Math 111 EXAM 2, SPRING 2018 **Name:** 60 points

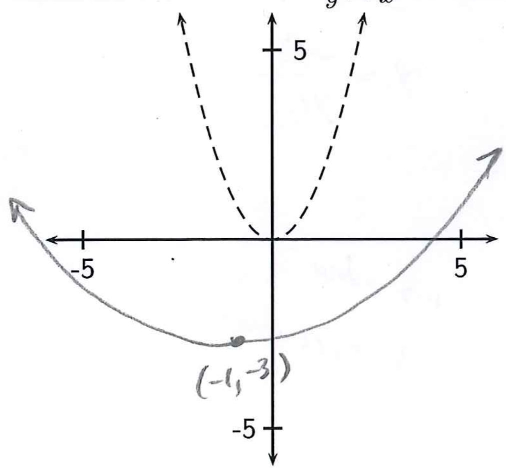
Each numbered exercise is worth six points. Be sure to label all answers to word problems with units.

Do **EXACTLY THREE** of exercises 1 - 4. **Cross out the one you DON'T want me to grade.**

1. **Sketch** the graph of

$$y = \tfrac{1}{2}(x+1)^2 - 3$$

on the coordinate grid to the right. The graph already shown is that of $y = x^2$. **Label the vertex with its coordinates and indicate clearly whether the graph is wider or narrower than that of $y = x^2$.**



$+1\tfrac{1}{2}$ each coord of vertex

$+1\tfrac{1}{2}$ open up

$+1\tfrac{1}{2}$ wider

2. Put $y = -2x^2 - 4x + 3$ into vertex form $y = a(x-h)^2 + k$ **by completing the square.**

$$y = -2(x^2+2x) + 3$$

$$y = -2(x^2+2x+1-1) + 3$$

$$y = -2(x^2+2x+1) + 2 + 3$$

$$\boxed{y = -2(x+1)^2 + 5}$$

2 2 2

or $+1\tfrac{1}{2}$ each + $1\tfrac{1}{2}$ idea

3. Determine the equation of the parabola having vertex $(3, 2)$ and one x-intercept of 1. $y=0$ (Notice that this is an x-intercept, not y-intercept!) Give your answer in any of the three forms of a quadratic function. **Show clearly how you get your answer.**

$$y = a(x-3)^2 + 2$$

$$0 = a(1-3)^2 + 2$$

$$-2 = 4a$$

$$a = -\tfrac{1}{2}$$

$$\boxed{y = -\tfrac{1}{2}(x-3)^2 + 2}$$

$+1\tfrac{1}{2}$ idea/form

$+1\tfrac{1}{2}$ $+1\tfrac{1}{2}$ $+1\tfrac{1}{2}$

4. Lucky Louie's Lemonade Stand is located in Nome, Alaska. Louie hires some consultants who find that Louie's profit is related to the number of cups of lemonade that he sells by the equation $P = -\frac{1}{2}x^2 + 12x - 40$, where x is the number of cups of lemonade sold and P is the profit *in cents*. Fill in the blanks of the sentence below, and **show in the space below how to get the answer in a more efficient way than just substituting various x values in.**

Louie's maximum profit is 32¢, which he gets when he sells 12 cups of lemonade.

$x = \frac{-12}{2(-\frac{1}{2})} = 12$

$P = -\frac{1}{2}(144) + 144 - 40 = 32$

+3 idea

+3 execution

Do **EXACTLY THREE** of exercises 5 - 8. **Cross out the one you DON'T want me to grade.**

5. Consider the function

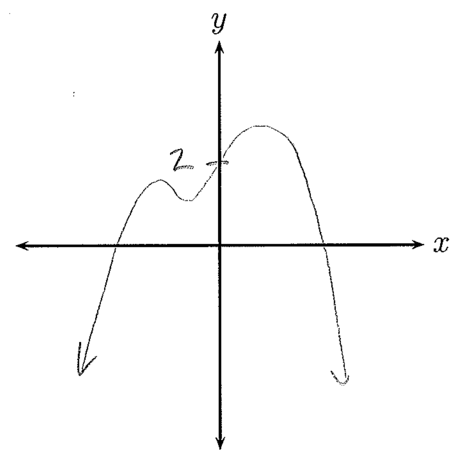
$$P(x) = -3x^4 + ax^3 + bx^2 + cx + 2\,,$$

where b, c and d are unknown constants. On the grid to the right, sketch what its graph might look like, to the best of your knowledge. **Label any intercepts that you can with their value(s).**

