shortest side is 10.5 inches long, the middle side is 11.9 inches long, and the longest side is 21 inches long.
14. Her cost is \$42.82.
15. The lengths of the sides of the triangle are 8, 15 and 17 units long.
16. (a) The area of the rectangle is 70 square inches.
    (b) The new area is 280 square inches.
    (c) No, the new area is *FOUR* times as large.
    (d) The new area is 630 square inches, nine times larger than the original area.

17. (a) You will have \$1416.00 if you take it out after four years.

(b) It will take 22.2 years to double your money.

18. $r = \dfrac{A-P}{Pt}$ or $r = \dfrac{A}{Pt} - \dfrac{1}{t}$ 19. $R = \dfrac{PV}{nT}$ 20. $b = \dfrac{ax+8}{x}$ or $b = a + \dfrac{8}{x}$

21. (a) $77°$F (b) $35°$C (c) $C = \dfrac{5F-160}{9}$ or $C = \frac{5}{9}F - 17\frac{7}{9}$ or $c = \frac{5}{9}(F-32)$

(d) $25°$C

22. $l = \dfrac{P-2w}{2}$ or $l = \frac{P}{2} - w$ 23. $r = \dfrac{C}{2\pi}$

24. (a) $y = -\frac{3}{4}x - 2$ (b) $y = -\frac{5}{2}x - 5$ (c) $y = \frac{3}{2}x + \frac{5}{2}$ (d) $y = \frac{3}{4}x - 2$

(e) $y = -\frac{3}{2}x + \frac{5}{2}$ (f) $y = \frac{5}{3}x + 3$ 25. $x = -\frac{10}{3}$

26. (a) $x = \dfrac{d-b}{a-c}$ (b) $x = \dfrac{8.99}{2.065} = 4.35$ (c) $x = \dfrac{-8}{a-b}$ (d) $x = \dfrac{bc-7}{a-b}$

27. $P = \dfrac{A}{1+rt}$

Section 1.4 Solutions Back to 1.4 Exercises

4. (a) 5 is the largest possible value of x, giving the solution $(5, -1)$.

(b) $x = 4,\ 1,\ -4$

7. (a) x-intercept: 2 y-intercepts: 1, 4 (b) x-intercepts: $-4,\ 0$ y-intercept: 0

8. (a) x-intercept: 4 y-intercepts: $-2,\ 2$

(b) x-intercepts: $-0.5,\ 4.5$ y-intercept: -1

(c) x-intercept: -3 y-intercept: ≈ 1.8

9. x-intercept: 2.5 y-intercept: 5

10. x-intercept: -3 y-intercept: 5 11. x-intercepts: $3,\ -1$ y-intercept: -3

12. x-intercept: $\frac{9}{2}$ or $4\frac{1}{2}$ y-intercept: 3

13. (a) The x-intercepts are $(0,0)$ and $(5,0)$. (b) The y-intercept is $(0,0)$.

14. (a) x-intercepts: 0 y-intercepts: 0, 2 (b) x-intercepts: -4 y-intercepts: 2

(c) x-intercepts: -3 y-intercepts: $\frac{3}{2}$ (d) x-intercepts: -3 y-intercepts: 9

(e) x-intercepts: $1, -1$ y-intercepts: 1 (f) x-intercepts: 0 y-intercepts: 0

(g) x-intercepts: 10 y-intercepts: 6 (h) x-intercepts: $3, -3$ y-intercepts: $4, -4$

(i) x-intercepts: -3 y-intercepts: 1 (j) x-intercepts: none y-intercepts: 4

(k) x-intercepts: -2 y-intercepts: 2

15. (a) The x-intercepts are $x = -3, 1, 2$ and the y-intercept is $y = 6$.

(b) The x-intercepts are $x = 5, -2, -4$ and the y-intercept is $y = 24$.

(c) The x-intercepts are $x = -2, 2$ and the y-intercept is $y = \frac{8}{3}$.

(d) The x-intercept is $x = 0$ and the y-intercept is $y = 0$.

