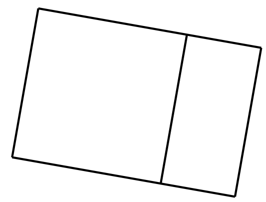
Math 251 ASSIGNMENT 7, FALL 2012

For each of the following, I'll suggest some reading or a video (or two) to watch before attempting each problem. If you feel confident in your abilities, you could try each problem *before* reading or watching, and only do the reading or watching if you need to. **Answers to the exercises that are not on the web are (or will be, by Monday) given at the end of this document**.

- Go to Read or Watch under *Maximizing area of a field. Minimizing fencing cost.* at the **Unit 6** web page. Do the reading OR watch maybe the second video. The first and fourth videos are relevant to this as well, but probably just one is enough.

1. A farmer is going to create a rectangular field with two "compartments", as shown to the right. He has 2400 feet of fence with which to do this. Determine the length and width that give the maximum area, and give the maximum area as well. Give units with all answers.

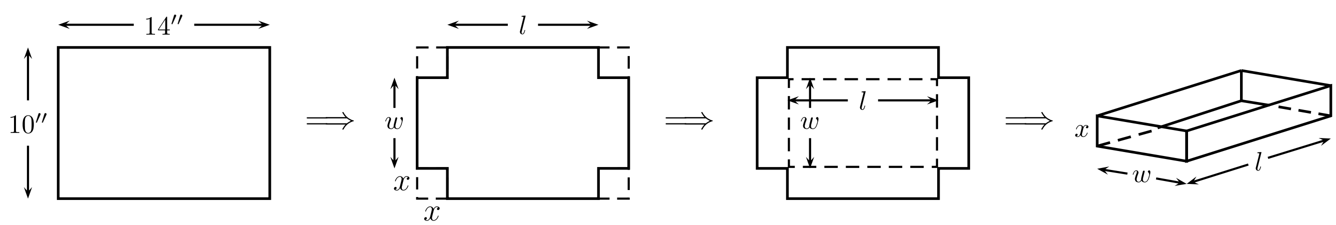


2. Under *Maximizing area of a field. Minimizing fencing cost.*, do the **Web Practice Problem 2** under Do.

- Go to Watch under *Maximizing area of a field. Minimizing fencing cost.*, and watch the third video.

3. You are building a rectangular dog kennel that is to have nicer fence on one side than the other three. The "nice" fence is \$8 per foot, and the other fence is \$5 per foot. The total area of the kennel is to be 240 square feet. Find the dimensions that minimize the cost *to the nearest tenth of a foot*, and tell which dimension the length of nicer fence is. Then give the cost to build the fence.

4. The pictures below show how we can start with a rectangular piece of cardboard that is 10 inches by 14 inches and make an open top box out of it. We first cut a square of side length x out of each corner, as shown in the second picture below. We then fold along the dotted lines to form a box, as shown in the last picture.



   (a) Suppose that $x = 1.5$ inches. What would w and l be in that case? What would the volume V be?

   (b) Find the volume when $x = 1$ and $x = 2$, paying close attention to how you do it.

   (c) What is the relationship between x, l and 14 inches? Write this as an equation. Repeat for x, w and 10 inches.

   (d) Using the fact that the volume of a box is length times width times height and your results from (c), give a formula for the volume V, as a function of x. Test it with your answers to (a) and (b), and "tweak it," if necessary, to make it work.

(e) What are the shortest and longest that x can physically be? Give your answer as an interval in which x can lie. What would the volume be for either of those two values?

(f) If you feel ready to, find the maximum volume possible, and the value of x that results in it. (Find both to the nearest hundredth.) If not, proceed to the bullet below, then come back and do this.

- Go to Read or Watch under *Maximizing volume of a box made by cutting out corners of a sheet.* Do the reading OR watch one of the videos.

5. Under *Maximizing volume of a box made by cutting out corners of a sheet.*, do the **Web Practice Problem 5** under Do,

- Go to Read *Paul's Online Notes Example 3* under *Maximizing volume of a box or can with given surface area. Minimizing surface area of a box or can with given volume.* OR Watch *Web Video 1.* **You may even wish to do both for this topic**.

6. Under *Maximizing volume of a box or can with given surface area. Minimizing surface area of a box or can with given volume.*, do the **Web Practice Problem 3** under Do.

7. An *open-topped* box has a base that is rectangular, with length twice the width. The volume of the box is to be 2400 cubic inches. Determine the dimensions of the box, to the nearest hundredth, that give the minimum surface area. Then give the minimum surface area, to the nearest tenth.

