$$\lim_{x \to \infty} \frac{x}{e^x} = \lim_{x \to \infty} \frac{1}{e^x}$$

$$h(x) = \frac{\sin(2x)}{x}$$

$$\lim_{x \to 0} \frac{\sin(2x)}{x} = \frac{\lim_{x \to 0} \sin(2x)}{\lim_{x \to 0} x}$$

$$\lim_{x \to 0} \frac{\sin(2x)}{x} = \lim_{x \to 0} \frac{2\cos(2x)}{1} = 2$$

$y = \sin(2x)$

$y = x$

$-0.1 \leq x \leq 0.1$

$$\lim_{x\to\infty} \frac{\sin(2x)}{x}$$

$$\lim_{x\to\infty} x = \infty$$

or undefined

or diverges

$$-1 \leq \sin(2x) \leq 1$$

$$-\frac{1}{x} \leq \frac{\sin(2x)}{x} \leq \frac{1}{x}$$

7 b) $\int_1^\infty \frac{1}{x^3}dx = \lim_{R\to\infty}\int_1^R \frac{1}{x^3}dx = \lim_{R\to\infty}\left(-\frac{1}{2R^2}+\frac{1}{2}\right) = \frac{1}{2}$

$\int_1^R \frac{1}{x^3}dx = \int_1^R x^{-3}dx = -\frac{1}{2}x^{-2}\Big|_1^R = -\frac{1}{2R^2}+\frac{1}{2}$

① Write out the first five terms of

$$\sum_{n=0}^{\infty} \frac{1}{3^n} = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$

② Find the first five partial sums in fraction form.

$S_0 = 1 = 1$

$S_1 = \frac{4}{3} = 1.\overline{3}$

$S_2 = \frac{13}{9} = 1.\overline{4}$

$S_3 = \frac{40}{27} = 1.\overline{481}$

$S_4 = \frac{121}{81} = 1.494\ldots$

Conjecture

$$\sum_{n=0}^{\infty} \frac{1}{3^n} = \lim_{n \to \infty} S_n = \frac{3}{2}$$

$$1 \qquad \frac{4}{3} \qquad \frac{13}{9} \qquad \frac{40}{27} \qquad \frac{121}{81}$$

$$S_0 \qquad S_1 \qquad S_2 \qquad S_3 \qquad S_4$$

$$\frac{\frac{9-1}{2}}{3^1} \qquad \frac{\frac{27-1}{2}}{3^2} \qquad \frac{\frac{81-1}{2}}{3^3} \qquad \frac{}{3^4}$$

$$\frac{\overset{3^3}{\cancel{27}}-1}{2\cdot 3^2}$$

$$\boxed{S_n = \frac{3^{n+1}-1}{2\cdot 3^n}}$$

$$\boxed{S_n = \frac{3^{n+1}-1}{2\cdot 3^n}}$$

$$\lim_{n\to\infty} \frac{3^{n+1}-1}{2\cdot 3^n} = \lim_{n\to\infty} \frac{\cancel{3^n}\left(3-\cancel{\frac{1}{3^n}}^{\to 0}\right)}{\cancel{3^n}\left(2\right)} = \frac{3}{2}$$

$$1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\cdots$$

$$1+r+r^2+r^3+r^4+\cdots=\frac{1}{1-r}$$

if $|r|<1$.

$$=\frac{1}{1-\frac{1}{3}}=\frac{1}{\frac{2}{3}}=\frac{3}{2}$$

Geometric series

$$5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\cdots$$

$$=5\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)$$

$$=5\cdot\frac{1}{1-\frac{1}{2}}$$