$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(2.7)^{2n-1}}{(2n-1)!} = \frac{2.7^{1}}{1!} - \frac{(2.7)^3}{3!} + \frac{(2.7)^5}{5!} - \frac{(2.7)^7}{7!} + \cdots = \sin 2.7$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

Integral test

* integrate wrt x, not n
* $\int \quad dx$
* $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} a(n) \neq \int_1^{\infty} a(x)\,dx$

a_n contains $\bigcirc^n$, try root test

$$\sum_{n=1}^{\infty}\left(\frac{3n+1}{5n-2}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{3n+1}{5n-2}\right)^n} = \lim_{n\to\infty}\frac{3n+1}{5n-2} = \lim_{n\to\infty}\frac{3\cancel{n}}{5\cancel{n}} = \frac{3}{5}$$

The series converges

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2} \quad \text{using nth root test}$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^2}} = \lim_{n\to\infty} \frac{2}{\sqrt[n]{n^2}} = \lim_{n\to\infty} \frac{2}{\left(n^{\frac{1}{n}}\right)^2} = 2$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \qquad \sqrt[n]{n^2} = \left(n^2\right)^{\frac{1}{n}} = \left(n^{\frac{1}{n}}\right)^2 \qquad \lim_{n\to\infty} n^{\frac{1}{n}} = 1$$

The series diverges.

$\int$, comparison, lim comp, ratio, root all apply to $\sum a_n$, a_n positive

Alternating Series

$$\sum (-1)^n a_n \text{ or } \sum (-1)^{n+1} a_n \quad a_n \text{ positive}$$

Alternating series test

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

a_n $\frac{1}{n}$ decreasing? $a_{n+1} \le a_n$

$n+1 \ge n$ $\frac{1}{n+1} \overset{?}{\le} \frac{1}{n}$

$\frac{1}{n+1} \le \frac{1}{n}$ Yes, $\frac{1}{n}$ is decreasing

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

The series converges

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

Does $\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n} \right|$ converge? If so, then we say $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges absolutely.

$|ab| = |a||b|$

$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$, which diverges.

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ does not converge absolutely, it is conditionally convergent.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$$

Converges absolutely? $\sum_{n=0}^{\infty} \left| \left(-\frac{1}{2}\right)^n \right| = \underbrace{\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n}_{\text{Converges, geometric, } r=\frac{1}{2}}$

Yes

implies that $\sum \left(-\frac{1}{2}\right)^n$ converges

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\dots=\ln 2$$

conditionally convergent

<u>Turn in by Friday:</u>

8.4: 17

8.5: 5, 15, 19

absolute convergence implies convergence