$$\sum_{n=0}^{\infty} \frac{1}{3^n} \qquad S_n = \frac{3^{n+1}-1}{2\cdot 3^n} = \frac{\cancel{3^n}\left(3-\frac{1}{3^n}\right)}{\cancel{3^n}\cdot 2}$$

$$S_n \longrightarrow \frac{3}{2} \quad \text{as } n \longrightarrow \infty$$

$$\sum_{n=0}^{\infty} \frac{1}{3^n} = \frac{3}{2}$$

$$\sum_{r=0}^{\infty} r^n = 1 + r + r^2 + r^3 + \cdots$$ is a geometric series.

$$S_n = 1 + r + r^2 + \ldots + r^{n-1} + r^n$$

$$r S_n = r + r^2 + r^3 + \ldots + r^n + r^{n+1}$$

$$S_n - r S_n = 1 - r^{n+1}$$

$$S_n(1-r) = 1 - r^{n+1}$$

$$S_n = \frac{1-r^{n+1}}{1-r}$$

$$|r| < 1$$

$$r = \frac{9}{10}$$

$$\left(\frac{9}{10}\right)^{n+1} = \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} \cdots$$

$$\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{5}{3^n} = \frac{5}{3} + \frac{5}{9} + \frac{5}{27} + \frac{5}{81} + \cdots$$

$$= \frac{5}{3}\underbrace{\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots\right)}_{\frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}}$$

$$= \frac{5}{\cancel{3}}\,\frac{\cancel{3}}{2} = \frac{5}{2}$$

$$\sum_{n=1}^{\infty}\left(\frac{5}{3}\right)^n = \frac{5}{3} + \frac{25}{9} + \frac{125}{27} + \frac{625}{81} + \cdots$$

times $\frac{5}{3}$ 〃 〃

Geometric, $r = \frac{5}{3}$ $|r| \geq 1$

diverges

$$\sum_{n=1}^{\infty} \cos(n\pi) = -1+1-1+1-1+\cdots$$

Geometric, $r=-1$ doesn't converge

n	S_n
1	-1
2	0
3	-1
4	0

12 $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{9}{10}\right)^n = -\frac{9}{10}+\frac{81}{100}-\frac{729}{1000}+\cdots$

$$= -\frac{9}{10}\left(1-\frac{9}{10}+\frac{81}{100}+\cdots\right)$$

$$\underbrace{1+r+r^2+\cdots}_{\frac{1}{1-r}}$$

$$= -\frac{9}{10}\left(\frac{10}{19}\right) \qquad \frac{1}{1-r} = \frac{1}{1-\left(-\frac{9}{10}\right)}$$

$$= -\frac{9}{19} \qquad = \frac{1}{1+\frac{9}{10}} = \frac{1}{\frac{19}{10}} = \frac{10}{19}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$ Harmonic Series

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\cdots+\frac{1}{15}+\frac{1}{16}+\cdots$$

$$1+\frac{1}{2}+\underbrace{\frac{1}{4}+\frac{1}{4}}_{\frac{1}{2}}+\underbrace{\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}_{\frac{1}{2}}+\underbrace{\frac{1}{16}+\cdots+\frac{1}{16}+\frac{1}{16}}_{\frac{1}{2}}+\cdots$$

$$a+ar+ar^2+ar^3+\cdots=\frac{a}{1-r}$$

Due Wednesday.

APEX 8.2: 13, 21 find what sums are