Section 1.5 Solutions

Back to 1.5 Exercises

1. (a) $-\frac{2}{3}$ (b) $\frac{2}{5}$ (c) 0 (d) undefined (e) 3 (f) $-\frac{1}{2}$

2. (a) $-\frac{5}{2}$ (b) 0 (c) $\frac{3}{4}$ (d) undefined (e) $\frac{4}{3}$ (f) $-\frac{1}{3}$

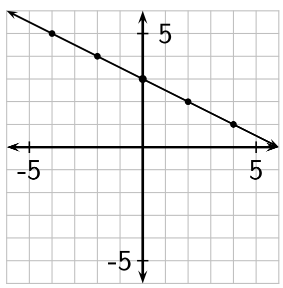
3. (a) $y = -\frac{5}{2}x + 3$ (b) $y = 3$ (c) $y = \frac{3}{4}x - 2$

(d) $x = 3$ (e) $y = \frac{4}{3}x$ (f) $y = -\frac{1}{3}x + 2$

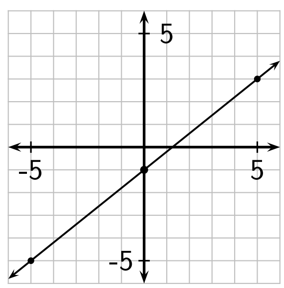
4. 3

5. $\frac{1}{2}$

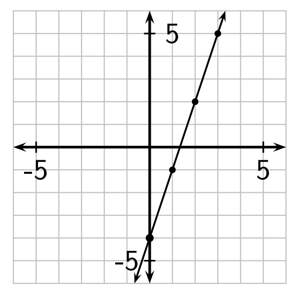
6. (a)



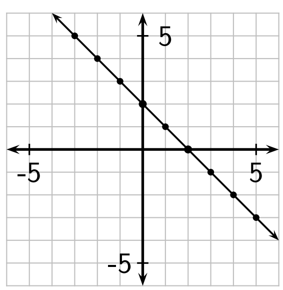
(b)



(c)



(d)



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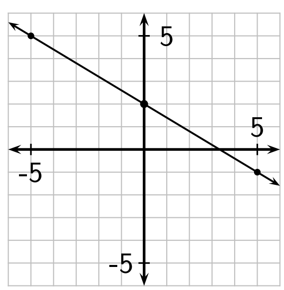
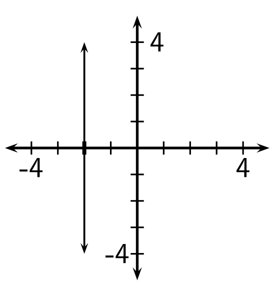
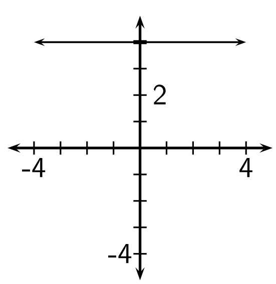


7. (a)



(b)



8. (a) $y = \frac{4}{3}x + 2$ (b) $y = -\frac{2}{3}x + 7$ (c) $y = 2$

(d) $y = \frac{1}{4}x - 5$ (e) $y = -\frac{3}{4}x + \frac{1}{2}$ (f) $y = 4$

9. (a) $m = \frac{5}{3}$ (b) $m = \frac{5}{3}$ (c) $m = -\frac{3}{5}$

10. $3x - 4y = 7 \Rightarrow y = \frac{3}{4}x - \frac{7}{4}$, $8x - 6y = 1 \Rightarrow y = \frac{4}{3}x - \frac{1}{6}$ The lines are neither parallel nor perpendicular.

11. $y = \frac{3}{2}x - 2$ 12. $y = \frac{3}{4}x + 3$ 13. $y = -5$ 14. $y = -\frac{4}{5}x - \frac{7}{5}$

15. $y = -\frac{2}{3}x + 6$ 16. $y = -5x + 10$ 17. $y = \frac{4}{3}x - 2$

18. The slope of the line through A and B is $\frac{3}{4}$ and the slope of the line through B and C is $\frac{6}{8} = \frac{3}{4}$. The three points lie on the same line.

19. $y = \frac{7}{5}x + 9$

Section 1.6 Solutions

Back to 1.6 Exercises

1. (a) $(3, 7)$ (b) $(3, 4)$ (c) $(13, -10)$

2. (a) $(1, -1)$ (b) $(-1, 4)$ (c) $(-2, \frac{1}{3})$

6. $(\frac{29}{22}, -\frac{5}{11})$ 7. $(8, 6)$

8. The plane's speed in still air is 540 mph and the wind speed is 60 mph.

9. There are 5 people ahead of you and 3 people behind you.

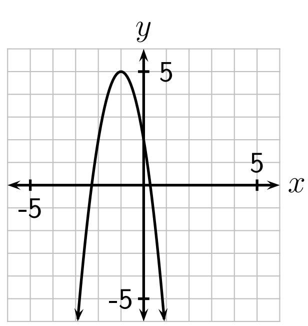
10. 3.9 million bushels are produced and sold at equilibrium.

