① Find $\underbrace{\sum_{n=2}^{\infty} 3\left(\frac{1}{5}\right)^n}_{\rightarrow \text{geometric}} = \underbrace{\frac{3}{25}}_{a} + \frac{3}{125} + \cdots = \frac{\frac{3}{25}}{1-\frac{1}{5}}$

$r = \frac{1}{5}$

$= \frac{3}{25-5}$

$= \frac{3}{20}$

② Find $\sum_{n=2}^{\infty} 3\left(-\frac{1}{5}\right)^n = \frac{3}{25} - \frac{3}{125} + \cdots$

$= \frac{\frac{3}{25}}{1-\left(-\frac{1}{5}\right)}$

$= \frac{\frac{3}{25}}{\frac{6}{5}}$

$= \frac{\cancel{3}^{1}}{\cancel{25}_{5}} \cdot \frac{\cancel{5}}{\cancel{6}_{2}} = \frac{1}{10}$

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \cdots$$

Converges if, and only if, $|r| < 1$

If it converges, converges to $S = \frac{a}{1-r}$

$r = 1$: $7 + 7 + 7 + \cdots$

P if, and only if, Q

Means:

1) If P is true, Q has to be.
2) If Q is true, P has to be.

$$P \Longleftrightarrow Q$$

Have $\sum a_n$.

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.

If P, the Q.

If P is true, Q has to be.

If Q is Q, we don't know about P.

If $\lim_{n\to\infty} a_n = 0$, $\sum a_n$ may converge, or it may diverge.

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n = 2$$

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots = \sum_{n=1}^{\infty}\frac{1}{n} \text{ diverges}$$

Harmonic Series

$$0 < f(x) << g(x) \quad \text{as} \quad x \to \infty$$

$$\frac{f(x)}{g(x)} \to 0$$

$$\lim_{x\to\infty} x^{50} e^{-x} = \lim_{x\to\infty} \frac{50x^{49}}{e^x} = \lim_{x\to\infty} \frac{50!}{e^x} = 0$$

$\sim 3 \times 10^{64}$

$$\lim_{x\to\infty} \frac{3x^2-7x+2}{x^2-5x+1} = \lim_{x\to\infty} \frac{3x^2}{x^2} = \lim_{x\to\infty} 3 = 3$$

$x << x^2$ as $x \to \infty$

As $n \to \infty$

$n^p << n^{p+s} << b^n << n!$ $\quad s, p > 0$

$b > 1$

$\sum_{n=1}^{\infty} \frac{n!}{n^5}$ converges? diverges?

$\lim_{n \to \infty} \frac{n!}{n^5} = \infty$

From Intro to Ser Exer, Part One

Turn In: 4, 5, 7 (Friday)

6 (Challenge (not to turn in))

Suggested: 8