Consider the two series

$$\sum_{n=0}^{\infty}\left(5+\left(\frac{2}{3}\right)^n\right) \qquad \sum_{n=0}^{\infty}\frac{2^n-5}{3^n}$$

$$\frac{a\pm b}{c}=\frac{a}{c}\pm\frac{b}{c}$$

Does each converge? Diverge? If converges, to what?

$$\sum_{n=0}^{\infty} \overset{a_n}{\left(5+\left(\tfrac{2}{3}\right)^n\right)} = 6 + 5\tfrac{2}{3} + 5\tfrac{4}{9} + \cdots$$

$\lim_{n\to\infty} a_n = 5$

$S_0 = 6$

$S_1 = 11\tfrac{2}{3}$

$\lim_{n\to\infty} S_n = \infty$

$S_2 = 16 + \tfrac{2}{3} + \tfrac{4}{9}$

The series divergences

$$\sum_{n=0}^{\infty} \frac{2^n-5}{3^n} = \frac{-4}{1} + \frac{-3}{3} + \frac{-1}{9} + \frac{3}{27} + \cdots$$

$$\sum_{n=0}^{\infty} \left(\frac{2^n}{3^n} - \frac{5}{3^n}\right) = \sum_{n=0}^{\infty} \frac{2^n}{3^n} - \sum_{n=0}^{\infty} \frac{5}{3^n}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

$$\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$$

$$= \sum_{n=0}^{\infty} \left(\overset{r}{\frac{2}{3}}\right)^n - 5\sum_{n=0}^{\infty} \left(\overset{r}{\frac{1}{3}}\right)^n$$

$$= \frac{1}{1-\frac{2}{3}} - 5 \cdot \frac{1}{1-\frac{1}{3}}$$

$$= 3 - 5 \cdot \frac{3}{2} = 3 - \frac{15}{2} = \boxed{-\frac{9}{2}}$$

$$\sum_{n=1}^{\infty} 3\left(\frac{2}{5}\right)^n = 3\cdot\frac{2}{5} + 3\left(\frac{2}{5}\right)^2 + 3\left(\frac{2}{5}\right)^3 + \cdots$$

$$= 3\cdot\frac{2}{5}\left(1 + \frac{2}{5} + \left(\frac{2}{5}\right)^2 + \cdots\right)$$

$$= \frac{6}{5}\cdot\frac{1}{1-\frac{2}{5}} = \frac{6}{5}\cdot\frac{1}{\frac{3}{5}} = \frac{6}{5}\cdot\frac{5}{3} = 2$$

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r} \quad \text{if } |r| < 1$$

$$\sum_{n=1}^{\infty} 3\left(\frac{2}{5}\right)^n = 3\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n$$

$$= 3\left[\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n - 1\right]$$

$$= 3\left[\frac{1}{1-\frac{2}{5}} - 1\right]$$

$$= 3\left[\frac{5}{3} - 1\right] = 3\cdot\frac{2}{3} = 2$$

$$\sum_{n=3}^{\infty}\frac{(-1)^{n+1}}{n}=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}-\frac{(-1)^{1+1}}{1}-\frac{(-1)^{2+1}}{2}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=\ln 2$$

$$=\ln 2-1+\frac{1}{2}$$

$$=\ln 2-\frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(n+(-1)^n\right)}{n^2} = \frac{(-1)^2\left(1+(-1)^1\right)}{1^2} = \frac{1+(-1)}{1}$$

$$+\frac{(-1)^3\left(2+(-1)^2\right)}{2^2} = \frac{-2-1}{4}$$

$$+\frac{(-1)^4\left(3+(-1)^3\right)}{3^2} = \frac{3-1}{9}$$

$$\sum_{n=1}^{\infty} -\frac{1}{n^2} = -\sum_{n=1}^{\infty}\frac{1}{n^2} = -\frac{\pi^2}{6}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(n+(-1)^n\right)}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n + (-1)^{n+1}(-1)^n}{n^2}$$

odd!
$2n+1$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n + (-1)}{n^2}$$

$$= \sum_{n=1}^{\infty} \left[\frac{(-1)^{n+1} n}{n^2} + \frac{-1}{n^2}\right]$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} - \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$= \ln 2 - \frac{\pi^2}{6}$$

Harmonic Series

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ diverges

Alternating Harmonic Series

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ converges to $\ln 2$

$a_{n+1} = (a_n)$, $a_0 =$ (or $a_1$)

$-2, -1, -\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, \dots$

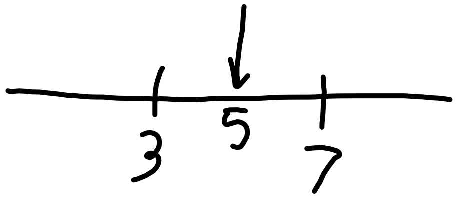
Recursive & Explicit Representations

$7, 3, 7, 3, 7, 3, \dots$

$1, \frac{4}{3}, \frac{9}{5}, \frac{16}{7}, \frac{25}{9}, \dots$

$a_n = \quad , n = \quad , \quad , \dots$

$1, 3, 7, 15, 31, \dots$



Evaluate, using "Important Series"

$$\sum_{n=4}^{\infty} \frac{1}{n^2} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$$