Section 1.7 Solutions

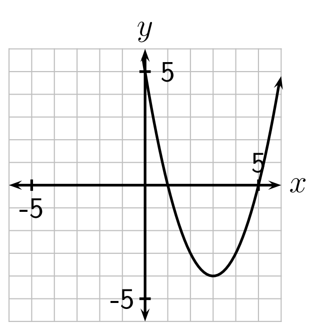
Back to 1.7 Exercises

1. (a) downward, $(-1, 5)$ (b) upward, $(3, -4)$ (c) upward, (-2,3)

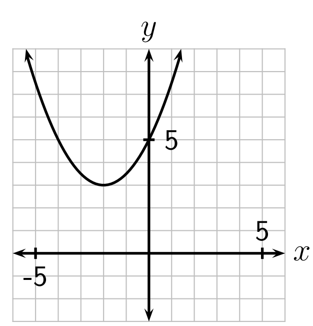
2. (a)



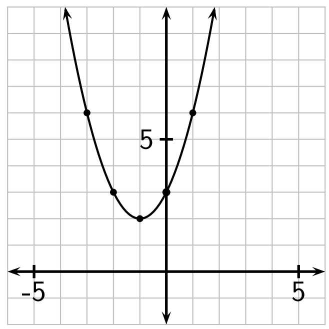
(b)



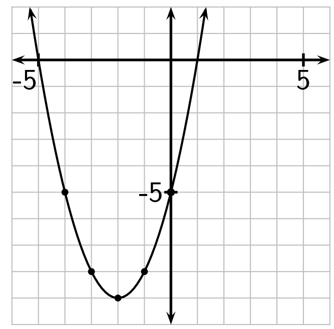
(c)



3. See graph below and to the left.



Exercise 3



Exercise 4

4. See graph above and to the right. The graph has the same x-intercepts as $y = -x^2 - 4x + 5$, but the the y-intercept and y-coordinate of the vertex are the opposites of what they are for $y = x^2 + 4x - 5$.

5. The graph of $y = ax^2 + bx + c$ can have 0, 1 or 2 x-intercepts.

6. (a) 6.5 (b) $(1, 4)$
   (c) The other x-intercept is 1 and the point $(-4, 5)$ is on the parabola.

7. 7

   upward (b) -7 (c) $-1,\ 7$

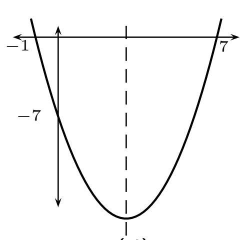
   (d) See below. The x-coordinate of the vertex is 3.

   (e) The y-coordinate of the vertex is -16. (f) See with Exercise 9 solutions.

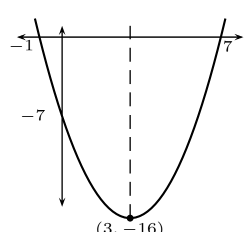
   (d) The minimum value of the function is -16, which occurs at $x = 3$.

8. (a) The x-intercept cannot be found. When we try to use the quadratic formula to find the x-intercepts, we get a negative number under a square root.
   (b) The parabola opens upward.
   (c) The vertex is $(3, 4)$.
   (d) Because the vertex is above the x-axis and the parabola opens upward, there are no x-intercepts.
   (e) $(2, 5),\ (4, 5),\ (1, 8),\ (5, 8),\ (0, 13),\ (6, 13)$, etc.

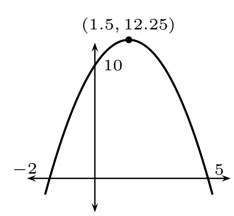
9. (a) $a = -1,\ b = 3,\ c = 10$ (b) See below.



7(d)

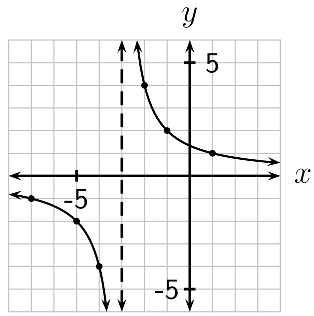


7(f)

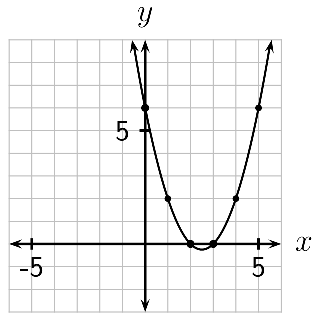


9(b)

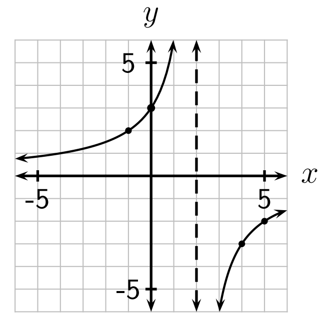
10. (a)



