Math 251 Practice Exam 1 F12

(1) a) $\lim_{x\to3} \frac{x^2-6x+9}{x^2-5x+6} = \lim_{x\to3} \frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x-2)} = \lim_{x\to3} \frac{x-3}{x-2} = \frac{0}{1} = 0$

b) $\lim_{x\to\infty} \frac{x^2-6x+9}{x^2-5x+6} = \lim_{x\to\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} 1 = 1$

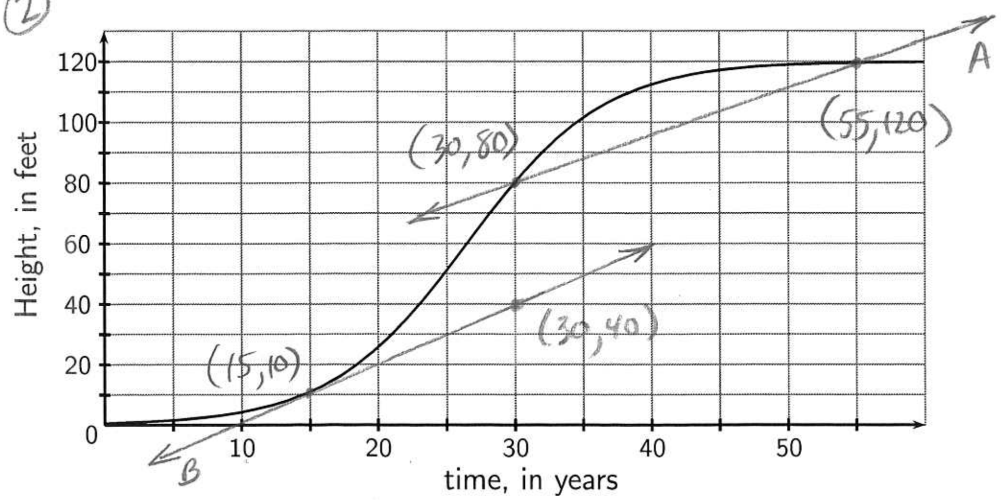
c) $\lim_{x\to3} \frac{x^2-9}{x-3} = \lim_{x\to3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \lim_{x\to3}(x+3) = 6$

d) $\lim_{x\to3} \frac{x+3}{x^2-9} = \lim_{x\to3} \frac{\cancel{x+3}}{\cancel{(x+3)}(x-3)} = \lim_{x\to3} \frac{1}{x-3}$ Hmmm...

$\lim_{x\to3^-} \frac{x+3}{x^2-9} = -\infty$, $\lim_{x\to3^+} \frac{x+3}{x^2-9} = \infty$ so $\lim_{x\to3} \frac{x+3}{x^2-9}$ DNE so

e) $\lim_{x\to\infty} \frac{x+3}{x^2-9} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$

(2)



a) $\frac{120-80}{55-30} = \frac{40}{25} = 1.6 \frac{\text{ft}}{\text{year}}$

b) $\frac{40-10}{30-15} = \frac{30}{15} = 2 \frac{\text{ft}}{\text{year}}$

c) least rapidly at 55, most rapidly at 25

(3) Ave ROC $= \frac{\Delta A}{\Delta t} = \frac{A(5)-A(0)}{5-0} = \frac{48.52-80}{5-0} = -6.3$ mg/hour

