Math 251 RECTILINEAR MOTION

Rectilinear motion refers to the movement of an object along a linear path; we'll think of a particle moving back and forth along the number line for a visual. (Assume that the number line is horizontal, with the positive end to the right.) We will be analyzing such motion in the case that the position of the particle at any time is a given function of time. The position is denoted by s, or $s(t)$ to indicate that it is a function of time. *Note that s DOES NOT stand for speed.* An example would be

$$s = -t^3 + 9t^2 - 15t - 4, \qquad t \geq 0$$

We'll use this example to illustrate the following things; assume that the position is in inches and the time is in seconds.

- The position of the particle at any time $t \geq 0$ is obtained by simply evaluating the function for that value of t. The position of the particle at five seconds is $s(5) = -(4)^3+9(4)^2-15(4)-4 = 16$ inches to the right of the origin (zero).
- The derivative function of s is velocity; $v(t) = s'(t) = -3t^2 + 18t - 15$. $v > 0$ means the particle is moving in the positive direction (to the right) and $v < 0$ means the particle is moving in the negative direction (to the left). $v = 0$ means the particle is stopped. Because $v(4) = -3(4)^2 + 18(4) - 15 = 9$ inches per second, the particle is moving nine inches per second to the right at time four seconds
- The speed of the particle is the absolute value of the velocity. The difference between speed and velocity is that velocity tells how fast something is moving AND the direction of motion. Speed only tells how fast it is moving.
- The derivative function of v is acceleration (which is also the second derivative of position): $a(t) = v'(t) = s''(t) = -6t + 18$. At time four the acceleration is $a(4) = -6(4) + 18 = -6$ inches per second, per second. Since it is negative, the acceleration is acting to the left.
- When the acceleration is acting in the same direction as the velocity, the particle is speeding up, and when the acceleration is acting opposite to the velocity the particle is slowing down. For our example, at four seconds the particle is moving to the right since the velocity $v(4)$ is positive, but it is slowing down because the acceleration $a(4) = -6$ is acting to the left.
- There are two main tools used in analyzing rectilinear motion, a sign chart for v and a, and a schematic diagram of the motion. To make the sign charts, do the following.
  - Make two number lines, one directly above the other. Put t just below the right end of each, and put v above the right end of the top one and a just above the right end of the bottom one.
  - Mark the t values where the velocity is zero on the velocity line, putting the t value below each mark and zero above. Above the line and between the zeros, indicate whether the velocity is positive or negative.
  - Repeat for the acceleration.
- A couple of pointers for the schematic diagram are
  - determine the positions of the particle at $t = 0$ and all times that the particle changes direction (use the velocity sign chart),
  - make a position (s) line with a scale that includes all positions just found,
  - make the schematic diagram, and label the motion curve with times for the starting point and all points where the direction changes and all points where the point goes from slowing down to speeding up, or vice-versa.

Math 251 RECTILINEAR MOTION PRACTICE PROBLEM **Do for October 24th class**

For this assignment you will consider a particle whose position s at any time t is given by

$$s = t^3 - 12t^2 + 21t - 9 \qquad \text{for} \qquad t \geq 0$$

Position is in inches and time is in seconds.

1. Give the velocity and acceleration functions.

2. Give the velocity at time two seconds, showing how you got it and by giving a sentence "The velocity at time..." Include units in your sentence, and give the direction as right or left, rather than using a sign.

3. Find the acceleration at time two seconds. Show how you got it and write another sentence stating the acceleration *AND* telling whether the particle is speeding up or slowing down at two seconds and why.

4. Create sign charts for the velocity and acceleration and use them to answer the following:

   (a) Give all times that the particle is stopped.

   (b) Give all time intervals in which the interval is moving to the left. Be sure to consider whether the intervals should be open or closed at their endpoints.

   (c) Give all intervals in which the particle is moving to the right.

   (d) Give all intervals on which the particle is speeding up.

   (e) Give all intervals on which the particle is slowing down.

5. Draw a schematic diagram of the motion of the particle, indicating each point where the velocity or acceleration is zero with a dot labeled with the time the particle is at that point. Include also a dot for the position at time zero. Above the curve and between each pair of dots, write whether the particle is speeding up or slowing down.

6. Find all times greater than zero where the *speed* is three inches per second, round to the nearest hundredth of a second. *You should get FOUR answers.*