+1½ each tail

+1½ 3 turning points

+1½ y-intercept



6. Sketch the graph of

$$P(x) = -\tfrac{1}{2}(x-1)(x-3)(x+2)^2$$

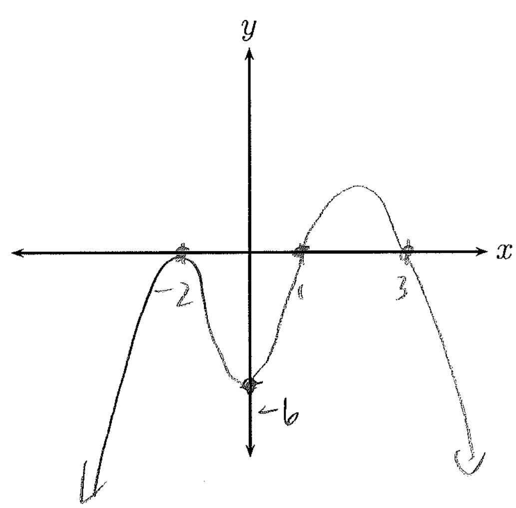
on the grid to the right. Label each intercept (both x and y) with its value.

+1½ x-intercepts

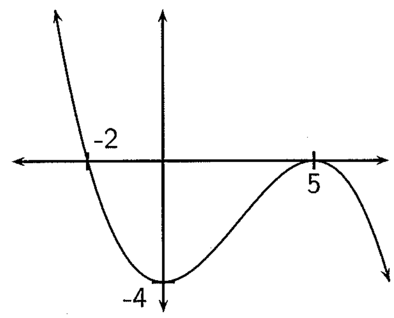
+1½ y-intercept

+1½ bounce

+1½ shape



7. Give the **equation** of a polynomial function (in factored form) that would have a graph like the one shown to the right.



$y = a(x+2)(x-5)^2$

$-4 = a(2)(25)$

$-\frac{4}{50} = a$

x2   x2   x2

$y = -\frac{2}{25}(x+2)(x-5)^2$

8. Fill in the blanks to describe the end behavior of the polygon in the previous exercise.

As $x \to$ $\underline{\ \infty\ }$, $y \to$ $\underline{\ -\infty\ }$ +1½ each blank   As $x \to$ $\underline{\ -\infty\ }$, $y \to$ $\underline{\ \infty\ }$

Do **EXACTLY FOUR** of the remaining exercises, including

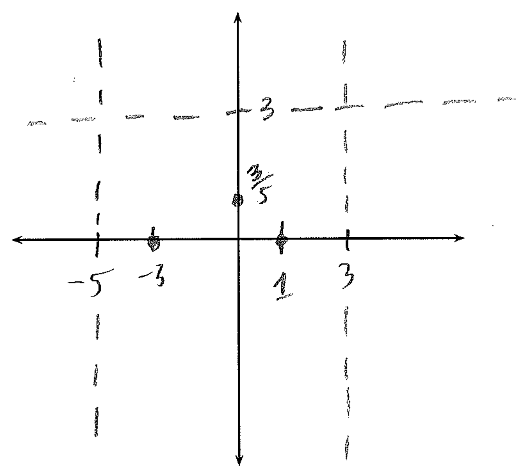
- at least one of Exercises 9 and 10
- at least one of exercises 14, 15, and 16

9. Add each of the following to the graph to the right for the function

$$f(x) = \frac{3x^2+6x-9}{x^2+2x-15} = \frac{3(x+3)(x-1)}{(x+5)(x-3)}.$$

$\frac{-9}{-15} = \frac{3}{5}$

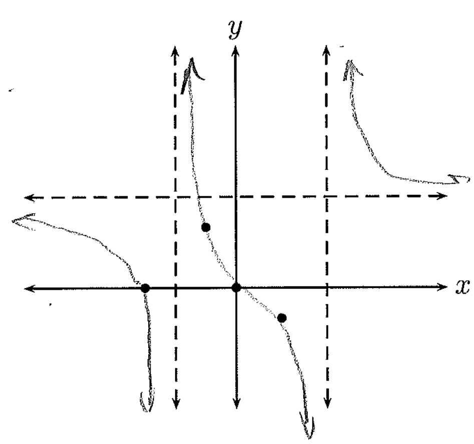
DO NOT graph the function, but label all important values on each axis with its numerical value.



