Math 251 Practice Exercises, Fall 2012 **October 12th**

For the following, use the following derivative rules. The first four are simply the chain rule as applied to the given functions.

$$(e^u)' = [e^u] \cdot u' \qquad (\sin u)' = [\cos u] \cdot u' \qquad (\cos u)' = [\sin u] \cdot u' \qquad (\ln u)' = \frac{1}{u} \cdot u'$$

Product Rule: $(uv)' = uv' + vu'$ **Quotient Rule:** $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

1. Calculate each derivative; Don't multiply out powers of polynomials, but simplify otherwise. Check your answers at Wolfram Alpha, *http://www.wolframalpha.com/* Look to see whether you entered the original function correctly, and remember that you may need to look down at the alternate forms.

   (a) $y = (x^2 - 5x + 2)^4$ (b) $f(t) = 3\sin(5t - 1)$ (c) $y = 5e^{x^2}$

   (d) $y = \ln(3x + 2)$ (e) $y = \sqrt{x^2 - 5x}$ (f) $y = -\cos(x^3)$

2. Consider the function $y = (3x - 1)^2$.

   (a) Rewrite the function by "foiling" the right side. Then find the derivative.

   (b) Find the derivative using the chain rule, then multiply out your result. You answer should be the same as what you got for (a). If not, find your error(s).

3. Check all of your answers to the following with Wolfram Alpha.

   (a) The function $y = \cos^2 x$ really means $y = (\cos x)^2$. Use this to find the derivative of $y = \cos^2 x$.

   (b) Now consider the function $f(\theta) = \sin^3(4\theta + \frac{\pi}{3})$. If you rewrite is as shown in part (a), you can see that this becomes a "three-layer" chain rule. You must do the derivative of the power first, then the sine function, then the argument of the sine function (that means what it is working on). Try it; you won't be able to check your answer with Wolfram Alpha, as you won't recognize the form it will give you. The answer is at the bottom of this document.

4. Sometimes you'll need to combine the chain rule with the product or quotient rule. Here are some exercises for that:

   (a) $y = (x^2 - 5x)^3(x + 3)^2$ (b) $y = \left(\dfrac{x-2}{x+1}\right)^2$ (c) $y = \frac{1}{x}\cos(3x - 5)$

5. The derivative of $g(x) = \dfrac{(x-1)^7}{5}$ requires the chain rule, *but it does not need the product rule.* Rewrite the function with the division as multiplication by a fraction instead. Then find the derivative.

6. The only trig functions that we have done the derivative of so far are sine and cosine. Let's figure out the derivatives of the others, using these facts:

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \csc u = \frac{1}{\sin u}$$

   (a) Using the above we can write $\sec x = (\cos x)^{-1}$. Use the chain rule to find the derivative of the second expression. Then show how it becomes $-\sec x \tan x$.

   (b) Use the same reasoning to show what the derivative of $\csc x$ is.

7. Use the quotient rule and the derivatives of sine and cosine to determine the derivative of the tangent function. Show clearly how this happens.

The answer to Exercise 3(b) is $f'(\theta) = 12\sin^2(4\theta + \frac{\pi}{2})\cos(4\theta + \frac{\pi}{3})$.