Find the 4th degree Mclaurin polynomials for $f(x)=e^{2x}$ and $g(x)=\frac{3}{x-5}$.

$f(x)=e^{2x} \rightarrow f(0)=1$
$f'(x)=2e^{2x} \rightarrow f'(0)=2$
$f''(x)=4e^{2x} \rightarrow f''(0)=4$
$f'''(x)=8e^{2x} \rightarrow f'''(0)=8$
$f^{(4)}(x)=16e^{2x} \rightarrow f^{(4)}(0)=16$

$p_4(x)=1+2x+\frac{4}{2!}x^2+\underbrace{\frac{8}{3!}x^3}_{n=3\text{ term}}+\frac{16}{4!}x^4$

Mclaurin series for e^{2x}

$\sum_{n=0}^{\infty}\frac{2^n}{n!}x^n$

May 15-9:54 AM

$g(x)=\frac{3}{x-5}$

$g(x)=3(x-5)^{-1} \Rightarrow g(0)=-\frac{3}{5}$
$g'(x)=-3(x-5)^{-2} \Rightarrow g'(0)=-\frac{3}{5^2}$
$g''(x)=6(x-5)^{-3} \Rightarrow g''(0)=-\frac{6}{5^3}$
$g'''(x)=-18(x-5)^{-4} \Rightarrow g'''=-\frac{18}{5^4}$
$g^{(4)}(x)=72(x-5)^{-5} \Rightarrow =-\frac{72}{5^5}$

3, 3, 6, 18, 72
1, 1, 2, 6, 24
0! 1!

$p_4(x)=-\frac{3}{5}-\frac{3}{5^2}x-\frac{6}{5^3(2!)}x^2-\frac{18}{5^4(3!)}x^3-\frac{72}{5^5(4!)}x^4$

$p_4(4)=-\frac{3}{5}\left(1+\frac{1}{5}x+\frac{2!}{5^2 2!}x^2+\frac{3!}{5^3 3!}x^3+\frac{4!}{5^4 4!}x^4\right)$

$=-\frac{3}{5}\left(1+\frac{1}{5}x+\frac{1}{5^2}x^2+\frac{1}{5^3}x^3+\frac{1}{5^4}x^4\right)$ $r=\frac{1}{5}x$

May 15-10:02 AM

Creating Mclaurin/Taylor series
* with "bare hands" from def (last resort)
* use alg and/or calculus with known series
* put in form $\frac{1}{1-r}$ for geometric series using algebraic "trickery"

May 15-10:19 AM

$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=\sum_{n=0}^{\infty}\frac{1}{n!}x^n$

Mclaurin Series for e^{2x}?

Substitution: $e^{2x}=\sum_{n=0}^{\infty}\frac{1}{n!}(2x)^n=\sum_{n=0}^{\infty}\frac{2^n}{n!}x^n$

May 15-10:06 AM

Mclaurin Series for $f(x)=\frac{3}{x-5}$ make look like $\frac{1}{1-x}$ x in here

$\frac{3}{x-5}=3\cdot\frac{1}{x-5}$

$=-3\cdot\frac{1}{5-x}$

$=-\frac{3}{5}\cdot\frac{1}{1-\frac{1}{5}x}$ geometric, $r=\frac{1}{5}x$ converges if $|\frac{1}{5}x|<1$

$=-\frac{3}{5}\left(1+\frac{1}{5}x+\frac{1}{5^2}x^2+\frac{1}{5^3}x^3+\dots\right)$ if $|x|<5$

$=-\frac{3}{5}\sum_{n=0}^{\infty}\frac{1}{5^n}x^n$ ROC $=5$

IOC $=(-5,5)$

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$\frac{1}{x+2}=\frac{1}{2+x}$ $\frac{1}{1-x}$

$=\frac{1}{2}\cdot\frac{1}{1+\frac{1}{2}x}$

$=\frac{1}{2}\cdot\frac{1}{1-\left(-\frac{1}{2}x\right)}$

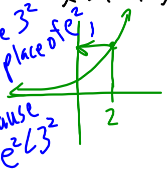
etc.

May 15-10:33 AM

Approximate e^2 from the 3rd deg Mclaurin polynomial. What is the error bound?

error bound $= \frac{M}{(n+1)!}|x-c|^{n+1}$ M is max of $|f^{(n+1)}(z_x)|$ for z_x btwn x and c

error bound $= \frac{M}{4!}|2-0|^4$ Use 3^2 in place of e^2



because $e^2 < 3^2$

$= \frac{9}{4!}2^4$

$= \frac{\cancel{9}^{3} \cdot \cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1}$

$= 6$

May 15-10:36 AM

$\sum_{n=0}^{\infty} n!\left(\frac{x}{10}\right)^n$ ROC? IOC?

$\lim_{n\to\infty} \frac{\left|\frac{(n+1)!\,x^{n+1}}{10^{n+1}}\right|}{\left|\frac{n!\,x^n}{10^n}\right|} = \lim_{n\to\infty} \frac{n+1}{10}|x| = \cancel{\text{something}}\ \infty \not< 1$ unless $x=0$

< 1 for convergence

The series converges only at $x=0$.

May 15-10:44 AM

Add to assignment:

Series for $\frac{5}{x-2}$ using alg "trickery"

May 15-10:49 AM