$$\int x(3x^2-7)^5dx = \frac{1}{6}\int 6x(3x^2-7)^5dx$$

$$= \frac{1}{6}\int u^5du$$

Let $u=3x^2-7$

$du=6x\,dx$

$$= \frac{1}{6}\cdot\frac{1}{6}u^6 + C$$

$$= \frac{1}{36}(3x^2-7)^6 + C$$

Integral Test

$\sum_{\boxed{n=1}}^{\infty} a_n$ converges if, and only if, $\int_{\boxed{1}}^{\infty} a(x)\,dx$ converges.

could be another number

n	a_n
1	2
2	4
3	6
4	8
⋮	

$a_n = 2n$

$a(n) = 2n$

$$\sum_{n=2}^{\infty} \frac{n}{\sqrt{3n^2-7}}$$

$u = 3x^2 - 7$
$du = 6x\,dx$

$$\int_2^{\infty} \frac{x}{\sqrt{3x^2-7}}\,dx = \lim_{R\to\infty} \int_2^R x(3x^2-7)^{-\frac{1}{2}}\,dx$$

$$= \lim_{R\to\infty} \frac{1}{6} \int_2^R 6x(3x^2-7)^{-\frac{1}{2}}\,dx$$

$$= \lim_{R\to\infty} \frac{1}{6} \int_2^R u^{-\frac{1}{2}}\,du$$

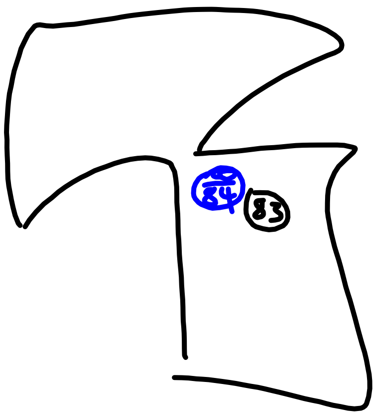
$$= \lim_{R\to\infty} \frac{1}{3} u^{\frac{1}{2}} \Big|_2^R$$

$$= \lim_{R\to\infty} \frac{1}{3}\sqrt{3x^2-7} \Big|_2^R$$

$$= \lim_{R\to\infty} \left(\frac{1}{3}\sqrt{3R^2-7} - \frac{1}{3}\sqrt{5} \right)$$

$$= \infty$$

$\sum_{n=2}^{\infty} \frac{n}{\sqrt{3n^2-7}}$ diverges


84
83

$$\ln n << n^p << n^{p+s} << b^n << n!$$

$$p > 0,\ s >,\ b > 1$$

$$\frac{n}{\sqrt{3n^2-7}} \overset{n\to\infty}{\approx} \frac{n}{\sqrt{3n^2}} = \frac{n}{\sqrt{3}\,n} = \frac{1}{\sqrt{3}}$$

$\sqrt{x^2} = x$ if $x \geq 0$

$$\lim_{n\to\infty} \frac{n}{\sqrt{3n^2-7}} = \frac{1}{\sqrt{3}}$$ informally

$$\sum_{n=1}^{\infty} \frac{2}{3^n+5}$$ behaves like $2\sum \frac{1}{3^n}$ geometric,
as n gets big
$r=\frac{1}{3}$
Converges

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n+1}}$$ like $\sum \sqrt{n}$

$$\sum_{n=2}^{\infty} \frac{n}{\sqrt{n-1}}$$ like $\sum \sqrt{n}$

$$\frac{n}{\sqrt{n}} = \frac{\sqrt{n}\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}}$$

$$\lim_{n\to\infty} \sqrt{n} = \infty$$

diverge, nth-term

Comparison Test

$0 \le a_n \le b_n$ for all n, so $\underbrace{\sum a_n \le \sum b_n}_{\text{start at same } n}$

If $\sum b_n$ converges, then $\sum a_n$ does too.

If $\sum a_n$ diverges, then $\sum b_n$ does too.

$4, -2, 4, -2, 4, \ldots$

$a_{n+1} = a_n$ Recursive

$a_n = n$ Explicit

$a_{n+1} = a_n + 6(-1)^n \quad a_0 = 4$

Consider $\sum_{n=1}^{\infty} \frac{2}{3^n+5}$

Compare with $\sum_{n=1}^{\infty} \frac{2}{3^n}$, which converges because it's geometric, with $r=\frac{1}{3}$.

$$0 < 3^n \leq 3^n+5$$

$$\frac{1}{3^n} \geq \frac{1}{3^n+5}$$

$$\frac{2}{3^n} \geq \frac{2}{3^n+5}$$

$\sum_{n=1}^{\infty} \frac{2}{3^n+5}$ converges by the comparison test.