$$\int_4^\infty e^{-3x}\,dx \overset{\text{def}}{=} \lim_{R\to\infty}\int_4^R e^{-3x}\,dx$$

For Monday, find

$$\int_2^\infty \frac{3}{x^5}\,dx$$

$$= \lim_{R\to\infty} -\frac{1}{3}e^{-3x}\Big|_4^R$$

$$= \lim_{R\to\infty}\left(-\frac{1}{3}\underbrace{e^{-3R}}_{\to 0} + \frac{1}{3}e^{-12}\right)$$

$$= \frac{1}{3}e^{-12}$$

8 c) $\left\{ \frac{3e^n - 1}{3e^n} \right\}$

$\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$

$$\lim_{n \to \infty} \frac{3e^n - 1}{3e^n} = \lim_{n \to \infty} \left(1 - \frac{1}{3e^n}\right) = 1$$

$$\sum_{n=1}^{10} \left(\frac{1}{10}\right)^n = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots$$

$$= \frac{1}{10}\left(1 + \frac{1}{10} + \frac{1}{100} + \cdots\right) \qquad r = \frac{1}{10}$$

$$= \frac{1}{10} \cdot \frac{1}{1-\frac{1}{10}}$$

$$= \frac{1}{10} \cdot \frac{10}{9}$$

$$= \frac{1}{9}$$

Factorial

* $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
* $0! = 1$
* $n! = n \cdot (n-1)(n-2) \cdots (3)(2)(1)$
* $x^5 \xrightarrow{\text{der}} 5x^4 \xrightarrow{\text{der}} 5 \cdot 4x^3 \xrightarrow{\text{der}} 5 \cdot 4 \cdot 3x^2 \cdots$

Factorial Tricks

$$\binom{7}{4} = {}_7C_4 = \frac{7!}{4!\cdot 3!}$$

$$= \frac{7\cdot \cancel{6}\cdot 5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{(\cancel{4}\cdot \cancel{3}\cdot 2\cdot 1)(\cancel{3}\cdot \cancel{2}\cdot \cancel{1})} = 35$$

$$\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)\cancel{n}\cancel{(n-1)}\cdots\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}$$

$$= (n+2)(n+1)$$

$$\frac{9!}{7!} = \frac{9\cdot 8\cdot\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdots}{\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdots}$$

$$n^p << n^{p+s} << b^n << n! \quad \text{as } n \to \infty$$

$$s, p > 0$$

$$b > 1$$

$$\sum_{n=3}^{\infty} \frac{1}{\sqrt{n^5}} = \sum_{n=3}^{\infty} \frac{1}{n^{\frac{5}{2}}} \quad C, \text{p-series}, p = \frac{5}{2}$$

$$\sum_{n=1}^{\infty} \frac{5(2)^n}{3^{n+1}} = \sum_{n=1}^{\infty} \frac{5}{3}\left(\frac{2}{3}\right)^n =$$

$$C, \text{geometric}, r = \frac{2}{3}$$

$$\sum_{n=0}^{\infty}\left(\frac{n!}{(n+1)!}\right)^2 = \sum_{n=0}^{\infty}\left(\frac{1}{n+1}\right)^2 = \sum_{n=0}^{\infty}\frac{1}{(n+1)^2}$$

C, p-series, $p=2$

$$\frac{n(n-1)(n-2)\cdots}{(n+1)(n)(n-1)\cdots}$$

$$\sum_{n=1}^{\infty} \frac{n^5}{n!}$$

$$\sum_{n=1}^{\infty} \frac{n!}{n^5} \qquad \lim_{n\to\infty} \frac{n!}{n^5} = \infty$$ The series diverges

nth term test: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^5}{n!} = 0$

$$n^p \ll \quad \ll \quad \ll n!$$

The series may or may not converge.