- all x-intercepts
- all y-intercepts
- all vertical asymptotes
- all horizontal asymptotes

+1½ each part

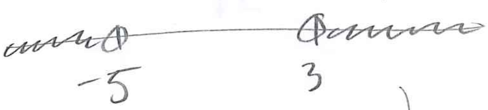
10. A student is graphing a rational function, and they (correctly!) plot the asymptotes and *all* intercepts as shown to the right. They also plot two more points that you can see. Based on this information alone, draw in what you would expect the graph to look like.



11. Solve the inequality $x^2 + 2x > 15$, **showing work indicating how you do it. Give your answer using interval notation.**

$$\left.\begin{aligned} x^2+2x-15&>0 \\ (x+5)(x-3)&>0 \end{aligned}\right\} +2$$

$\downarrow +2$



$\downarrow +2$

$(-\infty,-5)\cup(3,\infty)$

12. Solve the system of equations

$$\begin{aligned} x - y &= 1 \\ y^2 - 2y + x &= 3 \end{aligned}$$

$\rightsquigarrow x = y+1$

$y^2-2y+y+1=3$

$y^2-y-2=0$

$(y-2)(y+1)=0$

$y=-1,2$

$y=-1$: $x=0$ $\boxed{(0,-1)}$

$y=2$: $x=3$ $\boxed{(3,2)}$

+3 for y values

$+1\frac{1}{2}$ for x-values

13. $x^2 + y^2 - 6x + 2y = 39$ is the equation of a circle. Put it in standard form and give the center and radius.

center: $(3,-1)$ radius: 7

$+1\frac{1}{2}$ $+1\frac{1}{2}$

$x^2-6x+9+y^2+2y+1=39+9+1$

+3 $(x-3)^2+(y+1)^2=49$

Do in the space below whichever of the following that you choose to do. If you choose to do more than one and don't have room, use extra paper.

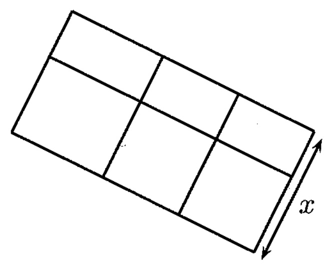
14. The length L of the skid mark left by a car is proportional to the square of the car's speed s at the start of the skid. This means that there is an equation $L = ks^2$ relating L and s, where k is some constant (fixed number). Suppose we know that a car traveling 80 miles per hour leaves a 138 foot skid mark. If another car leaves a skid mark that is 154 feet long, how fast was the car going?

15. A rectangle starts out with a width of 5 inches and a length of 8 inches. At time zero the width starts growing at a constant rate of 2 inches per minute, and the length begins growing by 3 inches per minute.

    (a) Find the area of the rectangle after four minutes, showing clearly how you do it. *2 points*

    (b) Give an equation for the area A as a function of how many minutes t it has been growing. *4 points*

16. A farmer is going to create a rectangular field with 6 compartments, as shown to the right. He has 6000 feet of fence with which to do this. Letting x represent the dimension shown on the diagram, write an equation for the total area A of the field as a function of x. Simplify your equation as much as possible. **Make no assumptions about the length and width of each compartment.**

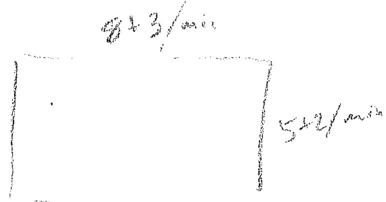


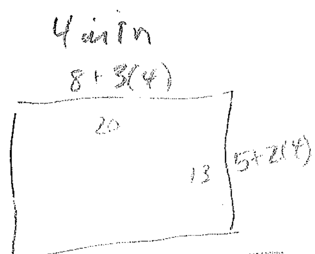
$$A = x\left(\frac{6000-4x}{3}\right) = x\left(2000 - \frac{4}{3}x\right)$$

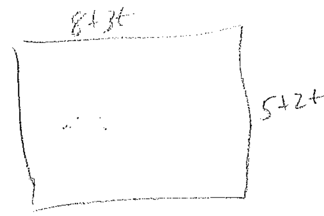
$$\boxed{A = 2000x - \frac{4}{3}x^2}$$

Simplify +1

(15)







a) $\boxed{A = 260 \text{ in}^2}$

b) $\boxed{A = (8+3t)(5+2t)}$

(14)

$138 = k(80)^2$

$k = \frac{138}{80^2} = 0.0216$

$154 = 0.0216 s^2$

$7142 = s^2$

$\boxed{s = 84.5 \text{ mph}}$