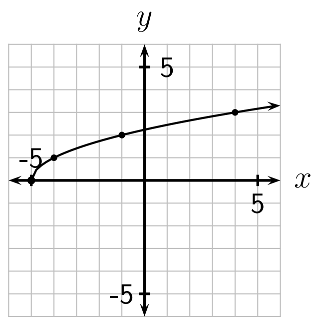
(b)



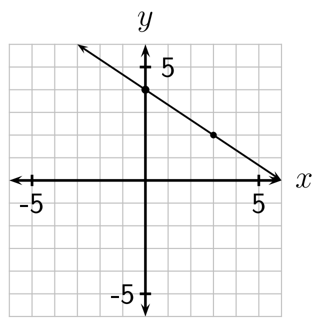
(c)



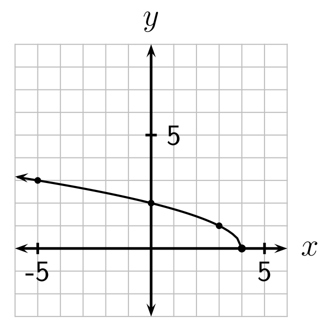
(d)



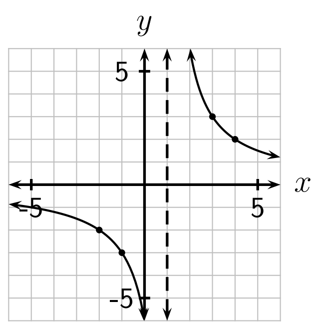
(e)



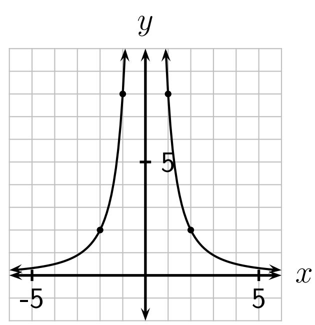
(f)



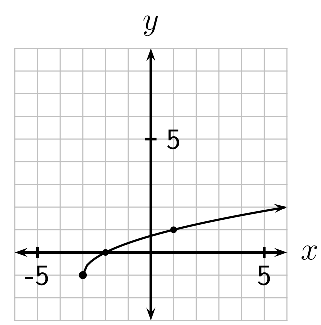
(g)



(h)



(i)



Section 1.8 Solutions

Back to 1.8 Exercises

1. (a) $x = -16$ (b) $x = 5$ (c) $x = 1, 9$ (d) $x = 3+\sqrt{2},\ 3-\sqrt{2},\ \ x = 1.4, 4.4$

   (e) $x = 0, \frac{4}{3}$ (f) no solution ($x = -4$ is not a solution) (g) $x = \frac{7}{3}$

   (h) $y = -4, 2$ (i) $x = 4$ (j) $x = -6, 6$

4. The length is 18.0 inches and the width is 10.6 inches.

5. The length is 11 units and the width is 8.5 units.

6. The price of the compact disc is $9.95.

7. (a) $y = -\frac{5}{2}x + 10$ (b) $y = \frac{2}{3}x - 2$ (c) $y = -\frac{3}{5}x - 2$

8. (a) $x = \dfrac{c}{a+b}$ (b) $x = \dfrac{-10}{a-c}$

9. (a) The projectile will be at a height of 20 feet at $t = \frac{1}{2}$ seconds and $t = 2\frac{1}{2}$ seconds.
   (b) The projectile hits the ground after 3 seconds.
   (c) At 1.92 seconds the projectile will be at a height of 33.2 feet.
   (d) The projectile will be at a height of 32 feet at $t = 1$ seconds and $t = 2$ seconds.
   (e) The projectile will be at a height of 36 feet at $t = 1\frac{1}{2}$ seconds. This is different in that it is at that height at only one time, because 36 feet is the highest point.
   (f) The projectile never reaches a height of 40 feet.

10. The price should be set at $6.50.

11. (a) x-intercepts: 0, 2 y-intercepts: 0 (b) x-intercepts: 4 y-intercepts: 2
    (c) x-intercepts: 1 y-intercepts: none (d) x-intercepts: $5, -5$ y-intercepts: $5, -5$
    (e) x-intercepts: 5 y-intercepts: -4 (f) x-intercepts: 2 y-intercepts: -4
    (g) x-intercepts: $-\frac{3}{2}, \frac{3}{2}$ y-intercepts: -3

12. (a) $y = 3x + 10$ (b) $x = 1$ (c) $y = 4x - 1$
    (d) $y = 2x + 5$ (e) $y = -\frac{1}{11}x - \frac{28}{11}$ (f) $y = \frac{5}{2}x$