(4) $f'(x) = \lim_{h\to0} \frac{[3(x+h)^2-(x+h)]-[3x^2-x]}{h}$

$= \lim_{h\to0} \frac{3(x^2+2xh+h^2)\cancel{-x}-h-3x^2\cancel{+x}}{h}$

$= \lim_{h\to0} \frac{\cancel{3x^2}+6xh+3h^2-h-\cancel{3x^2}}{h}$

$= \lim_{h\to0} \frac{6xh+3h^2-h}{h}$

$= \lim_{h\to0} (6x+3h-1)$

$= 6x-1$

(5) (a) $f'(x)=15x^4-14x+2$

(b) $y=6\sqrt{x}=6x^{\frac{1}{2}}$

$y'=3x^{-\frac{1}{2}}=\frac{3}{\sqrt{x}}$

(c) $g(x)=\frac{4}{x^3}=4x^{-3}$

$g'(x)=-12x^{-4}=-\frac{12}{x^4}$

(6) $y=x^2-5x \Longrightarrow \frac{dy}{dx}=2x-5 \Longrightarrow \left.\frac{dy}{dx}\right|_{x=-1}=2(-1)-5=-7$

(7) $\lim\limits_{x\to 0^-}\frac{\sin(3x)}{|x|}=-3$

x	$\frac{\sin(3x)}{\lvert x\rvert}$
-1	-0.14
$-.1$	$-2.9552\ldots$
$-.01$	$-2.9995\ldots$

(8) $\lim\limits_{x\to -4^+} f(x)=3$ $\quad\lim\limits_{x\to -4} f(x)=3$

$\lim\limits_{x\to -1} f(x)$ DNE $\quad\lim\limits_{x\to -1^-} f(x)=4$

$\lim\limits_{x\to 2} f(x)=3$ $\quad\lim\limits_{x\to 3^+} f(x)=-\infty$

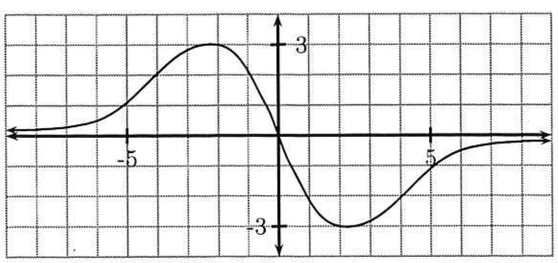
(9) $f(x)=x^2-5x+1$ $\quad f(2)=4-10+1=-5$

$f'(x)=2x-5$ $\quad y=-x+b \Rightarrow -5=-2+b$

$f'(2)=-1$ $\quad b=-3$

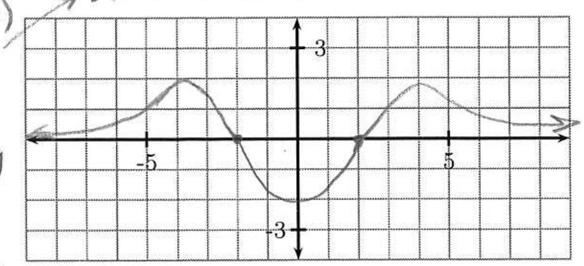
$\boxed{y=-x-3}$

(10)

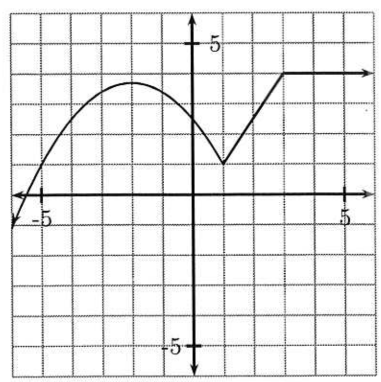


x	$f'(x)$
-8	pos
-2	0
-1	neg
0	neg
1	neg
2	0
3	pos
8	pos, but close to zero

but close to zero



11. The graph below and to the right is for a function $y=g(x)$.



(a) Give the values of the following derivatives at points. For any that do not exist, write DNE in the blank.

$f'(-2)=$ <u>0</u> $\qquad f'(1)=$ <u>DNE</u>

$f'(2)=$ <u>$\frac{3}{2}$</u> $\qquad f'(5)=$ <u>0</u>

(b) Give all intervals on which the derivative is negative. Assume that the behavior of the function continues beyond the edges of the graph as indicated by the arrowheads.

The derivative is negative from $x=-2$ to $x=1$, or on the interval $[-2,1]$