Exams Other stuff

$$\frac{372}{450} \text{ "+" } \frac{112}{112} = \frac{484}{562}$$

$$\frac{372}{450} \text{ "+" } \frac{23}{23} = \frac{395}{473}$$

Consider the sequence $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$

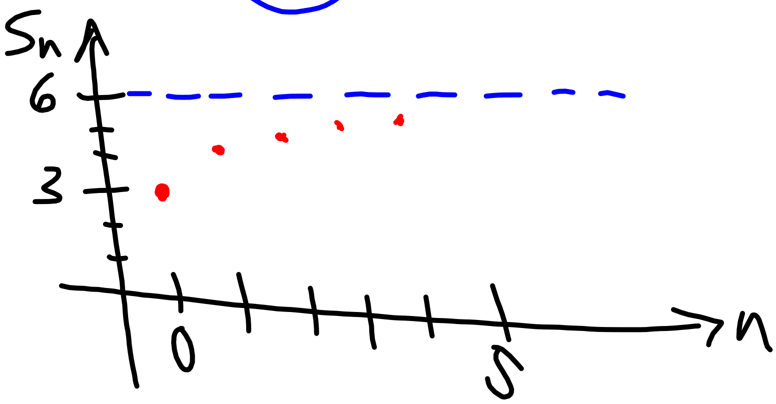
① Give a recursive def of the seq.

$a_0 = 3, \quad a_{n+1} = \frac{1}{2} a_n$

② Give an explicit " " "

$a_n = 3\left(\frac{1}{2}\right)^n, \quad n = 0, 1, 2, \ldots$

③ Graph the first five partial sums of $3 + \frac{3}{2} + \frac{3}{4} + \cdots$. You may find them as decimals.



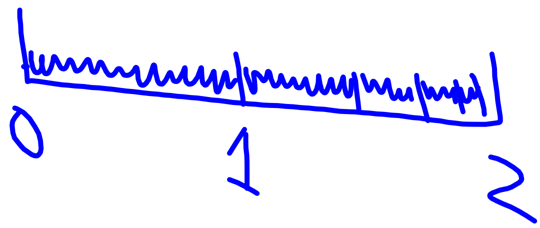
n	a_n	S_n
0	3	3
1	$\frac{3}{2}$	4.5
2	$\frac{3}{4}$	5.25
3	$\frac{3}{8}$	5.625
4	$\frac{3}{16}$	5.8125

$$3+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\dots=\sum_{n=0}^{\infty}3\left(\frac{1}{2}\right)^n=6$$

$$=3\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots\right)$$

$$\text{"="}\,3(2)$$

$$=6$$



Limits

① Try "plugging in"

$$\lim_{x\to\infty} \frac{x^2-3x}{x^2+5x} = \cancel{\left(\frac{\lim_{x\to\infty} x^2-3x}{\lim_{x\to\infty} x^2+5x}\right)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1-\frac{3}{x}}{1+\frac{5}{x}}$$

$$= \frac{\lim_{x\to\infty} 1-\frac{3}{x}}{\lim_{x\to\infty} 1+\frac{5}{x}}$$

$$= 1$$

$$\lim_{x \to 0} \frac{x^2-3x}{x^2+5x} = \lim_{\substack{x \to 0 \\ (x \neq 0)}} \frac{x(x-3)}{x(x+5)}$$

$$= \lim_{x \to 0} \frac{x-3}{x+5}$$

$$= -\frac{3}{5}$$

$$\lim_{x \to 0} \frac{\overset{0}{x^2 - 3x}}{\underset{0}{x^2 + 5x}} = \lim_{x \to 0} \frac{2x - 3}{2x + 5} = \frac{-3}{5}$$

$$\lim_{x \to \infty} \frac{\overset{\infty}{x^2 - 3x}}{\underset{\infty}{x^2 + 5x}} = \lim_{x \to \infty} \frac{\overset{\infty}{2x - 3}}{\underset{\infty}{2x + 5}} = \lim_{x \to \infty} \frac{2}{2} = 1$$

$$\lim_{x\to\infty} \underbrace{e^{-2x}\sin 3x}_{f(x)}$$

$$\underbrace{-e^{-2x}}_{p(x)} \le f(x) \le \underbrace{e^{-2x}}_{q(x)}$$

$$\lim_{x\to\infty} p(x) \le \lim_{x\to\infty} f(x) \le \lim_{x\to\infty} g(x)$$

$$0 \le \quad 0 \quad \le 0$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad \text{if } n \neq -1$$

improper integrals

$$\int_0^1 \frac{1}{\sqrt{x}} dx = \lim_{R \to 0^+} \int_R^1 x^{-\frac{1}{2}} dx = \lim_{R \to 0^+} 2x^{\frac{1}{2}} \Big|_R^1$$

$$= \lim_{R \to 0^+} \left[2 - 2R^{\frac{1}{2}} \right]$$

$$= 2$$

$$\int_1^{\infty} \frac{1}{x^2} dx$$