Volume and Surface Area of a Cylinder with radius r and height h:

$$V = \pi r^2 h \qquad\qquad S = 2\pi rh + 2\pi r^2$$

- Go to Read *Paul's Online Notes Example 4* under *Maximizing volume of a box or can with given surface area. Minimizing surface area of a box or can with given volume.* OR Watch *Web Video 2.* **You may even wish to do both for this topic**.

8. Under *Maximizing volume of a box or can with given surface area. Minimizing surface area of a box or can with given volume.*, do the **Web Practice Problem 4** under Do.

9. A cylindrical can is to have a surface area of 2000 cubic centimeters. Find the radius and height, to the nearest hundredth of a centimeter, that give the minimum surface area. Give the minimum surface area, to the nearest whole unit.

- Go to Read *Paul's Online Notes Example 2* under *Minimizing cost of a box or can with given surface area.* OR Watch one or both videos.

10. A box is constructed out of two different types of metal. The metal for the top and bottom, which are both square, costs $\$2/\text{ft}^2$ and the metal for the sides costs $\$3/\text{ft}^2$. Use calculus to find the dimensions that minimize the cost if the total volume is 40 ft^3. Then give the minimum cost for such a box.

11. Under *Minimizing cost of a box or can with given surface area.*, do the **Web Practice Problem 8** under Do.

12. A box with an open top is to have a volume of 10 ft^3. The length of its base is twice the width. Material for the base costs \$10 per square foot, and material for the sides costs \$6 per square foot. Find the cost of materials for the cheapest container, and give the dimensions resulting in that cost.

Answers:

1. The three sections of fence that are the same are 400 feet each, and the other two are 600 feet each. The area is 240,000 square feet.

3. The dimensions are 17.7 feet by 13.6 feet, with the nice side being 13.6 feet long. The cost is \$353.80.

4. (a) $w = 7$ inches, $l = 11$ inches, $V = (11)(7)(1.5) = 115.5$ cubic inches
   (b) $x = 1 \Rightarrow V = (12)(8)(1) = 96$ cubic inches, $x = 2 \Rightarrow V = (10)(6)(2) = 120$ cubic inches
   (c) $2x + l = 14$ or $l = 14 - 2x$, $2x + W = 10$ or $w = 10 - 2x$
   (d) $V = lwh = (14 - 2x)(10 - 2x)x = 4x^3 - 48x^2 + 140x$
   (e) $x = 0$ and $x = 5$, $[0, 5]$, $V = 0$ when $x = 0$ or $x = 5$
   (f) $V' = 12x^2 - 96x + 140$ Setting this equal to zero and solving with the quadratic formula gives $x = 1.92, 6.08$. Only the first of these is in $[0, 5]$, so the maximum volume is attained when $x = 1.92$ inches. The corresponding volume is $V = 120.16$ cubic inches.

7. $V = x(2x)(h) = 2x^2h = 1200$, $S = 2x^2 + 2(2xh) + 2(xh) = 2x^2 + 6xh$. From the first, $h = \frac{1200}{x^2}$, so $S = 2x^2 + \frac{7200}{x}$. Taking the derivative, setting equal to zero and solving gives $x = 12.16$ inches. The box ix then $12.16 \times 24.32 \times 8.12$, and the minimum surface area is 887.8 square inches.

9. $V = \pi r^2 h = 2000 \Rightarrow h = \frac{2000}{\pi r^2}$, $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{4000}{r}$. $S' = 4\pi r - \frac{4000}{r^2} = 0 \Rightarrow r = 6.83$ cm. $h = 13.65$ cm and $S = 879$ square centimeters.

10. $C = 2x^2(2) + 4xh(3) = 4x^2 + 12xh$, $x^2h = 40 \Rightarrow h = \frac{40}{x^2}$, so $C = 4x^2 + \frac{480}{x}$. $C' = 8x - \frac{480}{x^2} = 0$ when $x = 3.91$ feet. $h = 2.62$ feet, and the minimum cost is \$183.91.

12. $C = 2x^2(10) + 2xh(6) + 2(2x)h(6) = 20x^2 + 36xh$. $10 = 2x^2h \Rightarrow h = \frac{5}{x^2}$, so $C = 20x^2 + \frac{180}{x}$. $C' = 40x - \frac{180}{x^2} = 0$ when $x = 1.65$ feet. The dimensions that give the minimum cost are $1.65 \times 3.30 \times 1.84$, and the minimum cost is \$163.54.