

① Note that $n! = n(n-1)(n-2)(n-3)(n-4)\dots(3)(2)(1)$

Expand each in the same way:

$$(n+1)! \quad (2n+1)! \quad (2(n+1))!$$

② Simplify each: $\frac{(n+1)!}{n!}$ $\frac{(2n)!}{(2(n+1))!}$

③ Use your calculator to speculate what $\lim_{n \rightarrow \infty} n^{\frac{1}{n}}$ is.

$$\frac{(2n)!}{(2(n+1))!} = \frac{(2n)(2n-1)(2n-2) \dots}{(2n+2)(2n+1)(2n)(2n-1) \dots}$$

$$= \frac{1}{(2n+2)(2n+1)}$$

$$\lim_{n \rightarrow \infty} n^{\frac{1}{n}} = 1$$

Ratio test good when
 a_n contains !

Root test good when a_n
contains b^n .

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

nth-term

$$\lim_{n \rightarrow \infty} \frac{2^n}{n!} = 0 \quad \text{CT}$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \frac{2}{n+1}$$

$$\frac{a_{n+1}}{a_n} = a_{n+1} \cdot \frac{1}{a_n}$$

optional

$$\lim_{n \rightarrow \infty} \frac{2}{n+1} = 0, \text{ so } \sum_{n=1}^{\infty} \frac{2^n}{n!} \text{ converges}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1} \quad \left(\frac{1}{n^2}\right) \quad \text{Think it converges}$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 1, \text{ so ratio test is inconclusive}$$

$$n^2+1 \geq n^2$$

$$\frac{1}{n^2+1} \leq \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2.7^{2n-1}}{(2n-1)!} = \frac{2.7^1}{1!} - \frac{2.7^3}{3!} + \frac{2.7^5}{5!}$$

Find S_1, S_2, S_3

$$\left. \begin{array}{l} S_1 = 2.7 \\ S_2 = -.58 \\ S_3 = 0.62 \end{array} \right\} - \frac{2.7^7}{7!}$$

$$S_4 = 0.41$$