Find $\int_1^\infty \frac{1}{x^3}\,dx$.

$$\int_1^b \frac{1}{x^3}\,dx = \int_1^b x^{-3}\,dx = -\frac{1}{2}x^{-2}\Big|_1^b = -\frac{1}{2}b^{-2} - \left(-\frac{1}{2}(1)^{-2}\right)$$

$$= -\frac{1}{2b^2} + \frac{1}{2}$$

$$\int_1^\infty \frac{1}{x^3}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^3}\,dx = \lim_{b\to\infty}\left(-\frac{1}{2b^2} + \frac{1}{2}\right) = \frac{1}{2}$$

($\frac{1}{2b^2} \to 0$)

Give an explicit formula for each sequence, starting at n=0 or n=1.

I. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \ldots$ $a_n = \frac{(-1)^{n-1}}{n^2}$, $n = 1, 2, \ldots$

II. $5, 7, 9, 11, \ldots$ $a_n = 5 + 2n$, $n = 0, 1, 2, 3, 4, \ldots$

III. $\frac{2}{3}, \frac{8}{9}, \frac{26}{27}, \frac{80}{81}, \ldots$ $a_n = \frac{3^n - 1}{3^n}$, $n = 1, 2, 3, \ldots$

IV. $\frac{2}{3}, -\frac{8}{9}, \frac{26}{27}, -\frac{80}{81}, \ldots$ $a_n = (-1)^{n+1} \frac{3^n - 1}{3^n}$ $n = 1, 2, 3, \ldots$

* Alternating signs : $(-1)^{\square}$
* Evens $2n$, $2(n+1)$
* Odds $2n+1$, $2n-1$

I. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \ldots$

II. $5, 7, 9, 11, \ldots$

III. $\frac{2}{3}, \frac{8}{9}, \frac{26}{27}, \frac{80}{81}, \ldots$

IV. $\frac{2}{3}, -\frac{8}{9}, \frac{26}{27}, -\frac{80}{81}, \ldots$

V. $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, \ldots$

Increasing: II, III

Non-decreasing: II, III, V

Decreasing: none

Monotonic: II, III, V

Bounded, and bound:

I, $M = 1$

III, IV, $M = 1$

I. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \ldots$ $L = 0$

II. $\{5, 7, 9, 11, \ldots\}$ diverges

III. $\frac{2}{3}, \frac{8}{9}, \frac{26}{27}, \frac{80}{81}, \ldots$ $L = 1$

IV. $\frac{2}{3}, -\frac{8}{9}, \frac{26}{27}, -\frac{80}{81}, \ldots$ diverges

V. $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, \ldots$ diverges

Limits? $\left\{\frac{n^3}{n^4+1}\right\}, \left\{\frac{n^8}{3n^8+5}\right\}$

$$\lim_{x\to\infty} \frac{x^3}{x^4+1} = \lim_{x\to\infty} \frac{\frac{x^3}{x^4}}{\frac{x^4}{x^4}+\frac{1}{x^4}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{1}{x^4}} = 0$$

OK