Math 251 Assn 5 $\quad$ 3.10: 12 $\quad$ 4.3: 5, 21, 25, 29, 31 $\quad$ F12

(12) $xy^2 + x^2y^5 - x^3 = 3$ $\qquad$ Product rule for both xy^2 and x^2y^5

$$2xy\frac{dy}{dx} + y^2 + 5x^2y^4\frac{dy}{dx} + 2xy^5 - 3x^2 = 0$$

$$(2xy + 5x^2y)\frac{dy}{dx} = 3x^2 - y^2 - 2xy^5$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2 - y^2 - 2xy^5}{2xy + 5x^2y}}$$

(5) $y = x^3$ on $[-4, 5]$

$$\frac{f(5) - f(-4)}{5 - (-4)} = \frac{5^3 - (-4)^3}{5 - (-4)} = \frac{125 + 64}{9} = \frac{189}{9} = 21$$

$f'(x) = 3x^2 = 21$

$x^2 = 7$

$\boxed{x = \pm\sqrt{7}}$

(21) $$f'(x) = \frac{(x+1)(2x) - x^2}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2} = 0 \text{ when } x = 0, -2$$

$+ + + \; 0 \; - - - \; 0 \; + + +$ $\quad f'(x)$

$\quad -2 \qquad\quad 0 \qquad\quad x$

f has a local maximum at $x = -2$ and a local minimum at $x = 0$

(25) $y = x^3 - 12x^2$

$y' = 3x^2 - 24x = 3x(x - 8)$

$x = 0, 8$ are critical points

$+ + + + \; 0 \; - - - - - - \; 0 \; + + + + +$ $\quad f'(x)$

$\quad 0 \qquad\qquad 8 \qquad x$

$x = 0 \Rightarrow y = 0$

$x = 8 \Rightarrow y = -256$

Increasing on $(-\infty, 0)$ and $(8, \infty)$

Decreasing on $(0, 8)$

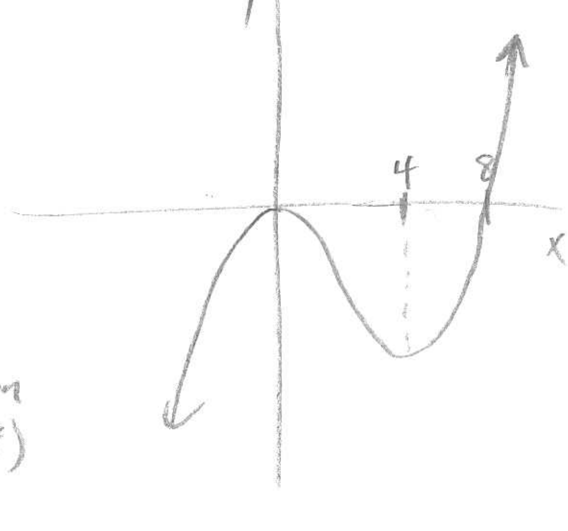
$y'' = 6x - 24 = 6(x - 4)$

$- - - - \; 0 \; + + + + +$

$\qquad 4$

Concave up on $(4, \infty)$

Concave down on $(-\infty, 4)$



Math 251 Assn 5

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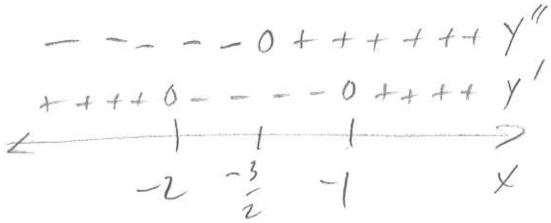


(29) $y = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + 4$

$y' = x^2 + 3x + 2 = (x+2)(x+1)$

$y'' = 2x + 3$

$x = -2: y = -\frac{8}{3} + 6 - 4 + 4 = \frac{10}{3}$

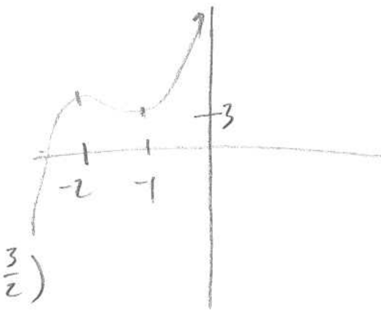
$x = -1: y = -\frac{1}{3} + \frac{3}{2} - 2 + 4 = -\frac{1}{3} + \frac{7}{2} = \frac{21}{6} - \frac{2}{6} = \frac{19}{6}$

The function is increasing on $(-\infty, -2)$ and $(1, \infty)$

The function is decreasing on $(-2, -1)$

There is a local max of $\frac{10}{3}$ at $x = -2$

local min of $\frac{19}{6}$ at $x = -1$

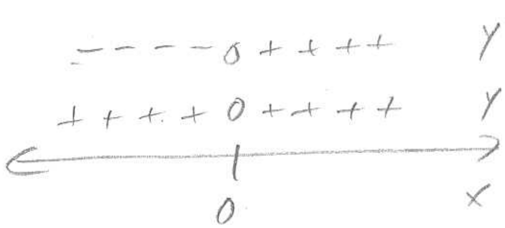


Concave up on $(-\frac{3}{2}, \infty)$, concave down on $(-\infty, -\frac{3}{2})$

(31) $y = x^5 + x^3 + 1$

$y' = 5x^4 + 3x^2 = x^2(5x^2 + 3) = 0$ when $x = 0$

$y'' = 20x^3 + 6x = x(20x^2 + 6) = 0$ when $x = 0$



Increasing on $(-\infty, 0)$ and $(0, \infty)$ Never decreasing

Concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$

No minima or maxima