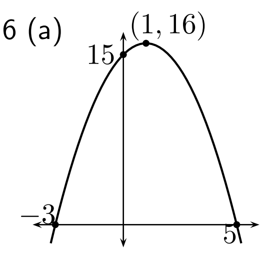
13. (a) $-2y + 7z = 16$ (b) $7y + 3z = -1$ (c) $z = 2$ (d) $y = -1$ (e) $x = 3$

14. (a) $(2, 0, 3)$ (b) $(1, \frac{1}{2}, \frac{9}{2})$ (c) $(4, 3, 2)$ (d) $(-2, 1, 4)$

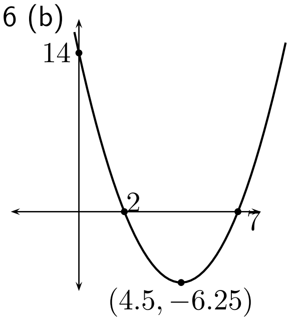
15. downward, $(\frac{3}{2}, \frac{49}{4})$

16. (a) Parabola opening downward, x-intercepts $(-3, 0)$, $(5, 0)$, y-intercept $(0, 15)$, vertex $(1, 16)$. See graph below. The function has an absolute maximum of 16 at $x = 1$.
    (b) Parabola opening upward, x-intercepts $(2, 0)$, $(7, 0)$, y-intercept $(0, 14)$, vertex $(4\frac{1}{2}, -6\frac{1}{4})$. See graph below. The function has an absolute minimum of $-6\frac{1}{4}$ at $x = 4\frac{1}{2}$.
    (c) Parabola opening downward, x-intercepts $(1, 0)$, $(9, 0)$, y-intercept $(0, -2\frac{1}{4})$, vertex $(5, 4)$. See graph below. The function has an absolute maximum of 5 at $x = 4$.
    (d) Parabola opening upward, x-intercept $(-3, 0)$, y-intercept $(0, 4\frac{1}{2})$, vertex $(-3, 0)$. See graph below. The function has an absolute minimum of 0 at $x = -3$.

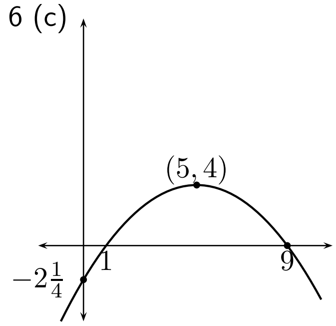
6 (a)



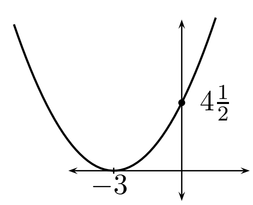
6 (b)



6 (c)



6 (d)

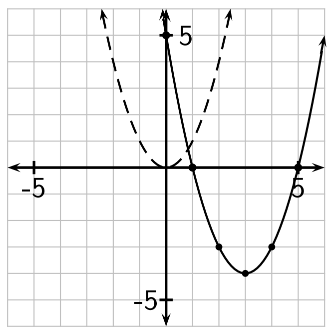


17. (a) $f(x) = x^2 - 6x + 5$

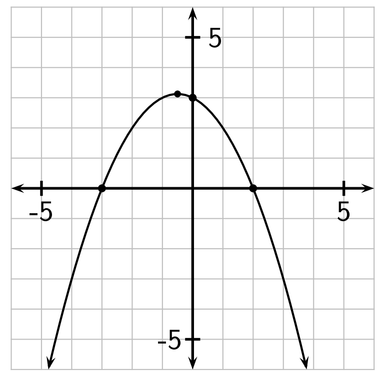
(b) $(3, -4)$ The x-coordinate is the value that makes $(x-3)$ zero, and the y-coordinate is the -4 at the end of the equation.

(c) The x-intercepts are 1 and 5, and the y-intercept is 5.

(d) See below.



Exercise 7(d)



Exercise 8(e)

18. (a) a would be $-\frac{1}{2}$, so the parabola opens down.

(b) The y-intercept is $y = -\frac{1}{2}(0-2)(0+3) = 3$.

(c) The x-intercepts are 2 and -3.

(d) The x-coordinate of the vertex is $x = \dfrac{2+(-3)}{2} = -\dfrac{1}{2}$.

The y-coordinate of the vertex is $y = -\frac{1}{2}(-\frac{1}{2}-2)(-\frac{1}{2}+3) = -\frac{1}{2}(-\frac{5}{2})(\frac{5}{2}) = \frac{25}{8} = 3\frac{1}{8}$.

(e) See above.