Find the ROC and IOC for each:

$$\sum_{n=0}^{\infty} \frac{n!}{2^n} x^n \qquad \sum_{n=0}^{\infty} \frac{2^n}{n!} x^n$$

$$\lim_{n\to\infty} \frac{\left|\frac{(n+1)!\,x^{n+1}}{2^{n+1}}\right|}{\left|\frac{n!\,x^n}{2^n}\right|} = \lim_{n\to\infty} \frac{n+1}{2}|x| = \infty \not< 1$$

if $x \neq 0$

ROC $= 0$ IOC is $[0,0]$

For other, $\lim_{n\to\infty} \frac{|\,|}{|0|} = 0 < 1$ for all x ROC is infinite

IOC is $(-\infty, \infty)$

OR $\mathbb{R}$

$$7-14(x+1)+28(x+1)^2-56(x+1)^3+\cdots$$

$$1-2+4-8+16+\cdots$$ geometric $r=-2$

$\times -2$ $\times -2$ $\times -2$

$$7-14+28-56+\cdots \quad r=-2$$

$\times -2$ $\times -2$

Geometric

$r=-2(x+1)$

$f(x) = 7 - 14(x+1) + 28(x+1)^2 - 56(x+1)^3 + \cdots$

geometric, $r = -2(x+1)$, converges if $|-2(x+1)| < 1$

$|-2||x+1| < 1$

$|x+1| < \frac{1}{2}$

$|x-(-1)| < \frac{1}{2}$ → x is within $\frac{1}{2}$ of $c = -1$

ROC

$\left(-\frac{3}{2}, -\frac{1}{2}\right)$ IOC

$|x-3| < 5$

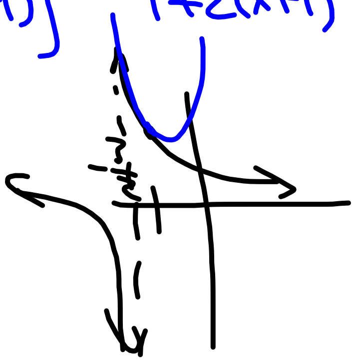
$7-14(x+1)+28(x+1)^2-56(x+1)^3+\cdots$

Geometric, $r=-2(x+1)$ $S=\frac{1}{1-r}$ if $|r|<1$

$f(x)=7\frac{1}{1-[-2(x+1)]}=7\frac{1}{1+2(x+1)}=\frac{7}{2x+3}$

$2x+3=0$
$2x=-3$
$x=-\frac{3}{2}$

same



$$7-14(x+1)+28(x+1)^2-56(x+1)^3+\cdots$$

$x=-\frac{1}{2}$: $7-14\left(\frac{1}{2}\right)+28\left(\frac{1}{4}\right)$

$7-7+7-7+\cdots$ doesn't converge

$x=-\frac{3}{2}$: $7-14\left(-\frac{1}{2}\right)+28\left(-\frac{1}{2}\right)^2-56\left(-\frac{1}{2}\right)^3+\cdots$

$7+7+7+\cdots$

$$h(x) = \frac{3}{7-x} \quad \text{centered at } c = 2$$

$$= \frac{3}{7-x+2-2}$$

$$\cancel{= \frac{3}{9-x-2}}$$

$$\cancel{= \frac{3}{9-(x+2)}}$$

$$\frac{3}{9-(x-2)}$$

$$= \frac{3}{5-x+2}$$

$$= \frac{3}{5-(x-2)}$$

$$= \frac{3}{5} \cdot \frac{1}{1-\frac{1}{5}(x-2)}$$

$$\frac{1}{1-r}$$

$$r = \frac{1}{5}(x-2)$$

$$\sum_{n=0}^{\infty} r^n$$

$$= \frac{3}{5} \sum_{n=0}^{\infty} \frac{1}{5^n} (x-2)^n$$

$$y'' + 2y' - 3y = 0$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots$$

$$y'' = 2a_2 + 3\cdot 2a_3 x + 4\cdot 3a_4 x^2 + 5\cdot 4a_5 x^3 + 6\cdot 5a_6 x^4 + \cdots$$

$$\begin{array}{llllll} y'' & 2a_2 + & \overset{6}{\cancel{3\cdot 2}}a_3 x + & \overset{12}{\cancel{4\cdot 3}}a_4 x^2 + & \overset{20}{\cancel{5\cdot 4}}a_5 x^3 + & \overset{30}{\cancel{6\cdot 5}}a_6 x^4 + \\ +2y' & = 2a_1 + & 4a_2 x + & 6a_3 x^2 + & 8a_4 x^3 + & 10a_5 x^4 \\ -3y & -3a_0 - & 3a_1 x - & 3a_2 x^2 - & 3a_3 x^3 - & 3a_4 x_4 + \cdots \end{array}$$

optional $\Longrightarrow$ $= (2a_2 + 2a_1 - 3a_0) + (6a_3 + 4a_2 - 3a_1)x + \cdots = 0$

$2a_2 + 2a_1 - 3a_0 = 0$ $\quad$ $6a_3 + 4a_2 - 3a_1 = 0$

a_0 and a_1 will "stay themselves."

$a_2 = \frac{3}{2}a_0 - a_1$

$$a_3 = -\frac{4}{6}a_2 + \frac{3}{6}a_1$$
$$= -\frac{2}{3}a_2 + \frac{1}{2}a_1$$
$$= -\frac{2}{3}\left(\frac{3}{2}a_0 - a_1\right) + \frac{1}{2}a_1$$
$$= -a_0 + \frac{2}{3}a_1 + \frac{1}{2}a_1 = -a_0 + \frac{7}{6}a_1$$

Lots more...

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$= a_0 + a_1 x + \left(\frac{3}{2}a_0 - a_1\right)x^2 + \left(-a_0 + \frac{7}{6}a_1\right)x^3 + \cdots$$

$$= a_0 + \frac{3}{2}a_0 x^2 - a_0 x^3 + \cdots$$

$$+ a_1 x - a_1 x^2 + \frac{7}{6}a_1 x^3 + \cdots$$

Turn in Tues:

Repeat Assn 15, #1 for

$$3+\frac{3}{5}(x-2)+\frac{3}{25}(x-2)^2+\frac{3}{125}(x-2)^3+\cdots$$