Comparison Test

$0 < a_n \leq b_n$ for all n (or at least all greater than some N)

$\sum a_n \leq \sum b_n$

If $\sum a_n$ diverges, then $\sum b_n$ diverges

If $\sum b_n$ converges, then $\sum a_n$ converges

$$\sum_{n=2}^{\infty} \frac{n}{\sqrt{n+1}}$$ compare with $\sum \frac{n}{\sqrt{n}} = \sum \sqrt{n}$, which diverges, n-th term

$$n+1 \geq n$$

$$\sqrt{n+1} \geq \sqrt{n}$$

$$\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$$

$$\frac{n}{\sqrt{n+1}} \leq \frac{n}{\sqrt{n}} \rightarrow$$ Inconclusive!

$\sum_{n=2}^{\infty} \frac{n}{\sqrt{n-1}}$ compare with $\sum \frac{n}{\sqrt{n}} = \sum \sqrt{n}$,
which diverges, n-th term

$n-1 \leq n$

$\sqrt{n-1} \leq \sqrt{n}$

$\frac{1}{\sqrt{n-1}} \geq \frac{1}{\sqrt{n}}$

$\frac{n}{\sqrt{n-1}} \geq \frac{n}{\sqrt{n}}$ $\rightarrow$ $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n-1}}$ diverges by comparison test

$$c \ll \ln n \ll n^p \ll n^{p+s} \ll b^n \ll n!$$

as $n \to \infty$, $p, s > 0$, $b > 1$

constant

$$\sum_{n=1}^{\infty} \frac{2}{3^n - n}$$ compare with $\sum \frac{2}{3^n}$, which converges, geometric, $r = \frac{1}{3}$

$$\frac{2}{3}+\frac{2}{9}+\cdots$$
$$\frac{2}{3}\left(1+\frac{1}{3}+\frac{1}{9}+\right)$$
$$\frac{2}{3}\cdot\frac{1}{1-\frac{1}{3}} = 1$$

$$\lim_{n\to\infty} \frac{\frac{2}{3^n-n}}{\frac{2}{3^n}} = \lim_{n\to\infty} \frac{3^n}{3^n-n}$$ algebra

$$= \lim_{n\to\infty} \frac{3^n}{3^n}$$ $n << 3^n$

$$= \lim_{n\to\infty} 1 = 1$$ algebra, lim of a constant is the constant

$$\sum_{n=2}^{\infty} \frac{2}{3^n-n}$$ converges

$$3^n - n \le 3^n$$

$$\frac{1}{3^n - n} \ge \frac{1}{3^n}$$

$$\frac{2}{3^n - n} \ge \frac{2}{3^n}$$

$\sum a_n$ converges if, and only if,
$\sum b_n$ converges

Conditional statement:

If P, then Q. $P \rightarrow Q$

Can't say If Q, then P.

If not Q, then not P.

Biconditional Statement

If $P \to Q$ and $Q \to P$, we write $P \longleftrightarrow Q$

not

$P \to Q$, $\sim Q \to \sim P$

$Q \to P$, $\sim P \to \sim Q$

$\sim P \longleftrightarrow \sim Q$

P if, and only if, Q.

$\sum a_n$ converges if, and only if, $\sum b_n$ converges.

$$\lim_{n\to\infty} \frac{\frac{2}{3^n - n}}{\frac{2}{3^n}} \xrightarrow[\text{algebra}]{} = \lim_{n\to\infty} \frac{3}{3^n - n} \xrightarrow[n << 3^n]{} = \lim_{n\to\infty} \frac{3^n}{3^n} = \dots$$

Add to Assignment 7:

#11. Use the limit comparison test on the series from #2.