The sixth degree Mclaurin polynomial for $\cos x$ is

$$p_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$$

a) Use to approximate $\cos 1$ to 5 places past the decimal. $\cos 1 \approx 0.54028$

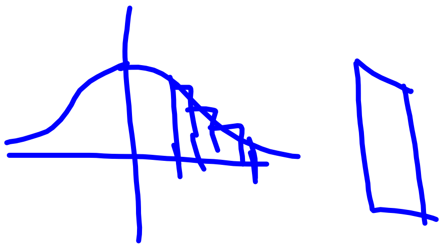
b) Find $\cos 1$ to 5 places past the decimal.

$\cos x = 0.54030$

c) How far off is the approximation from the actual value?

$\text{error} = |\text{approx} - \text{actual}| = 0.00002$

$$\int_1^2 e^{-x^2}\,dx = \Big|_1^2$$



$f(x)$, create Taylor poly deg n

centered at c

$$\cancel{P_n(x)}\ f(x) = f(c) + \frac{f'(c)}{1!}(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \underbrace{R_n(x)}_{\text{remainder}}$$

$$R_n(x) = \frac{f^{(n+1)}(z_x)}{(n+1)!}(x-c)^{n+1}$$

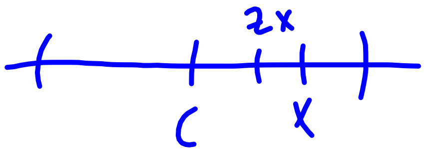
z_x is a number btwn x and c

$$\text{error} = \left|p_n(x) - f(x)\right| = \left|R_n(x)\right|$$

$$= \left|\frac{f^{(n+1)}(z_x)}{(n+1)!}(x-c)^{n+1}\right|$$

$$= \frac{\left|f^{(n+1)}(z_x)\right|}{(n+1)!}\left|x-c\right|^{n+1}$$

Assume $c < z_x < x$



$$\text{error bound} = \frac{M}{(n+1)!}\left|x-c\right|^{n+1}$$

M is the maximum of $\left|f^{(n+1)}(z)\right|$ for z between x and c (including x and c)

or M is any larger value.

Find error bound when using the 6th deg Mclaurin series for cos x to approximate cos 1.

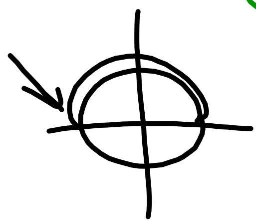
(centered at 0)

$$\text{error bound} = \frac{|f^{(n+1)}(z)|}{(6+1)!}|1-0|^{6+1} \leq \frac{1}{7!} 1$$

$$= 1.98 \times 10^{-4}$$

$$= 0.000198$$

Find $\underset{=n}{4\text{th}}$ deg T.P. for cos @ $\underset{=c}{3\pi}$



$$f(x) = \cos x \rightarrow f(3\pi) = -1$$
$$f'(x) = -\sin x \rightarrow f'(3\pi) = 0$$
$$f''(x) = -\cos x \rightarrow f''(3\pi) = 1$$
$$f'''(x) = \sin x \rightarrow f'''(3\pi) = 0$$
$$f^{(4)}(x) = \cos x \rightarrow f^{(4)}(3\pi) = -1$$

$$p_4(x) = -1 + \frac{1}{2!}(x-3\pi)^2 - \frac{1}{4!}(x-3\pi)^4$$

$p_4(10) \doteq 0.83912$

$\cos 10 \doteq -0.83907$

4th deg, centered @ 3π, $\cos x$ at $x=10$

$$\text{error bound} = \frac{|f^{(n+1)}(z_x)|}{(n+1)!}|x-c|^{n+1}$$

$$\leq \frac{1}{5!}|10-3\pi|^5 = 0.000525$$